Number Systems
Exercises

Robin Hirsch

Exercises

1. $1010 + 1101$Answer: 10111
2. $1011 + 111$Answer: 10010
3. $1111 - 1010$Answer: 101
4. $1110 - 101$ Answer: 1001
5. 1101×101 Answer: 1000001
6. $1001 \div 101$ Answer: $1r101$ or $1.11001100\ldots$ which we write $1.\dot{1}\ 10\ \dot{0}$
7. What is the biggest binary number you can write with 5 bits? Answer: 31
8. What is the biggest binary number you can write with n bits? Answer: $2^n - 1$
9. Roughly, how many bits do you need to write the number n in binary? Answer: roughly $\log_2 n$, exactly $\lceil \log_2(n+1) \rceil$
10. Write $\frac{3}{4}$ in binary, using a "binary point" 0.??. Answer: 0.11
11. Write $\frac{2}{3}$ in binary. Answer: $0.101010\ldots = 0.\ \dot{1}\dot{0}$
12. Which fractions recur infinitely in binary and which terminate? Answer: if demoninator of fraction in lowest form is power of 2 then it terminates, else not.

Exercises

13. Convert the binary number 1101101111110101 to hex. Answer: $DBF5$
14. Convert the hex number $ABC7$ to binary. Answer: 101010110111
15. In hex, $2BFC + 54A7$. Answer: $80A3$
16. In hex, $AC74 - B3F$. Answer: $A135$
17. If a number has k digits in hex, how many digits (bits) does it have in binary? Answer: $\frac{k}{4}$

18. If a number has k digits in decimal, roughly how many digits does it have in binary? Answer: roughly $\frac{k}{\log_2 10} \cong \frac{k}{3}$

19. If a number has k digits base a, roughly how many digits does it have in base b? Answer: roughly $\frac{k}{\log_b a} = k \times log_a b$

Exercises Write the following decimal numbers in eight bit two's complement, do the addition/subtraction, convert your answer back to decimal.

20. $^-3 +^+ 11$ Answer: $-3 = 11111101$. So $^-3 +^+ 11 = 11111101 + 00001011 = 00001000$

21. $^+125 +^- 40$ Answer: $125 = 01111101$, $^-40 = 11011000$ so $125 +^- 40 = 01010101 = 85$

22. $^+5 +^- 7$Answer: $00000101 + 11111001 = 11111110 =^- 2$

23. $^-23 -^- 18$. Answer: $11101001 + 11101110 = 11010111 =^- 41$

For n bit two's complement what is the range of numbers you can represent? Answer: Biggest positive numer is $0\overbrace{11\ldots1}^{n-1} = 2^{n-1} - 1$. Negative numbers go down from $-1 = \overbrace{1\ldots11}^{n}$ down to $1\overbrace{00\ldots0}^{n-2}1 =^- 2^{n-1} - 1$. But negative numbers can go one lower still: least negative number is $1\overbrace{00\ldots0}^{n-1} =^- 2^{n-1}$. So range of numbers is $[^-2^{n-1},^+ 2^{n-1} - 1]$.

Exercises Write the following fractions in binary. Use one sign bit and four bits after the binary point.

24. $\frac{5}{8}$ Answer: 0.1010

25. $-\frac{5}{8}$ Answer: negate previous answer, i.e. flip all bits and add one to lsb, so 1.0110

Convert the following binary fractions to ordinary fractions.

26. 0.1000 Answer: $\frac{1}{2}$

27. 1.0001 Answer: $^-\frac{15}{16}$

28. 0.1111 Answer: $\frac{15}{16}$

29. 1.1111 Answer: $^-\frac{1}{16}$

What fractions can be represented as binary fractions with one sign bit and four bits after the binary point? Answer: $[^-1,^+ \frac{15}{16}]$

Exercises Using 5 bits for the mantissa and 5 bits for the exponent, write the following numbers in twos complement binary.

30. $\frac{5}{16}$ Answer: 0.0101 0000, mantissa represents $\frac{5}{16}$ exponent represents $2^0 = 1$

31. $101\frac{1}{4}$ Answer: Bad example. This number is $\frac{21}{4}$, so mantissa should be $\frac{21}{32} = 0.10101$, but four available bits are not enough to represent this. Can approximate by losing last bit, 0.1010 0011, i.e. $\frac{5}{8} \times 2^3 = 5$, so the quarter gets lost.

32. $\frac{1}{1024}$ Answer: 0.1000 10111

33. $-\frac{3}{512}$ Answer: 0.0011 11001