Auction Mechanisms for Demand-Side Intermediaries in Online Advertising Exchanges

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ABSTRACT

Motivated by the online advertising exchange marketplace where demand-side intermediaries conduct local upstream auctions and participate in the exchanges' real-time auctions, we study the revenue and efficiency effects of three different auction mechanisms for such intermediaries. Specifically, we consider the widely-used first-price sealed-bid auction and two variations of the Vickrey auction (termed *pre-* and *post-award*), in a single-exchange single-item setting. We show that, for a homogeneous population of intermediaries with captive buyers competing at the exchange, the three mechanisms yield different expected profits for the intermediaries and revenue for the exchange, but a complete ranking for all mechanisms cannot be attained. We also demonstrate that the optimal reserve price of the exchange increases with the number of buyers and/or intermediaries, and that the social welfare decreases, compared to classical auctions without intermediaries. Moreover, we show that pre-award Vickrey auctions are less efficient than the other mechanisms. Finally, we compare the two Vickrey variations in a duopoly setting with non-captive buyers, and show that all buyers always select the post-award mechanism.

Categories and Subject Descriptors

J.4 [**Social and Behavioral Sciences**]: Economics

Keywords

Ad Auction; Intermediary; Auction Design; Competition

1. INTRODUCTION

Online advertising constitutes one of the major sources of revenue for businesses on the web. In the U.S. alone, the total advertising money spent for 2012 was \$ 37 billions [10]. One of the most widespread ways of trading online advertisements (ads) are auctions, a paradigm that successfully started with sponsored search [3] and has been recently adopted in the display advertising market. This became true with the introduction of the advertising (ad) exchanges [7], technology platforms that bring together advertisers and publishers in a centralized market, using real-time auctions to trade display ads. Billions of such auctions are conducted daily, making the use of autonomous agents indispensable for the prosperous operation of this complex marketplace.

One of the most important players in these markets are supply- and demand-side intermediaries that provide the technical infrastructure, relevant tools, as well as a centralized point of access to the various ad exchanges, acting as brokers and executing orders on behalf of their customers. The focus of this work is on the demand-side intermediaries of the market. More specifically, these intermediaries usually have more than one interested advertiser for each available impression and have to submit a single bid at the exchange. To make such a decision, intermediaries typically implement local auctions among their interested advertisers, making a profit by arbitraging between advertisers' received payments and corresponding payments to the exchange. Hence, they act both as auctioneers and bidders, and have to carefully design their mechanisms to maximize their profit and attract customers, while competing with other such intermediaries.

Motivated by the ad exchange paradigm, in this paper, we study the revenue and efficiency implications of the intermediaries' choice of mechanism in a single-exchange single-item setting. More specifically, we consider a seller (called the *center*) that auctions off a single good to a population of buyers via a number of intermediaries using a Vickrey auction with a reserve price. In this setting, we consider three commonly-used mechanisms for the intermediaries, namely the first-price sealed-bid auction and two variations of the Vickrey auction that we term *pre-* and *post-award* Vickrey auctions, depending on the timing of the decision about intermediary payments. We show that pre-award Vickrey auctions are less efficient than the other mechanisms. Remarkably, we also show that the center's revenue and intermediaries' profits generated are not the same for the three intermediary mechanisms, and that the optimal reserve price of the center (even when there is lack of competition between intermediaries) depends on the number of buyers and/or intermediaries. To the best of our knowledge, this is the first time that this effect has been thoroughly studied in the context of online advertising. Mansour et al. [4] briefly discuss various intermediary mechanisms without offering further analysis. Feldman et al. [1] have studied the optimal auction design problem for intermediaries and the center, but their analysis is focused on the limiting case of one buyer per intermediary, where they consider reserve prices for the intermediaries. In contrast, we do not make these assumptions, but our focus is on comparing different mechanisms, illustrating the trade-offs in terms of revenue and efficiency.

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Our contributions are as follows. We first show that, even with a single intermediary, the optimal reserve price for the center increases with the number of buyers, in contrast to classical results in optimal auction design and the results of Feldman et al., and that the social welfare decreases compared to an auction with no intermediaries. We then study homogeneous populations of intermediaries with captive buyers that implement the aforementioned mechanisms. We demonstrate that the optimal reserve price of the center depends both on the number of intermediaries and buyers per intermediary for pre-award Vickrey auctions, but only on the latter number for the other mechanisms. We also show that pre-award Vickrey auctions yield lower social welfare than the other auctions. We then go on to demonstrate that the expected revenue/profits are generally different for the different mechanisms. More specifically, we prove that the center's expected revenue is always higher for post-award Vickrey auctions compared to the other two mechanisms. Furthermore, we show that first-price sealed-bid auctions yield higher expected profit for the intermediaries than post-award Vickrey auctions. However, there cannot be a complete revenue/profit ranking of the three mechanisms. Finally, we extend our study to the setting with competing intermediaries and non-captive buyers; in a duopoly scenario, we prove that, when buyers strategically select their intermediary, they all prefer the more efficient post-award Vickrey auction than its pre-award counterpart.

The rest of the paper is structured as follows. We discuss related work in Section 2 and define our model in Section 3; Section 4 presents our findings for a single intermediary. Our results for competing intermediaries with captive buyers are presented in Section 5, and the study of non-captive buyers is given in Section 6. Finally, Section 7 concludes.

2. RELATED WORK

The most relevant to our work is the paper by Feldman et al. [1] who consider pre-award Vickrey auctions with reserve prices, and who focus on the optimal auction design problem for both the intermediaries and the center, under the limiting assumption that each intermediary has only a single buyer in its market. The authors show that intermediaries use randomized reserve prices in equilibrium, whereas the center sets a reserve price that decreases with the number of intermediaries. However, it is not clear whether this effect stems from the very presence of intermediaries or from their equilibrium reserve prices for this setting. The authors also demonstrate that their result for the intermediaries extends to the case with multiple buyers per intermediary, but do not manage to explicitly characterize the equilibrium distribution. Another work that builds on a similar model is that by Stavrogiannis et al. [11], that deals with the buyers' intermediary selection problem, but ignores the resulting intermediaries' profits and the center's revenue.

Our work is also related to the literature on procurement auctions with subcontracting, where contractors bid on a project and then subcontract its parts with smaller firms, called the subcontractors. Contractors typically run local auctions before or after the central auction to subcontract parts of the project. Traditional works in this area deal with the agency relation of a contractor with its subcontractors, focusing on how to optimally divide the project (see e.g. [5]). The closest to our problem are the works by Wambach [13] and Nakabayashi [9] who study the problem of subcontracting with pre-award auctions. The former takes a mechanism design approach for an intermediary, but the author considers the competition from other intermediaries as exogenous and disregards the effect of the reserve price at the center. Nakabayashi, on the other hand, considers competition between contractors for first-price sealed-bid and pre-award Vickrey auctions. However, in contrast to our work, in his setting, contractors incur private costs for the good, which complicates the equilibrium analysis, and results in different properties of the two auction types.

Finally, our study of non-captive buyers is related to the literature on competing auctions [6], where sellers compete to attract customers and maximize their revenue. However, this stream deals with independent auctioneers that compete for buyers, whereas in our scenario the auctioneers additionally compete as bidders in a central auction.

3. MODEL

Consider a seller, called the *center*, who is auctioning an indivisible good to $K \in \mathbb{N}$ *ex ante* symmetric, utility-maximizing buyers via a number $n \in \mathbb{N}$ of intermediary auctioneers s_j, $j = 1, ..., n$. We assume that the center and the intermediaries have no value for the good and that the preferences of the buyers and auctioneers are described by von Neumann and Morgenstern utility functions. Buyers have independent private valuations, v_i, $i = 1, ..., K$, i.i.d. drawn from a commonly-known distribution F with a continuous, positive density f, and support $V = [0, 1]$. The center runs a Vickrey auction with a reserve price $\rho \in V$ and a fair tie-breaking rule, and each intermediary is allowed to submit a single bid[1]. Hence, the center's revenue equals the maximum of the second-highest submitted bid and ρ, if there is at least a bid above ρ, and is otherwise zero. Each intermediary, s_j, runs a *contingent* auction among its set of $k_j \geq 1$ buyers (where $\sum_{j=1}^{n} k_j = K$) to determine the price to be paid from the winning buyer, conditional on it winning at the central auction, as well as the bidding amount to be submitted to the center. The intermediary's profit is the difference between the payment it receives from its winning buyer and the price it pays at the center, whereas the surplus of a buyer is the difference between her valuation and the price paid at the intermediary. In more detail, the game proceeds as follows:

1. The center announces its reserve price, ρ, to the intermediaries, which announce it to the population of buyers.
2. Buyers learn their valuations for the good, (optionally) select their preferred intermediary, s_j, and submit a bid to that intermediary.
3. Intermediaries run auctions among their buyers and submit their (single) bids (if any) to the center.
4. The center runs its auction with the intermediaries' bids, transfers the good to the winning intermediary (if any) and receives payment from that intermediary.
5. The winning intermediary (if any) transfers the good to its winning buyer and receives payment from that buyer.

We consider three different intermediary mechanisms:

– *Pre-award Vickrey auction* (`PRE`). In this mechanism, the intermediary runs a local Vickrey auction and agrees to be paid the maximum of the second-highest local bid and

[1]This is the predominant mechanism used in ad exchanges.

the center's reserve price, ρ, if it wins at the center. Given that both the center's and intermediary's auctions are dominant-strategy incentive compatible (DSIC), buyers submit their true valuations, and the intermediary submits its second-highest local bid to the center, which is its payment from the buyer if it wins [11].

- *Post-award Vickrey auction* (`POST`). In this mechanism, the intermediary runs a local Vickrey auction, forwards the highest local bid to the center, and its payment is determined after the central auction as the maximum of ρ, the local second-highest bid and the intermediary's payment to the center. This mechanism is also DSIC for the buyers, and, as we show later, is more efficient than its pre-award counterpart.
- *First-price sealed-bid auction* (`FPSB`). In this mechanism, the intermediary uses a first-price sealed-bid auction for its local buyers and submits its highest local bid (if it is higher than ρ), i.e. its payment, to the center. As we will show, this mechanism is also more efficient than the pre-award Vickrey auction, and sometimes even yields higher expected profits; however, buyers in this mechanism follow Bayes-Nash equilibrium (BNE) bidding strategies.

In the following section, we start our analysis with a simple case with a single intermediary. As we show, even when there is lack of competition at the intermediary level, the optimal reserve price of the center depends on the number of buyers, in contrast to the setting without intermediaries.

4. SINGLE INTERMEDIARY

We start by showing that, even when only one intermediary is introduced, the center's reserve price increases with the number of buyers and the social welfare decreases, compared to a setting without intermediaries. Thus, these changes occur because of the very presence of the intermediaries, and not only due to their competition.

Feldman et al. [1] have shown that, for single-buyer intermediaries with reserve prices, the center's reserve price *decreases* with the number of intermediaries. As the authors notice, this is in contrast with the results of Myerson [8] for a classical setting with no intermediaries, who has shown that the optimal reserve price, ρ^*, satisfies the equation $\rho^* = \frac{1-F(\rho^*)}{f(\rho^*)}$, i.e. it is independent of the number of buyers. In contrast, we show that, when there is no competition between intermediaries, the opposite happens, i.e. the optimal reserve price increases with the number of buyers[2]. We consider a scenario where the center offers a take-it-or-leave-it price, ρ, to one intermediary representing K buyers. Given the lack of competition between intermediaries, all standard auctions where the good is given to the highest bidder yield the same expected center's revenue, intermediary's profit and buyers' surplus. Let us assume that the intermediary runs a `PRE` auction. The center's expected revenue equals ρ times the probability that there is at least one buyer that is willing to accept it:

$$revenue(\rho) = \rho[1 - F^K(\rho)] \tag{1}$$

which is maximized by setting an optimal ρ^* as:

$$\rho^* = \frac{1 - F^K(\rho^*)}{KF^{K-1}(\rho^*)f(\rho^*)} \tag{2}$$

[2]This is true even if the intermediary sets a reserve price.

The intermediary's ex ante expected profit is the expected difference of the second-highest bid and ρ^*:

$$profit(\rho) = \int_{\rho^*}^{1} (y - \rho^*) f_2^{(K)}(y)dy = 1 - \rho^* - \int_{\rho^*}^{1} F_2^{(K)}(y)dy \tag{3}$$

where $f_2^{(K)}, F_2^{(K)}$ are the p.d.f. and c.d.f., respectively, of the second-highest-order statistic among K samples i.i.d. drawn from f, F. To illustrate these observations, we consider an example with buyers whose valuations are drawn from a uniform distribution $U(0,1)$. Then, (2) yields:

$$\rho^* = \frac{1}{(K+1)^{\frac{1}{K}}} \tag{4}$$

which increases with the number of buyers[3]. Figure 1 shows the center's expected revenue and the social welfare with and without the intermediary, when the center sets the optimal ρ^*, as the number of buyers increases. We can see that the social welfare decreases compared to the classical Myerson setting. This is due to the double marginalization effect from the presence of the intermediary, i.e. the intermediary obtains some of the center's revenue, so, in response, the center increases its reserve price and that reduces the demand of the buyers [12]. Finally, it can be seen that the intermediary's expected profit decreases with the number of buyers, as ρ^* increases as well.

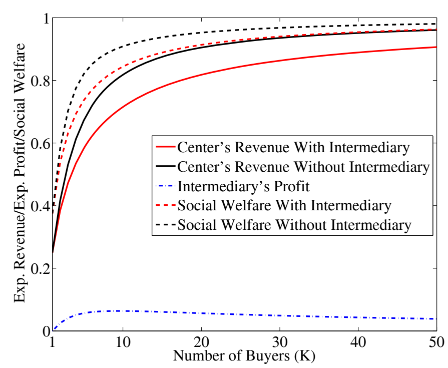

Figure 1: Expected center's revenue and social welfare with and without a single intermediary and the latter's expected profit as a function of the number of buyers whose valuations are i.i.d. drawn from $U(0,1)$.

5. MULTIPLE INTERMEDIARIES

We now consider a scenario with a homogeneous population of $n > 1$ intermediaries. In line with Feldman et al., we assume that buyers are uniformly allocated to the intermediaries, such that each intermediary has exactly k buyers in its market, i.e. $k_j = k$ for all $j = 1, ..., n$ and $K = nk$, and that buyers cannot move between intermediaries (i.e. they are captive). We study the aforementioned three mechanisms for the intermediaries where we show that the former yield different intermediaries' expected profits and center's revenue, although it is not possible to provide a complete

[3]It is easy to see that $\lim_{K\to\infty}(K+1)^{-\frac{1}{K}} = 1$.

ranking of the three. Finally, we show that the social welfare decreases further for PRE auctions due to the inefficiency resulting from misallocations. In what follows, we first characterize the expected profit and revenue for each intermediary and the center, respectively, under each mechanism.

5.1 Pre-Award Vickrey Auctions

We first consider a setting with PRE intermediary auctions. The center's expected revenue in this case can be written as:

$$\begin{aligned} revenue_{PRE}(\rho) &= \rho[G^n(\rho) + n(1-G(\rho))G^{n-1}(\rho) - F^{nk}(\rho)] + \\ &+ \int_\rho^1 y g_2^{(n)}(y)dy = 1 - \rho F^{nk}(\rho) - \int_\rho^1 G_2^{(n)}(y)dy = \\ &= 1 - \rho H^n(\rho) - \int_\rho^1 G_2^{(n)}(y)dy \end{aligned} \tag{5}$$

where $G = F_2^{(k)}$ is the c.d.f. of the second-highest-order statistic among k bids, $G_2^{(n)}, g_2^{(n)}$ are, respectively, the c.d.f. and the p.d.f. of the second-highest-order statistic among the n submitted bids of the intermediaries, and $H_1^{(n)} = H^n$ is the c.d.f. of the highest-order statistic among n bids i.i.d. drawn from $H = F^k$ (i.e. the c.d.f. of the highest-order statistic among k bids). More specifically, the center expects to be paid ρ with probability that no intermediary's second-highest bid is greater than ρ (first term) but when there is at least one bid higher than ρ (third term), or when there is only one intermediary whose second-highest bid is greater than ρ (second term). In any other case, the center receives the expected second-highest among the intermediaries' submitted bids (fourth term). This means that, by taking the first order condition on equation (5), the optimal center's reserve price will satisfy:

$$\rho^* = \frac{G_2^{(n)}(\rho^*) - H^n(\rho^*)}{h_1^{(n)}(\rho^*)} \tag{6}$$

where $h_1^{(n)}$ is the p.d.f. of the highest-order statistic among n bids i.i.d. drawn from $H = F^k$. Hence, we can see that the center's optimal ρ not only depends on the number of buyers per intermediary but also on the number of participating intermediaries. An intermediary's profit in this case equals:

$$\begin{aligned} profit_{PRE}(\rho) &= \int_\rho^1 f_2^{(k)}(y) \int_\rho^y (F_2^{(k)}(x))^{n-1} dx dy = \\ &= \int_\rho^1 G^{n-1}(y)[1-G(y)]dy \end{aligned} \tag{7}$$

which is the expectation over the distribution of the second-highest-order statistic of the probability of winning against $n-1$ bids. The c.d.f. of each such bid corresponds to that of the second-highest-order statistic over k samples i.i.d. drawn from F since intermediaries submit their local second-highest bids. Finally, a buyer with $\upsilon \in [\rho, 1]$ expects surplus:

$$\begin{aligned} \Pi_{PRE}(\upsilon,\rho) &= \int_\rho^\upsilon (\upsilon - y) f_1^{(k-1)}(y) G^{n-1}(y)dy + \\ &+ (\upsilon-\rho)F^{k-1}(\rho)\sum_{i=0}^{n-1}\{\frac{\binom{n-1}{i}}{i+1}F^{(n-1-i)k}(\rho)[kF^{k-1}(\rho)(1-F(\rho))]^i\} \end{aligned} \tag{8}$$

More specifically, a buyer expects positive surplus if his bid is the highest in the intermediary's auction and the second-highest bid is higher than the bids submitted at the center and the reserve price (first term). Finally, the buyer wins the good at the center's reserve price when all other intermediaries' bids are less than ρ or when i other intermediaries also submit their reserve prices, winning with a probability of $\frac{1}{i+1}$ (second term).

This mechanism always guarantees positive profit for the intermediary that wins at the center. However, besides the inefficiency due to the center's reserve price, this auction induces an additional (misallocation) inefficiency when more than one intermediary is present. To see this, consider a setting with two intermediaries, s_1, s_2, a population of four buyers so that $\upsilon_1 > \upsilon_2 > \upsilon_3 > \upsilon_4$ where buyers 1 and 4 are allocated to s_1 and buyers 2 and 3 to s_2. Given that s_1, s_2 submit υ_4, υ_3 respectively, s_2 wins and the good is allocated to buyer 2, although buyer 1's valuation is higher. In the next subsection, we present an alternative mechanism for the intermediaries, which keeps the incentive compatibility property and does not suffer from this type of inefficiency.

5.2 Post-Award Vickrey Auctions

Given that POST auctioneers submit their highest local bid, the highest overall bidder always wins. Hence, there are no misallocation inefficiencies. However, compared to the previous Vickrey auction, there is an apparent trade-off: intermediaries increase their probability of winning by submitting higher bids, but also decrease the number of times they make a positive profit (they make zero profit even if they win but their local second-highest bid is smaller than their payment at the center). In this case, an intermediary's expected payment to the center will be:

$$\begin{aligned} payment_{POST}(\rho) &= \int_\rho^1 f_1^{(k)}(y)[\rho H^{n-1}(\rho) + \int_\rho^y x h_1^{(n-1)}(x)dx]dy = \\ &= \rho H^{n-1}(\rho)[1-H(\rho)] + \int_\rho^1 x h_1^{(n-1)}(x)[1-H(x)]dx \end{aligned} \tag{9}$$

which is the expectation over the distribution of the intermediary's highest submitted bid of the payment for any submitted bid y. Then the center's expected revenue is:

$$revenue_{POST}(\rho) = n \cdot payment_{POST}(\rho) = 1 - \rho H^n(\rho) - \int_\rho^1 H_2^{(n)}(y)dy \tag{10}$$

where $H_2^{(n)}, h_2^{(n)}$ are, respectively, the c.d.f. and the p.d.f. of the second-highest-order statistic among the n intermediaries' bids. Hence, the optimal center's ρ will satisfy:

$$\rho^*_{POST} = \frac{1 - H(\rho^*_{POST})}{h(\rho^*_{POST})} \tag{11}$$

From this, we can see that the optimal reserve price for the latter only depends on the number of buyers per intermediary and is independent of the number of intermediaries. Then, each intermediary's expected profit is:

$$\begin{aligned} profit_{POST}(\rho) &= F^{(n-1)k}(\rho)\int_\rho^1 (y-\rho) f_2^{(k)}(y)dy + \\ &+ \int_\rho^1 \int_\rho^{y_2} (y_2 - x_1) f_1^{(n-1)k}(x_1) f_2^{(k)}(y_2) dx_1 dy_2 = \\ &= \int_\rho^1 F^{(n-1)k}(y)[1 - F_2^{(k)}(y)]dy = \int_\rho^1 H^{n-1}(y)[1-G(y)]dy \end{aligned} \tag{12}$$

More specifically, an intermediary expects to receive the difference between his local second-highest bid and ρ only when there are at least two buyers with bids above ρ and all other opponent bids are less than ρ (first term). The second term is the expected profit in the other case where the highest opponent of $(n-1)k$ bids is lower than the second highest among the winning intermediary's k bids. Finally, the expected surplus of a buyer whose valuation is $\upsilon \in [\rho, 1]$ is the same as with a Vickrey auction with nk buyers and a reserve price of ρ:

$$\Pi_{POST}(\upsilon) = (\upsilon - \rho)F^{nk-1}(\rho) + \int_{\rho}^{\upsilon} (\upsilon - y)f_1^{(nk-1)}(y)dy = \\ = \int_{\rho}^{\upsilon} F^{nk-1}(y)dy \tag{13}$$

5.3 First-Price Sealed-Bid Auctions

Finally, intermediaries often employ a FPSB auction, usually for reasons of transparency. This mechanism also avoids the misallocation inefficiency of the first mechanism, but the strategies of buyers are no longer DSIC. Moreover, given that the allocation is the same as with the POST auctions, the total revenue generated is the same, but, as will be shown, the profit share of the intermediaries will be different. When intermediaries implement FPSB auctions, then a buyer i with private valuation υ wins only if his bid, b_i, is the highest submitted bid among all buyers' bids, i.e. if only $b_i \geq max_{j \neq i} b_j$. Hence, if buyers use the symmetric, increasing bidding strategy $\beta(\cdot) : [\rho, 1] \to [\rho, 1]$, buyer i wins if $b_i \geq \beta(Y_1^{(nk-1)})$, where $Y_1^{(nk-1)}$ is the highest-order statistic among the other $nk-1$ valuations. Using standard equilibrium analysis (see [2]), it is easy to show that buyers' symmetric BNE strategy is the same as in a FPSB auction without intermediaries, a reserve price ρ and $nk = K$ buyers:

$$\beta(\upsilon) = \upsilon - \frac{\int_{\rho}^{\upsilon} F^{nk-1}(x)dx}{F^{nk-1}(\upsilon)}, \upsilon \geq \rho \tag{14}$$

Then, if $F_\beta(\cdot) = F(\beta^{-1}(\cdot))$ is the c.d.f. of the submitted bids in each intermediary, and $H_\beta = F_\beta^k$, the c.d.f. of the highest-order statistic of the k local bids, the ex ante expected payment of an intermediary to the center is:

$$payment_{FPSB}(\rho) = \int_{\rho}^{\beta(1)} f_{\beta_1}^{(k)}(u)[\rho H_\beta^{n-1}(\rho) + \int_{\rho}^{u} y h_{\beta_1}^{(n-1)}(y)dy]du \\ = \rho H_\beta^{n-1}(\rho)[1 - H_\beta(\rho)] + \int_{\rho}^{\beta(1)} y h_{\beta_1}^{(n-1)}(y)[1 - H_\beta(y)]dy \tag{15}$$

Hence the expected revenue for the center is:

$$revenue_{FPSB}(\rho) = n\rho H_\beta^{n-1}(\rho)[1 - H_\beta(\rho)] + \int_{\rho}^{\beta(1)} y h_{\beta_2}^{(n)}(y)dy = \\ = 1 - \rho H^n(\rho) - \int_{\rho}^{1} F^{nk-1}(x)dx - \int_{\rho}^{1} H_2^{(n)}(x)\beta'(x)dx \tag{16}$$

where, if $y = \beta(x) \Rightarrow dy = \beta'(x)dx$, and $H_\beta(\beta(x)) = F_\beta^k(\beta(x)) = F^k(x)$. The ex ante expected profit of an intermediary is:

$$profit_{FPSB}(\rho) = \int_{\rho}^{\beta(1)} f_{\beta_1}^{(k)}(y) \int_{\rho}^{y} H_\beta^{n-1}(u)dudy = \\ = \int_{\rho}^{\beta(1)} H_\beta^{n-1}(u)[1 - H_\beta(u)]du = \int_{\rho}^{1} H^{n-1}(y)[1 - H(y)]\beta'(y)dy \tag{17}$$

Finally, a buyer expects the same surplus as with a POST auction, given that the allocation in both mechanisms is the same, i.e. $\Pi_{FPSB}(\upsilon) = \Pi_{POST}(\upsilon)$ for all $\upsilon \in [0, 1]$, and also the optimal reserve price is the same, i.e. $\rho^*_{FPSB} = \rho^*_{POST}$. In what follows, we provide a comparison of the aforementioned intermediary mechanisms, combining our theoretical insights with numerical results.

5.4 Comparison

Having expressed the expected utilities for all scenarios, in this section, we compare, both theoretically and numerically, the resulting intermediaries' expected profits, center's expected revenue and social welfare under the three mechanisms for homogeneous populations of intermediaries. First, we numerically show that, for an example with buyers' valuations i.i.d. drawn from the uniform distribution $U(0, 1)$, the social welfare is smaller for PRE intermediary mechanisms than the other mechanisms, due to the former's misallocation inefficiency, that is more apparent for large n. For the intermediaries, we show that FPSB auctions always yield higher expected profit than POST auctions, but our numerical results depict that it is not possible to provide a complete ranking of the three mechanisms. More specifically, FPSB auctions yield higher expected profit for a small number of intermediaries, whereas PRE auctions are better for intermediaries in the opposite case. What's more, it is shown that, for the same reserve price, PRE (POST) intermediaries have no strong unilateral incentive to switch to a POST (PRE) auction. For the center, we show that its expected revenue is higher when the intermediaries implement POST auctions than when they employ the other two auctions. Finally, it is shown that the center's optimal reserve price for PRE auctions is always higher than the other two mechanisms. To show this, we start with the following lemma.

LEMMA 1. *When all intermediaries implement pre-award Vickrey auctions, the center's optimal reserve price is always higher than the optimal reserve price when all intermediaries implement post-award Vickrey or first-price sealed-bid auctions.*

PROOF. Let ρ^*_{PRE}, ρ^*_{POST} be the optimal reserve prices for equations (6) and (11), respectively. Taking the first order derivative with respect to ρ in equation (5) yields:

$$\frac{drevenue_{PRE}(\rho)}{d\rho} = -H^n(\rho) - n\rho H^{n-1}(\rho)h(\rho) + G_2^{(n)}(\rho) \tag{18}$$

Now, applying the condition of (11) in the equation above, we get:

$$\frac{drevenue_{PRE}(\rho)}{d\rho}|_{\rho=\rho^*_{POST}} = -H^n(\rho^*_{POST}) - \\ - n\frac{1 - H(\rho^*_{POST})}{h(\rho^*_{POST})}H^{n-1}(\rho^*_{POST})h(\rho^*_{POST}) + G_2^{(n)}(\rho^*_{POST}) = \\ = (n-1)H^n(\rho^*_{POST}) - nH^{n-1}(\rho^*_{POST}) + G_2^{(n)}(\rho^*_{POST}) = \\ = G_2^{(n)}(\rho^*_{POST}) - H_2^{(n)}(\rho^*_{POST}) = \\ = nG^{n-1}(\rho^*_{POST}) - (n-1)G^n(\rho^*_{POST}) - \\ - [nH^{n-1}(\rho^*_{POST}) - (n-1)H^n(\rho^*_{POST})] > 0 \tag{19}$$

since $G(y) - H(y) = kF^{k-1}(y) - (k-1)F^k(y) - F^k(y) = (k-1)F^{k-1}(y)[1 - F(y)] \geq 0 \Rightarrow G(y) \geq H(y)$ for any $y \in [0, 1]$, and the function $nx^{n-1} - (n-1)x^n$ is an increasing function of x. Hence, since for the existence of an optimal reserve price, the function $revenue_{PRE}(\rho)$ should be concave, the above equation means that $\rho^*_{POST} < \rho^*_{PRE}$ and since $\rho^*_{FPSB} = \rho^*_{POST} \Rightarrow \rho^*_{FPSB} < \rho^*_{PRE}$. $\square$

This last result allows us to compare the center's expected revenue for the two Vickrey auctions.

Theorem 1. *If all intermediaries implement post-award Vickrey auctions, the center's optimal expected revenue is always not less than the expected revenue when all intermediaries implement pre-award Vickrey auctions.*

Proof. Taking the difference of (5) and (10), for the same reserve price, ρ, we obtain that:

$$revenue_{POST}(\rho) - revenue_{PRE}(\rho) = \int_{\rho}^{1} [G_2^{(n)}(y) - H_2^{(n)}(y)]dy \geq 0 \tag{20}$$

where $G_2^{(n)} = nG^{n-1} - (n-1)G^n$, $H_2^{(n)} = nH^{n-1} - (n-1)H^n$. This is since $G(y) \geq H(y)$ and the function $nx^{n-1} - (n-1)x^n$ is a strictly increasing function of x. We should also notice that the inequality is strict for any reasonable reserve, $\rho \in [0, 1)$.

If ρ^*_{PRE}, ρ^*_{POST} are the optimal reserve prices for equations (6) and (11) respectively, from the previous result, we have that $revenue_{POST}(\rho^*_{POST}) \geq revenue_{POST}(\rho^*_{PRE}) \geq revenue_{PRE}(\rho^*_{PRE})$. This, combined with the result of Lemma 1, concludes the proof. □

We now start our analysis for the intermediaries' expected profits, starting with the following proposition.

Proposition 1. *For any reserve price, ρ, of the center, the expected profits of pre-award Vickrey intermediary auctions are always not less than the corresponding profits of post-award Vickrey intermediary auctions, when all intermediaries implement the same mechanism.*

Proof. This happens if $G^{n-1}(y) \geq H^{n-1}(y)$ from equations (7) and (12), which is true (see proof of Lemma 1). □

Then, keeping the reserve price of the center fixed, we show that no intermediary has a strict incentive to deviate from homogeneous `PRE` (`POST`) to `POST` (`PRE`) auctions.

Proposition 2. *For any reserve price, ρ, of the center, an intermediary has no strict incentive to switch from a pre-award (post-award) to a post-award (pre-award) Vickrey auction when all other intermediaries implement pre-award (post-award) Vickrey auctions.*

Proof. First, assume that $n-1$ intermediaries implement `PRE` auctions, and one intermediary switches to a `POST` auction. Then the deviator's expected profit will be:

$$\begin{aligned} profit_{dev.POST}(\rho) &= G(\rho)^{n-1} \int_{\rho}^{1} (y - \rho) f_2^{(k)}(y)dy + \\ &+ \int_{\rho}^{1} \int_{\rho}^{y_2} (y_2 - x_1) g_1^{(n-1)}(x_1) g_2(y_2) dx_1 dy_2 = \\ &= \int_{\rho}^{1} G^{n-1}(x)(1 - G(x))dx \end{aligned} \tag{21}$$

which is the same as when implementing pre-award payments. In contrast, if $n-1$ intermediaries use `POST` auctions, then a deviating intermediary's expected profit when implementing a `PRE` auction will be:

$$\begin{aligned} profit_{dev.PRE}(\rho) &= \int_{\rho}^{1} g(y) \int_{\rho}^{y} H^{n-1}(x)dxdy = \\ &= \int_{\rho}^{1} H^{n-1}(x)(1 - G(x))dx \end{aligned} \tag{22}$$

which is again the same as with `POST`. □

This last result means that, for the same reserve price of the center, both auctions constitute non-strict Nash equilibrium mechanisms. Finally, we look at the case when intermediaries implement `FPSB` auctions and compare their expected profit with the more efficient `POST` auctions.

Theorem 2. *The expected profits of intermediaries implementing first-price sealed-bid auctions are always higher than the corresponding profits for post-award Vickrey auctions.*

Proof Sketch. Let us assume that the buyers' private valuations, υ_i, are known, and that $\upsilon_i > \upsilon_j$ when $i < j$, and that $\rho = 0$. If all intermediaries implement `POST` auctions, then an intermediary s_ℓ has buyer with υ_1 in its market with probability $\frac{1}{n}$, both buyers $1, 2$ with probability $\frac{1}{n^2}$ and pays υ_3 with probability $\frac{1}{n^2}(1 - \frac{1}{n})$, pays υ_4 with probability $\frac{1}{n^3}(1 - \frac{1}{n})$ and so on, paying υ_{k+1}, with probability $\frac{1}{n^k}(1 - \frac{1}{n})$. Hence his expected profit w.r.t. the buyers' allocation can be written as:

$$profit_{POST} = \sum_{j=3}^{k+1} (\upsilon_2 - \upsilon_j) \frac{1}{n^{j-1}} (1 - \frac{1}{n}) \tag{23}$$

If all intermediaries implement `FPSB` auctions, then the expected profit over the buyers' allocation of an intermediary can be similarly expressed as

$$profit_{FPSB} = \sum_{j=2}^{k+1} (\beta(\upsilon_1) - \beta(\upsilon_j)) \frac{1}{n^{j-1}} (1 - \frac{1}{n}) \tag{24}$$

Taking their difference yields:

$$\begin{aligned} profit_{FPSB} - profit_{POST} &= (\beta(\upsilon_1) - \beta(\upsilon_2)) \frac{1}{n} (1 - \frac{1}{n}) + \\ &+ \sum_{j=3}^{k+1} (\beta(\upsilon_1) - \upsilon_2 + \upsilon_j - \beta(\upsilon_j)) \frac{1}{n^{j-1}} (1 - \frac{1}{n}) \end{aligned} \tag{25}$$

The expectation of this difference w.r.t. the joint distribution of the buyer's valuations is always positive since $\beta(\upsilon_1) > \beta(\upsilon_2)$, $\upsilon_j \geq \beta(\upsilon_j)$ and $\beta(\upsilon_1) = \upsilon_2$ in expectation (it is easy to show that $\int_0^1 \int_0^{y_1} (\beta(y_1) - y_2) f_{1,2}^{(nk)}(y_1, y_2,) dy_1 dy_2 = 0$). The results are similar when the optimal reserve price, ρ^*, is used by the center, given that $\rho^*_{FPSB} = \rho^*_{POST}$ and buyers in `FPSB` auctions will bid more aggressively. □

Given this last result, and that both `FPSB` and `POST` auctions yield the same allocation of the good, generating the same total utility for the center and the intermediaries, we have the following corollary.

Corollary 1. *The expected revenue of the center is always higher when intermediaries implement post-award Vickrey auctions than when the latter implement first-price sealed-bid auctions.*

Our theoretical analysis shows that the center prefers `POST` auctions for the intermediaries, however no ranking between `FPSB` and `PRE` auctions has been provided. Similarly, it is not clear which mechanism is better for the intermediaries. When each intermediary has a single buyer, we can see that the `FPSB` auction is the only mechanism that yields positive profit, but for $k > 1$ buyers per intermediary, it is not possible to obtain a general ranking of the expected profits for the intermediaries. To show this, we numerically evaluate the proposed auctions for a population of buyers whose

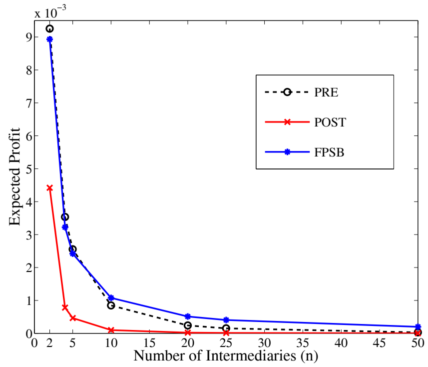

Figure 2: Intermediaries' expected profits for the three different mechanisms with increasing number of opponents for a fixed number of $K = 100$ buyers whose valuations are i.i.d. drawn from $U(0, 1)$.

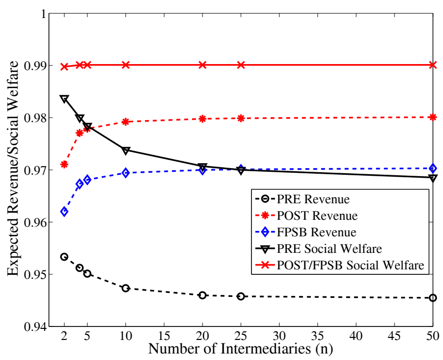

Figure 3: Expected center's revenue and social welfare for the three different mechanisms with an increasing population of intermediaries and a fixed number of $K = 100$ buyers whose valuations are i.i.d. drawn from $U(0, 1)$.

private valuations are i.i.d. drawn from a uniform distribution $U(0, 1)$. More specifically, we consider a population of $K = 100$ buyers and vary the number of intermediaries ($n = 2, 5, 10, 20, 25, 50$), keeping the total number of buyers fixed. Figure 2 shows the resulting expected profits for the intermediaries. As shown, in this example, PRE auctions yield higher expected profits for a small number of intermediaries, whereas FPSB auctions yield higher expected profits in the remaining cases. POST auctions seem to perform worse in terms of profit than their PRE counterpart, but one can verify that for $n = 2$, $k = 2$, the opposite happens, so a general ranking of the two is not possible.

Finally, Figure 3 illustrates the center's expected revenue and corresponding social welfare for the mechanisms for the same example. As can be seen, the social welfare for the more efficient FPSB and POST auctions slightly increases with the number of intermediaries (getting very close to the social welfare of 0.9901 for the setting without intermediaries), whereas the opposite effect happens for the PRE mechanism. The latter is due to the fact that, as the number of buyers per intermediary decreases, the misallocation inefficiency increases, thus further decreasing the corresponding social welfare. Regarding the revenue of the center, it slightly increases with the number of intermediaries for the more efficient FPSB and POST mechanisms, whereas the opposite effect happens for the setting of PRE mechanisms.

Our analysis suggests that FPSB performs well both in terms of profit and efficiency; buyers, however, need to employ BNE strategies. Moreover, FPSB auctions are known for suffering from stability issues in repeated settings, such as the ones we observe in ad exchanges. It then makes sense to focus on DSIC mechanisms. From the experiments with uniform distribution, we see that, when buyers are captive, POST auctions generally yield lower expected profit than their counterpart, and so are less likely to be adopted in this scenario. Interestingly, next we show that, when buyers strategically select their intermediary, the opposite holds.

6. BUYERS' STRATEGIC SELECTION OF INTERMEDIARIES

Until now we have assumed that buyers are captive. However, buyers often strategically select the intermediary that yields the highest expected surplus. In this section, we study the effect of the resulting intermediary selection for the buyers on the mechanisms of the intermediaries in a duopoly setting with Vickrey auctions. We analyze the more interesting case where buyers *single-home*, i.e. select only one intermediary. When buyers multi-home, then it is clear that the post-award mechanism always wins as its submitted bid will always be higher than that of the PRE auction. In the next theorem, we show that the same is true for single-homing buyers.

THEOREM 3. *There exists a unique symmetric Bayesian pure-strategy Nash equilibrium in the duopoly buyer intermediary selection problem where all buyers select the intermediary implementing a post-award Vickrey auction, when the other intermediary implements a pre-award Vickrey auction.*

PROOF. Let $\theta : V \mapsto [0, 1]$ denote the function that maps a buyer's valuation to the probability of selecting the PRE intermediary. Then, if all opponent buyers select the latter intermediary (intermediary 1) with probability $\theta \in [0, 1]$, the expected surplus of a buyer whose valuation is $\upsilon \geq \rho$ from this intermediary can be expressed as:

$$\Pi_1(\upsilon) = (\upsilon - \rho)F^{K-1}(\rho) + \int_{\rho}^{\upsilon} (\upsilon - y)\theta(y)f_1^{(K-1)}(y)dy \qquad (26)$$

More specifically, the buyer expects positive surplus when all other buyers' valuations are below ρ (first term) or when the highest opponent bid is less than the buyer's bid and is submitted in the same intermediary auction (second term). On the other hand, the expected surplus from the POST intermediary (intermediary 2) is:

$$\Pi_2(\upsilon) = (\upsilon - \rho)F^{K-1}(\rho) + (\upsilon - \rho)\int_0^{\rho}\int_{\rho}^{1} \theta(y_1)f_{1,2}^{(K-1)}(y_1, y_2)dy_1dy_2$$
$$+ \int_{\rho}^{\upsilon} (\upsilon - y)(1 - \theta(y))f_1^{(K-1)}(y)dy +$$
$$+ \int_{\rho}^{\upsilon}\int_{y_2}^{1} (\upsilon - y_2)\theta(y_1)f_{1,2}^{(K-1)}(y_1, y_2)dy_1dy_2 \qquad (27)$$

More specifically, the buyer expects positive surplus when all other buyers' valuations are below ρ (first term) or when there is only one buyer with bid above ρ submitted in the other intermediary auction (second term), paying the center's reserve price, ρ. Moreover, the buyer pays the highest opponent bid when it is below υ and submitted in the same auction (third term) as well as the second-highest opponent bid, wherever this is submitted, as long as it is above ρ, below υ, and, at the same time, the highest opponent bid is submitted in the opponent intermediary.

Taking the difference of the expected surplus from both intermediaries yields:

$$\Pi_2(\upsilon) - \Pi_1(\upsilon) = (K-1)(\upsilon-\rho)F^{K-2}(\rho)\int_\rho^1 \theta(y)f(y)dy + \\ + \int_\rho^\upsilon (\upsilon - y)(1-2\theta(y))f_1^{(K-1)}(y)dy + \\ + \int_\rho^\upsilon \int_{y_2}^1 (\upsilon - y_2)\theta(y_1)f_{1,2}^{(K-1)}(y_1,y_2)dy_1dy_2 \quad (28)$$

The partial derivative of this difference w.r.t. υ is:

$$\frac{\partial \Pi_2(\upsilon) - \Pi_1(\upsilon)}{\partial \upsilon} = (K-1)F^{K-2}(\rho)\int_\rho^1 \theta(y)f(y)dy + \\ + \int_\rho^\upsilon (1-2\theta(y))f_1^{(K-1)}(y)dy + \int_\rho^\upsilon \int_{y_2}^1 \theta(y_1)f_{1,2}^{(K-1)}(y_1,y_2)dy_1dy_2 \quad (29)$$

However, we can write:

$$\int_\rho^\upsilon \int_{y_2}^1 \theta(y_1)f_{1,2}^{(K-1)}(y_1,y_2)dy_1dy_2 = \\ = \int_\rho^\upsilon \int_{y_2}^\upsilon \theta(y_1)f_{1,2}^{(K-1)}(y_1,y_2)dy_1dy_2 + \\ + \int_\rho^\upsilon \int_\upsilon^1 \theta(y_1)f_{1,2}^{(K-1)}(y_1,y_2)dy_1dy_2 = \\ = (k-1)[F^{k-2}(\upsilon)\int_\upsilon^1 \theta(y)f(y)dy - F^{k-2}(\rho)\int_\rho^1 \theta(y)f(y)dy] + \\ + \int_\rho^\upsilon \theta(y)f_1^{(k-1)}(y)dy \quad (30)$$

Hence, (29) can be written as:

$$\frac{\partial \Pi_2(\upsilon) - \Pi_1(\upsilon)}{\partial \upsilon} = (k-1)F^{k-2}(\upsilon)\int_\upsilon^1 \theta(y)f(y)dy + \\ + \int_\rho^\upsilon (1-\theta(y))f_1^{(K-1)}(y)dy \geq 0 \quad (31)$$

Hence, given that $\Pi_2(\rho) = \Pi_1(\rho)$ and $\Pi_2(\cdot)$ grows faster than $\Pi_1(\cdot)$ for every $\upsilon > \rho$, it should always be $\Pi_2(\upsilon) > \Pi_1(\upsilon)$, so the only equilibrium strategy is $\theta(y) = 0$ □

7. CONCLUSIONS AND FUTURE WORK

In this paper, we studied the effects of three different widely-used mechanisms for demand-side intermediaries in online advertising exchanges. As we have shown, the introduction of the intermediaries radically changes the optimal reserve price of the center which was shown to increase with the number of buyers and/or the number of intermediaries, in contrast to previous results on classical auctions. Moreover, we have illustrated that the introduction of the intermediaries decreases the social welfare of the system compared to scenarios without intermediaries, an effect that is more evident for intermediaries with pre-award Vickrey auctions. Then we considered the revenue/profit effects of the mechanisms. More specifically, we have shown that the center's expected revenue is higher when intermediaries adopt post-award Vickrey auctions, that first-price sealed-bid auctions yield higher expected profits than post-award intermediary mechanisms, and that both are more efficient than the pre-award Vickrey auctions. Although it is not possible to obtain a complete revenue/profit ranking of the three mechanisms, first-price sealed-bid auctions seem to provide a good trade-off between revenue and efficiency, but their strategies are in BNE which are known for their stability issues in repeated settings, as is the case in ad exchanges. Finally, for a setting with non-captive buyers, we proved that, when one intermediary implements a pre-award Vickrey auction against a post-award intermediary, buyers always select the more efficient, latter mechanism. This result is despite the fact that, for captive buyers, the pre-award mechanism was empirically shown to outperform the post-award one.

This paper constitutes a first attempt to study the aforementioned effects of intermediaries on the advertising ecosystem. We observed that no single mechanism always outperforms others in homogeneous populations of intermediaries. Thus, as a next step, we aim to study the likelihood of adopting each of the mechanisms, using techniques from evolutionary game theory. Also, letting the intermediaries set reserve prices is a theoretically interesting direction. However, previous results show that the problem is technically challenging [1, 11]. Finally, another interesting extension is comparing first-price sealed-bid auctions to the other mechanisms in the case of non-captive buyers, which involves computing the buyers' BNE bidding strategies for this setting.

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