Resources, Concurrency and Local Reasoning

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Part I

An Illustrative Example

$$
\begin{array}{lcl}
\texttt{semaphore } \mathit{free} := 1;\ \mathit{busy} := 0 & & \\
\vdots & & \vdots \\
P(\mathit{free}); & & P(\mathit{busy}); \\
[10] := m; & \| & n := [10]; \\
V(\mathit{busy}); & & V(\mathit{free}); \\
\vdots & & \vdots
\end{array}
$$

Recall: $V(s)$ bumps s up by 1; $P(s)$ decrements by 1 if > 0, waiting otherwise

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\begin{array}{l}
\texttt{semaphore } \mathit{free} := 1;\ \mathit{busy} := 0 \\
\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
\{\texttt{emp}\} \\
P(\mathit{free}); \qquad\qquad\qquad\qquad\qquad\qquad\quad P(\mathit{busy}); \\
\\
[10] := m; \qquad\qquad \| \qquad\qquad\quad n := [10]; \\
\\
V(\mathit{busy}); \qquad\qquad\qquad\qquad\qquad\quad V(\mathit{free}); \\
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\{\texttt{emp}\} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \{\texttt{emp}\} \\
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[10] := m; \qquad\qquad \| \qquad\qquad n := [10]; \\
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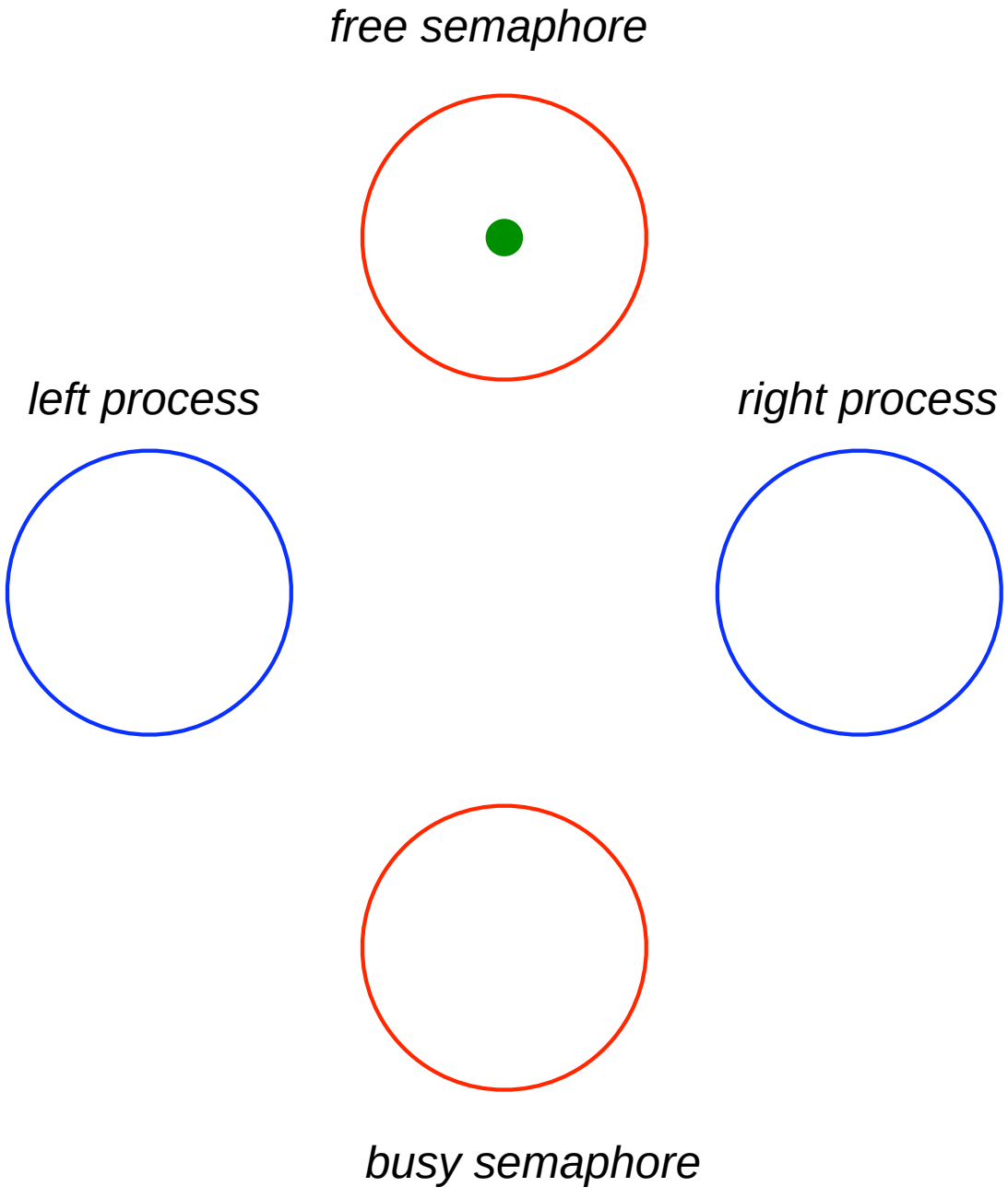
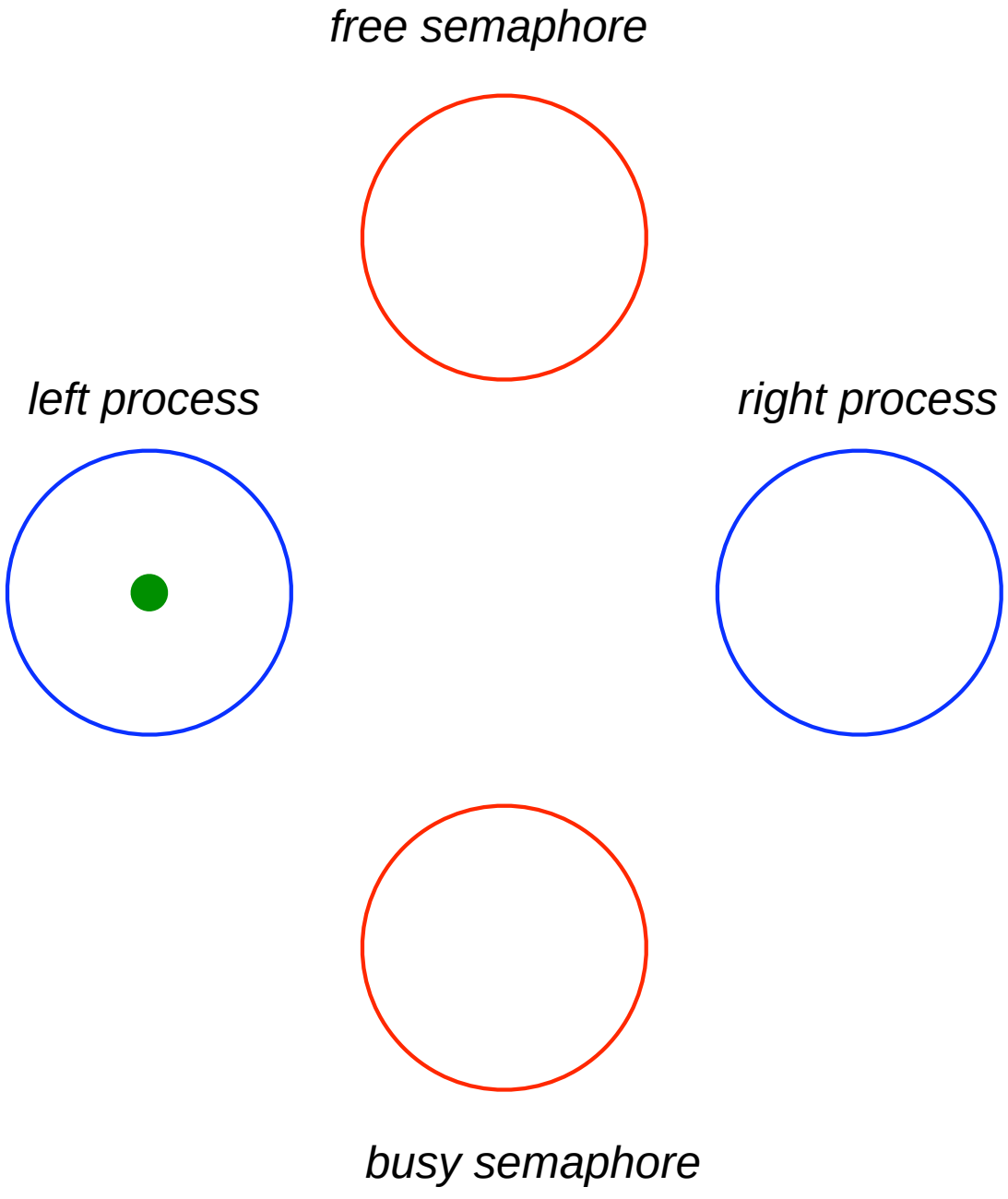
\vdots
\end{array}
$$

$$RI_{free} \stackrel{\text{def}}{=} (\mathit{free} = 0 \wedge \mathtt{emp}) \vee (\mathit{free} = 1 \wedge 10 \mapsto -)$$

$$RI_{busy} \stackrel{\text{def}}{=} (\mathit{busy} = 0 \wedge \mathtt{emp}) \vee (\mathit{busy} = 1 \wedge 10 \mapsto -)$$


free semaphore
left process
right process
busy semaphore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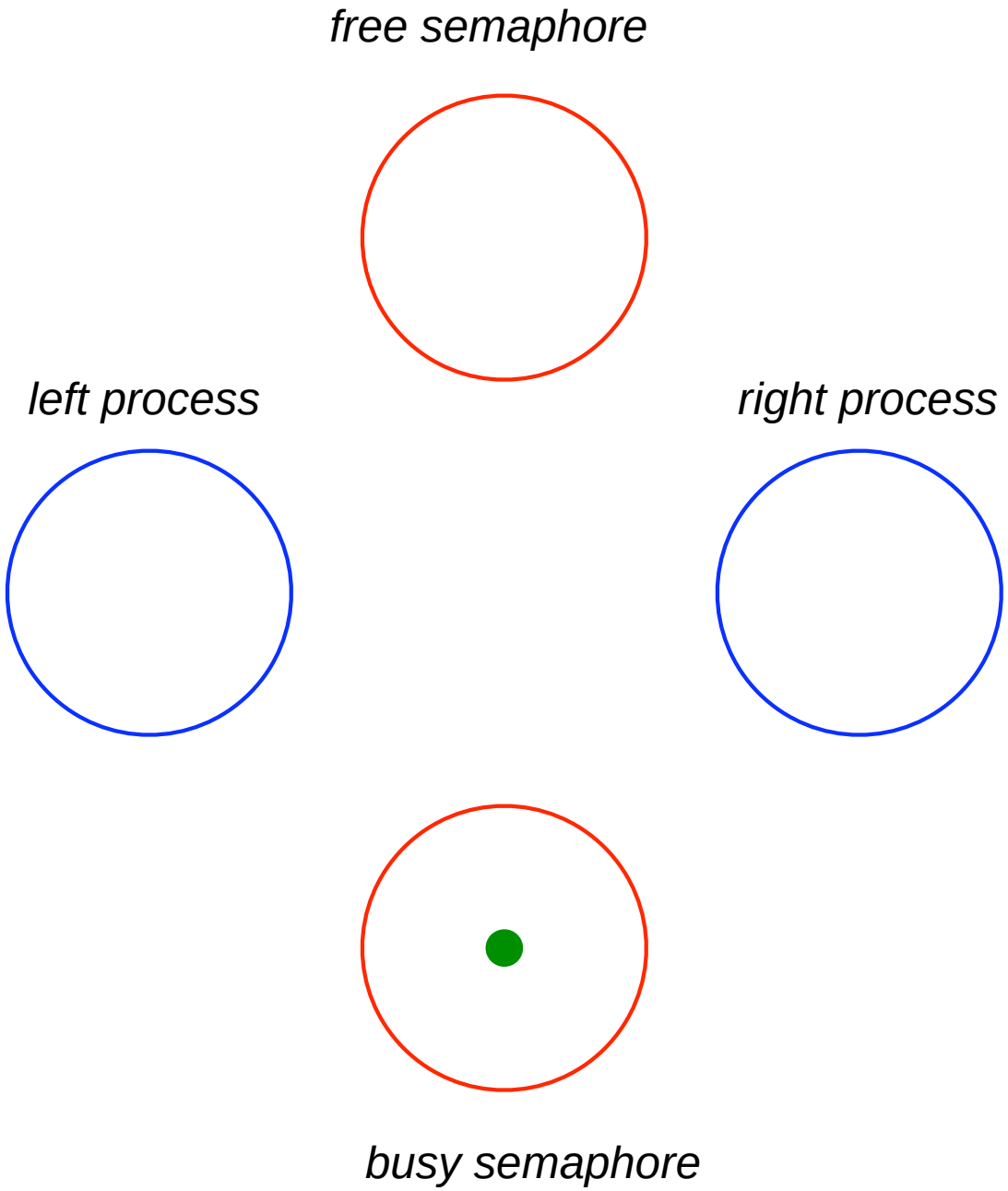
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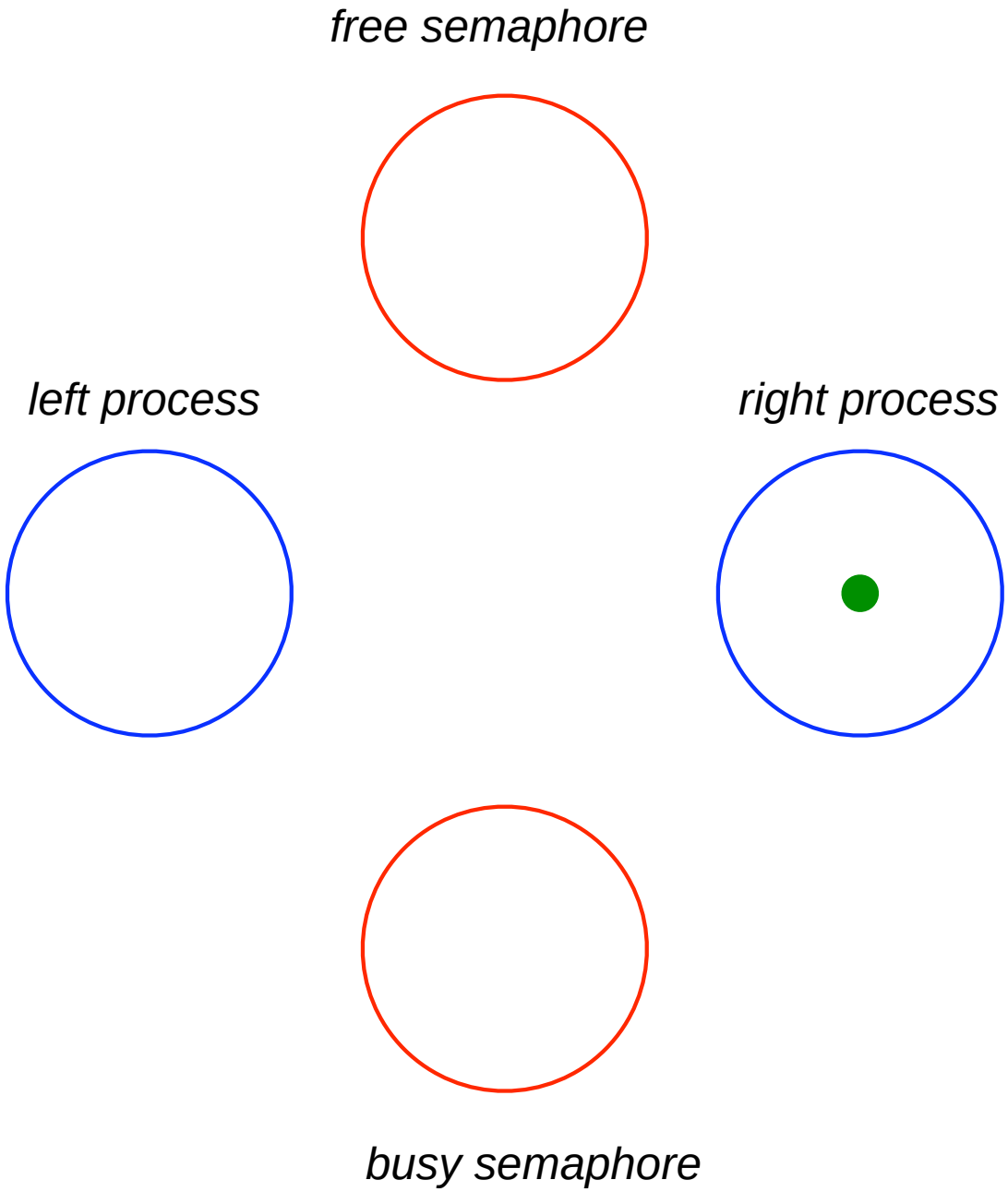
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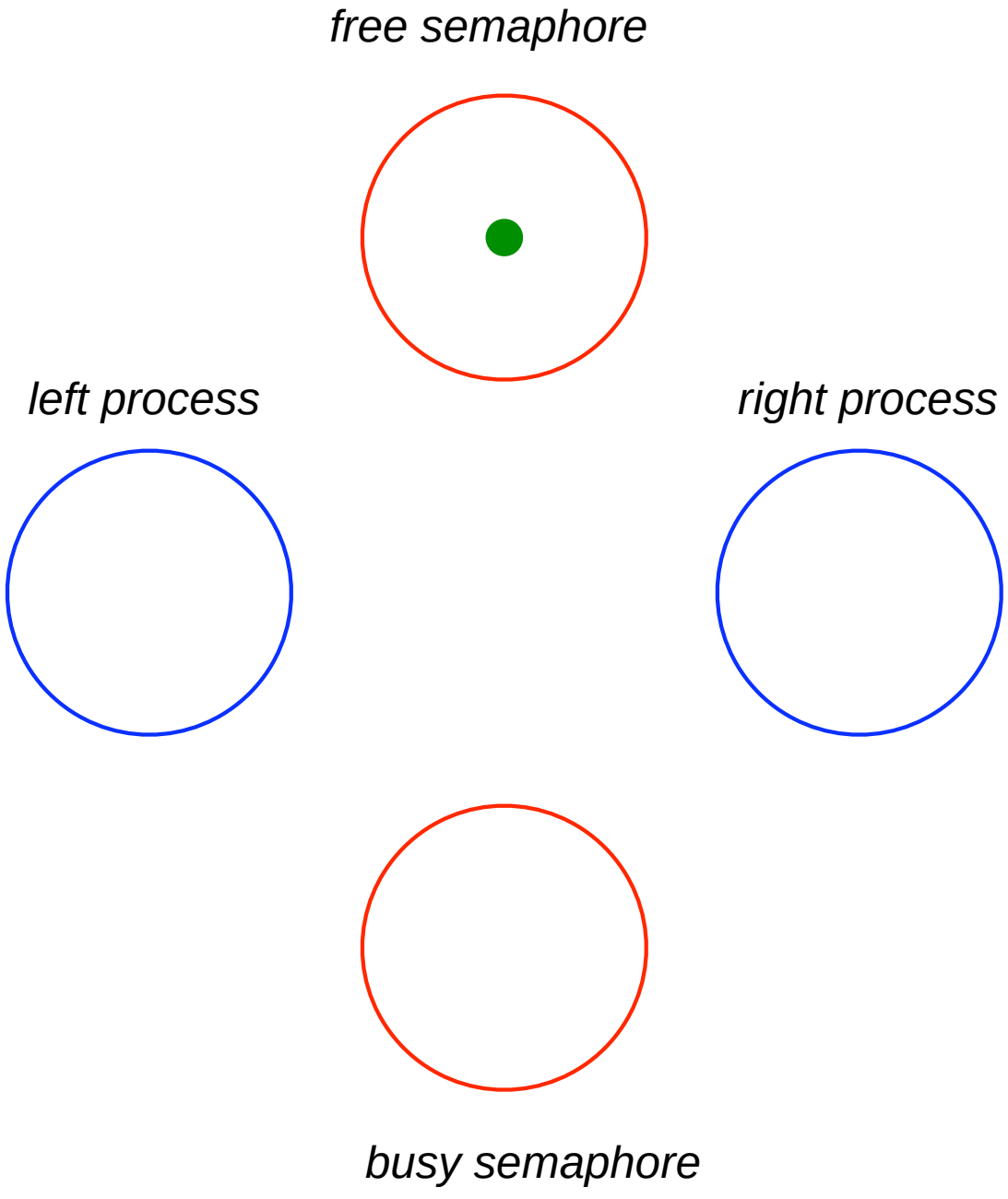
free semaphore
left process
right process
busy semaphore

$\{10 \mapsto -\}$

semaphore $free := 1,\ busy := 0;$

$\{RI_{free} * RI_{busy} * \mathtt{emp} * \mathtt{emp}\}$

$\{\mathtt{emp}\}$		$\{\mathtt{emp}\}$
`while true do`		`while true do`
produce m		$P(busy);$
$P(free);$		$n := [10];$
$[10] := m;$	$\|$	$V(free);$
$V(busy);$		*consume n*
$\{\mathtt{false}\}$		$\{\mathtt{false}\}$

$\{\mathtt{false}\}$

$$RI_{free} \stackrel{\text{def}}{=} (free = 0 \wedge \mathtt{emp}) \vee (free = 1 \wedge 10 \mapsto -)$$

$$RI_{busy} \stackrel{\text{def}}{=} (busy = 0 \wedge \mathtt{emp}) \vee (busy = 1 \wedge 10 \mapsto -)$$

$\{10 \mapsto -\}$

semaphore $free := 1,\ busy := 0;$

$\{RI_{free} * RI_{busy} * \mathtt{emp} * \mathtt{emp}\}$

$\{\mathtt{emp}\}$	$\{\mathtt{emp}\}$
while true do	while true do
$\{\mathtt{emp} \wedge \mathtt{true}\}$	$\{\mathtt{emp} \wedge \mathtt{true}\}$
produce m	$P(busy);$
$\{\mathtt{emp}\}$	$\{\mathtt{emp}\}$
$P(free);$	$n := [10];$
$\{10 \mapsto -\}$	$\{10 \mapsto -\}$
$[10] := m;$ $\quad \|$	$V(free);$
$\{10 \mapsto -\}$	$\{10 \mapsto -\}$
$V(busy);$	*consume n*
$\{\mathtt{emp}\}$	$\{\mathtt{emp}\}$
$\{\mathtt{false}\}$	$\{\mathtt{false}\}$

$\{\mathtt{false}\}$

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- Each semaphore invariant talks only about itself, not about the other semaphore or the processes.

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- Each assertion within a process talks about only its own state, not the state of the other process or even the semaphores.
- We don't maintain $0 \leq \mathit{free} + \mathit{busy} \leq 1$ as a global invariant.
- Semaphores are "logically attached" to resource. P and V are ownership transformers.

Part II

Context

Resource Logics

- Linear Logic (Girard, 1987)
- BI (O'Hearn, Pym, 1999)
- Separation Logic (Reynolds, O'Hearn,... 2000...)

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- Linear Logic (Girard, 1987)
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- Resource is a central concern in concurrent programming
- A natural question: Could resource logic be used?

Local Reasoning

- It should be possible for reasoning and specification to be confined to the resources that a program actually accesses (it's *resource footprint*.[1]

[1] O'Hearn, Reynolds, Yang. Local Reasoning about Programs that Alter Date Structures. CSL'01

Local Reasoning

- It should be possible for reasoning and specification to be confined to the resources that a program actually accesses (it's *resource footprint*.[1]

- Separation Logic's *frame rule*

$$\frac{\{P\}\,C\,\{Q\}}{\{P * R\}\,C\,\{Q * R\}}\ \mathrm{Modifies}(C) \cap \mathrm{Free}(R) = \emptyset$$

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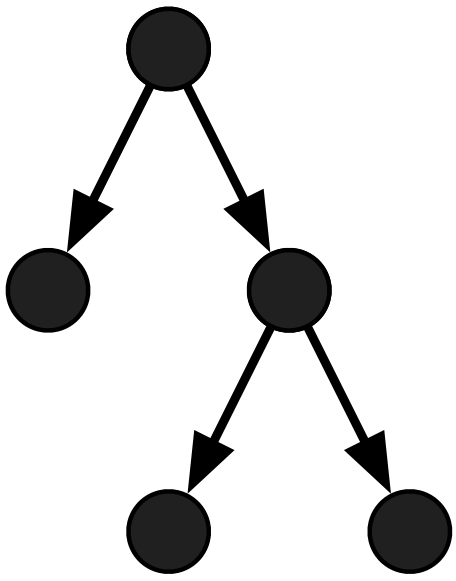
From

{ tree(z) } dispose tree(z) {emp}

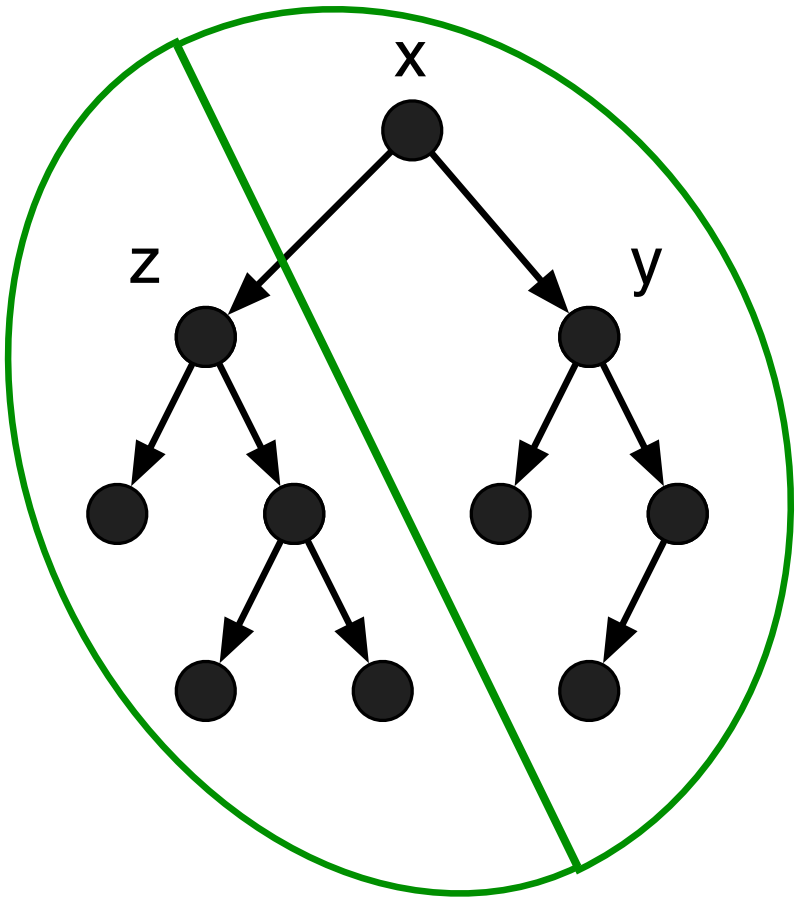
z

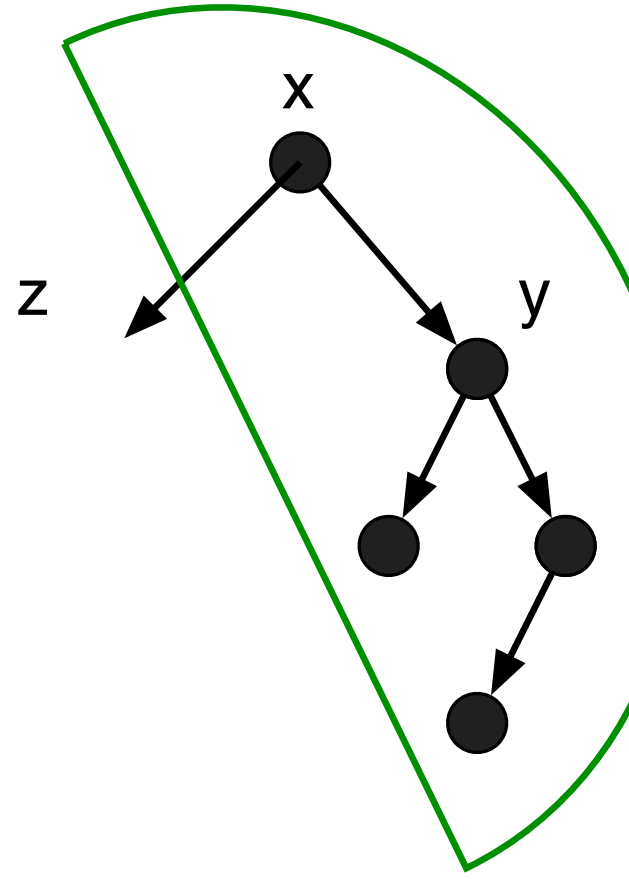


z

We Can Infer

{ tree(z) * tree(y) * xl-y,z} disposetree(z) {tree(y) * xl-y,z}





Parallel Disposetree

- Disjoint Concurrency

$$\frac{\{P_1\}\,C_1\,\{Q_1\} \quad \{P_2\}\,C_2\,\{Q_2\}}{\{P_1 * P_2\}\,C_1 \parallel C_2\,\{Q_1 * Q_2\}}$$

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- Main Part of Proof

$$\begin{array}{c} \{tree(y) * tree(z)\} \\ \{tree(y)\} \qquad\qquad \{tree(z)\} \\ \texttt{disposetree}(y) \quad \parallel \quad \texttt{disposetree}(z) \\ \{\texttt{emp}\} \qquad\qquad \{\texttt{emp}\} \\ \{\texttt{emp} * \texttt{emp}\} \\ \{\texttt{emp}\} \end{array}$$

- Specification: $\{tree(x)\}\texttt{disposetree}(x)\{\texttt{emp}\}$

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- Specification: $\{tree(x)\}\texttt{disposetree}(x)\{\texttt{emp}\}$
- No need for a rely (no one touches my tree)

Parallel Disposetree

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- Main Part of Proof

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- Specification: $\{tree(x)\}\texttt{disposetree}(x)\{\texttt{emp}\}$
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- Or a guarantee (I touch only my own tree)

Reasoning about (shared-variable) Concurrency

- Owicki-Gries, Rely-Guarantee, Temporal Logic...

Reasoning about (shared-variable) Concurrency

- Owicki-Gries, Rely-Guarantee, Temporal Logic...
- Hoare, 1972: Towards a Theory of Parallel Programming
  - revolves around "spatial separation"
  - extremely modular
  - limited, but...

Cautious and Daring Concurrency

- A program is *cautious* if, whenever concurrent processes access the same piece of state, they do so only within the same grouping of mutual exclusion. Otherwise, the program is *daring*.

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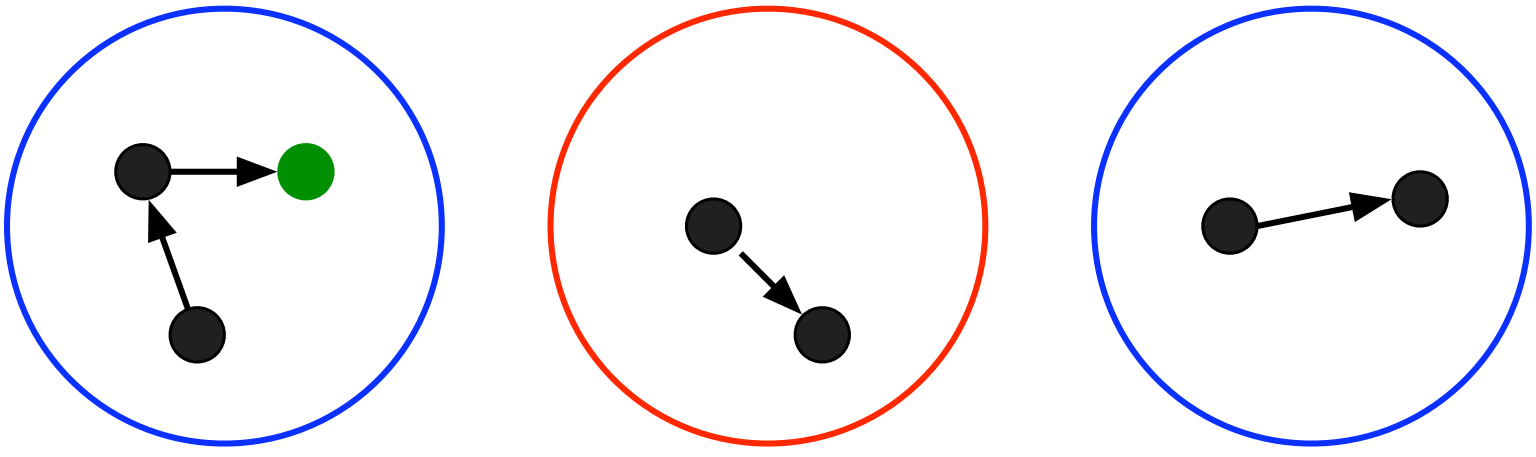
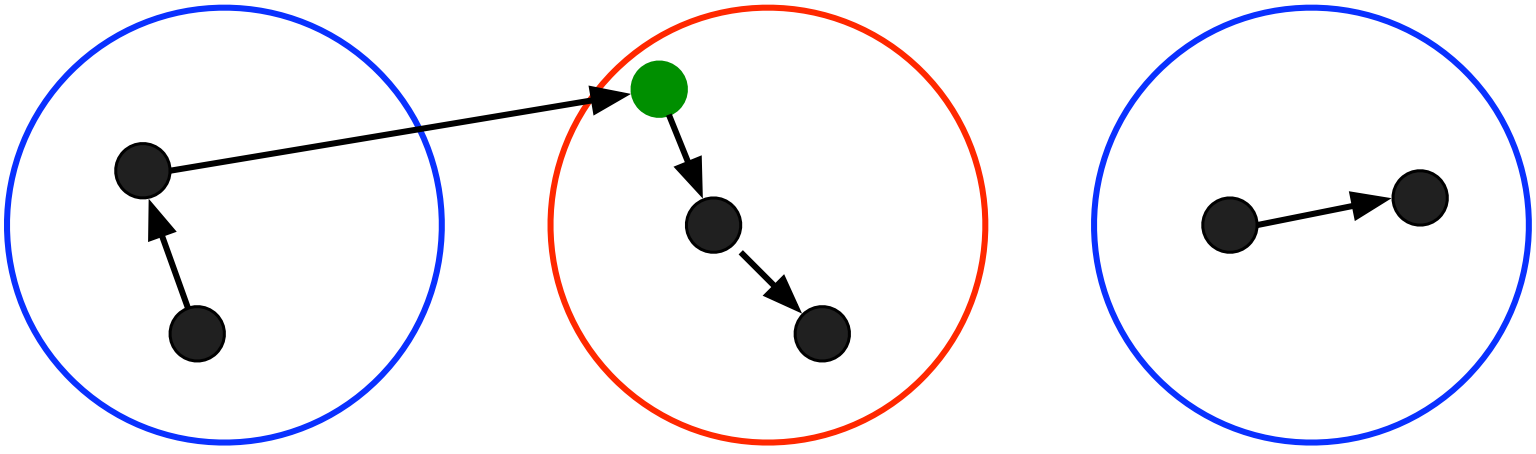
- Daring programs are many
  - Memory Managers, Thread Pools, Connection Pools
  - Efficient Message Passing (copy avoiding)
  - Double-buffered I/O
  - Microkernel OS designs
  - Essentially all semaphore programs

Dynamic Partitioning and Ownership Transfer



Dynamic Partitioning and Ownership Transfer



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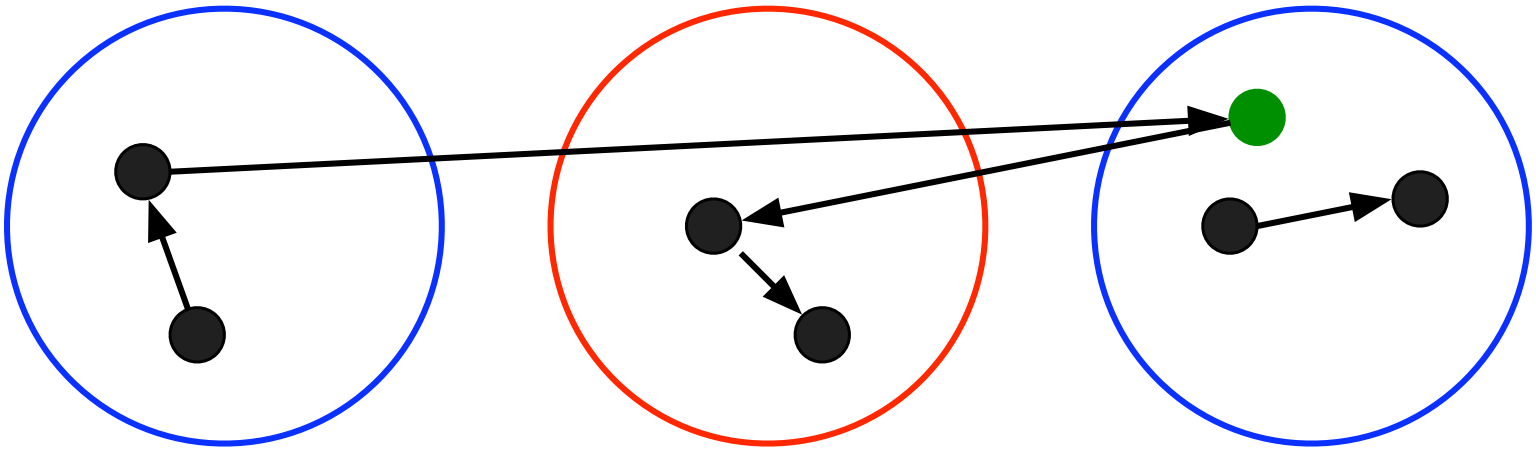


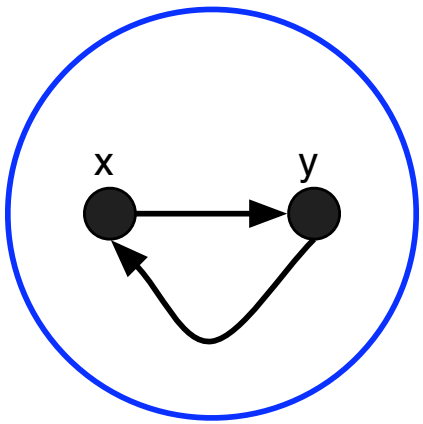
Part III

Separation Logic[2]

[2]Separation Logic: A logic for shared mutable data structure. John Reynolds, LICS'02.. + papers by Reynolds, O'Hearn, Yang...

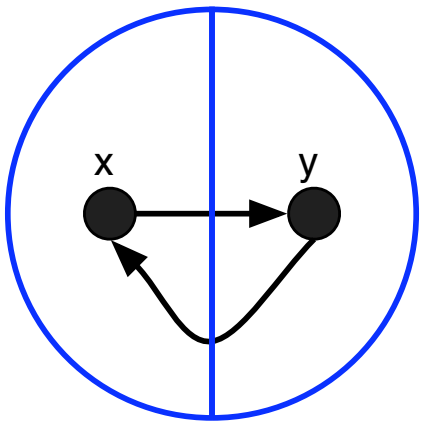
Separation Logic

$$x|\!\!\to y \;*\; y|\!\!\to x$$



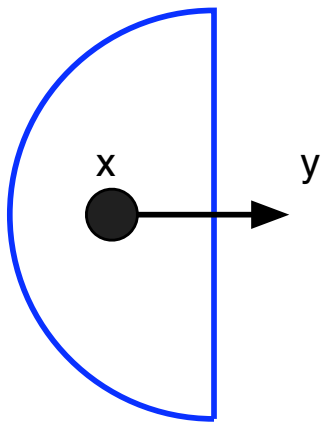
Separation Logic

xl->y * yl-> x



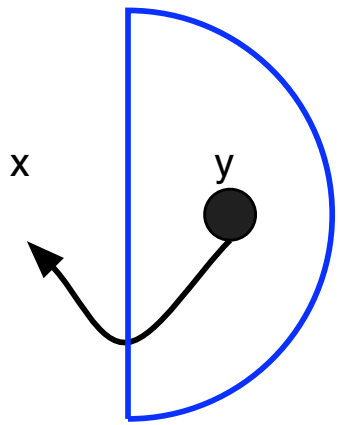
Separation Logic

x|->y



Separation Logic

y|-> x



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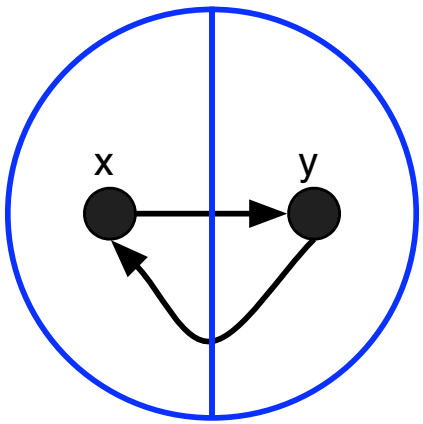

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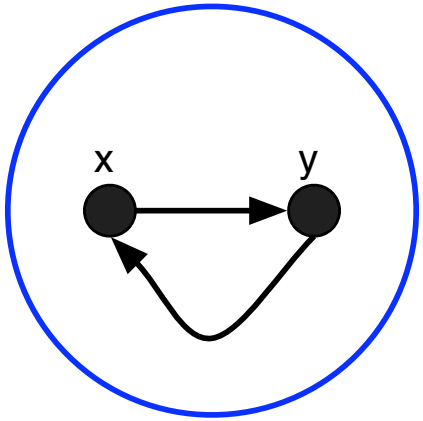


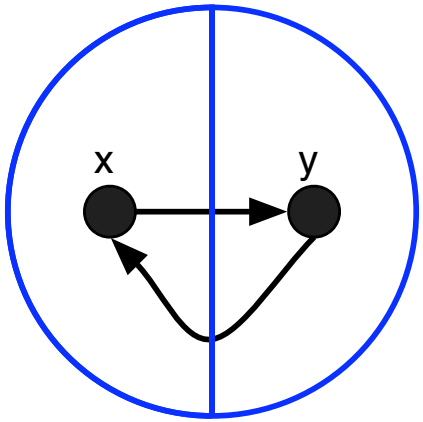
x=10

y=42

10	42
42	10

Separation Logic

xl->y * yl-> x



x=10

y=42

10

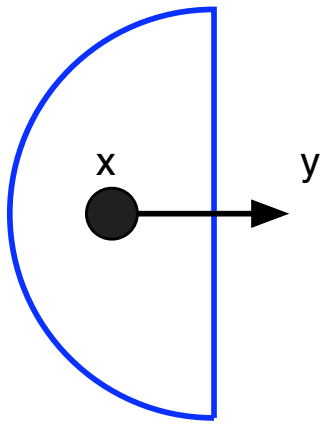
42

42

10

Separation Logic

x|->y



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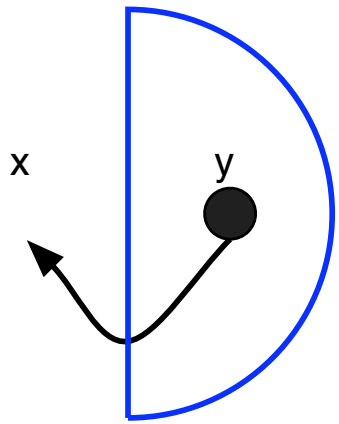
Separation Logic

y|-> x



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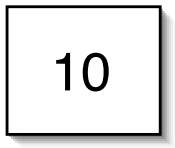

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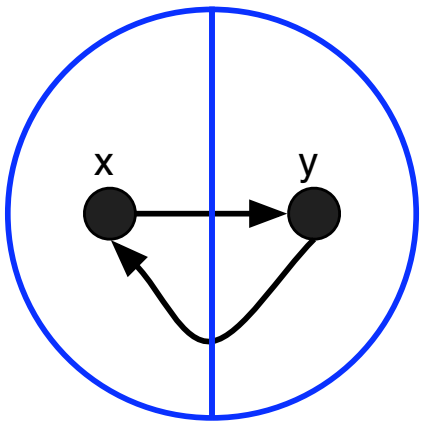

The Assertion Language and Model

Assertions

$$P, Q, R \quad ::= \quad \texttt{emp} \mid E \mapsto F \mid P * Q \mid P \rightarrow Q \mid \forall x.P \mid \cdots$$

Semantics: $s, h \models P$, where

$$s \in S = \texttt{Variables} \rightharpoonup \texttt{Val} \qquad h \in H = \texttt{Addresses} \rightharpoonup \texttt{Val}$$

RAM Model: `Addresses`=Naturals, `Val`=Integers.

$*$ splits h component, unioning heaps with disjoint domains. H has a partial commutative monoid structure.

The powerset of H is a boolean algebra, *and* a residuated monoid. (A Boolean BI Algebra.)[3]

[3] Possible Worlds and Resources, the Semantics of BI. Pym, O'Hearn, Yang, TCS'04. And Pym monograph on BI.

Well specified programs don't go wrong

$\{(x \mapsto -) * P\}\ [x] := 7\ \{(x \mapsto 7) * P\}$

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$\{(x \mapsto -) * P\}\ [x] := 7\ \{(x \mapsto 7) * P\}$

$\{\texttt{true}\}\ [x] := 7\ \{???\}$

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$\{(x \mapsto -) * P\}\ [x]{:=}\,7\ \{(x \mapsto 7) * P\}$

$\{\texttt{true}\}\ [x]{:=}\,7\ \{???\}$

$\{P * (x \mapsto -)\}\ \texttt{dispose}(x)\ \{P\}$

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$\{P * (x \mapsto -)\}\ \texttt{dispose}(x)\ \{P\}$

$\{\texttt{true}\}\ \texttt{dispose}(x)\ \{???\}$

Well specified programs don't go wrong

$\{(x \mapsto -) * P\}\ [x]{:=}\,7\ \{(x \mapsto 7) * P\}$

$\{\texttt{true}\}\ [x]{:=}\,7\ \{???\}$

$\{P * (x \mapsto -)\}\ \texttt{dispose}(x)\ \{P\}$

$\{\texttt{true}\}\ \texttt{dispose}(x)\ \{???\}$

$\{P\}\ \ x = \texttt{cons}(a, b)\ \ \{P * (x \mapsto a, b)\}\ \ \ (x \notin \mathit{free}(P))$

Part IV

Concurrency Proof Rules[4]

[4] Following Hoare, and Owicki-Gries

Process Interaction:

Conditional Critical Regions (Hoare, 72)

init;
`resource` $r_1(\text{variable list}), \ldots, r_m(\text{variable list})$
$C_1 \parallel \cdots \parallel C_n$

`with` r `when` B `do` C `endwith`.

–if a variable belongs to a resource, it cannot appear in a parallel process except in a critical section for that resource

–if a variable is changed in one process, it cannot appear in another unless it belongs to a resource.

$$\frac{\{P\}\,init\,\{RI_{r_1} * \cdots * RI_{r_m} * P'\} \qquad \{P'\}\,C_1 \parallel \cdots \parallel C_n\,\{Q\}}{}$$

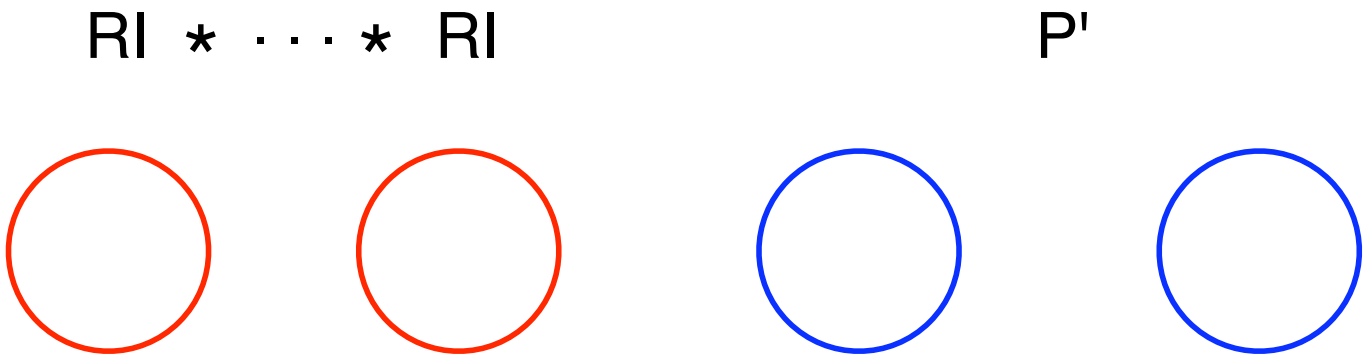
$\{P\}$

init;

`resource` r_1(variable list), . . . , r_m(variable list)

$C_1 \parallel \cdots \parallel C_n$

$\{RI_{r_1} * \cdots * RI_{r_m} * Q\}$



$$\frac{\{P\}\, init\, \{RI_{r_1} * \cdots * RI_{r_m} * P'\} \qquad \{P'\}\, C_1 \parallel \cdots \parallel C_n \{Q\}}{}$$

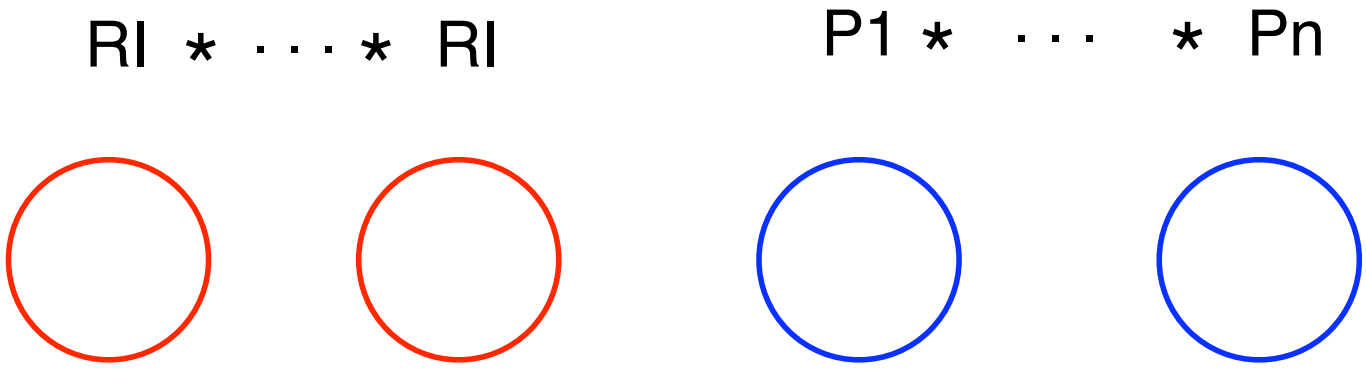
$\{P\}$

init;

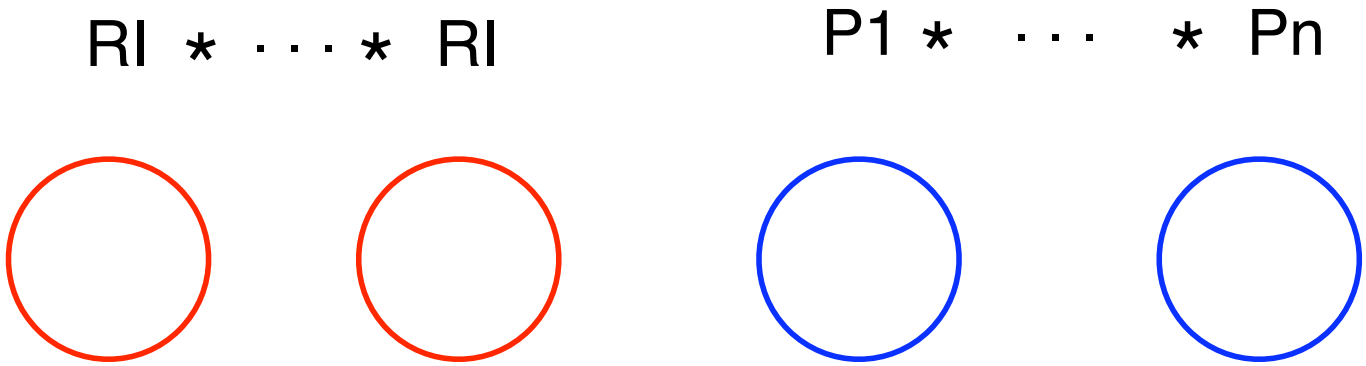
`resource` r_1(variable list), ..., r_m(variable list)

$C_1 \parallel \cdots \parallel C_n$

$\{RI_{r_1} * \cdots * RI_{r_m} * Q\}$



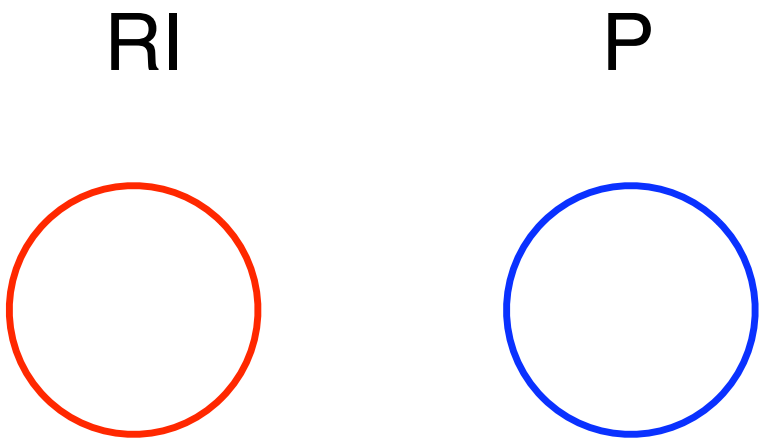
$$\frac{\{P_1\}\, C_1\, \{Q_1\} \;\cdots\; \{P_n\}\, C_n\, \{Q_n\}}{\{P_1 * \cdots * P_n\}\, C_1 \parallel \cdots \parallel C_n\, \{Q_1 * \cdots * Q_n\}}$$



– no variable free in P_i or Q_i is changed in C_j when $j \not= i$.

– any modified variable free in RI_r must occur only within a critical region for r.

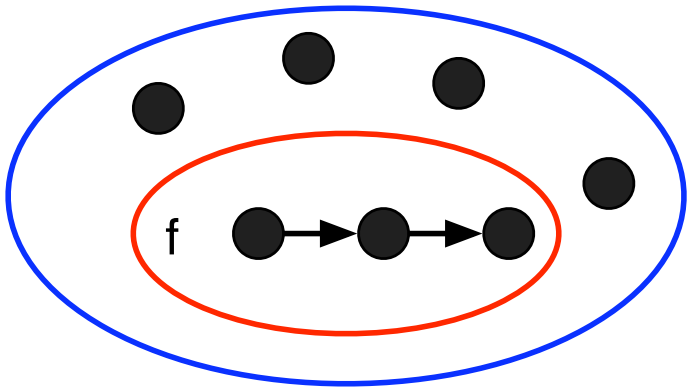
$$\frac{\{(P * RI_r) \wedge B\}\ C\ \{Q * RI_r\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \texttt{endwith}\ \{Q\}}$$



- No other process modifies variables in P or Q

Brookes's Semantic Analysis

- **Soundness Theorem**. The proof rules are sound, as long as resource invariants are *precise*.
- Reynolds counterexample shows unsoundness for imprecise invariants.
- A predicate is precise if, for every state, there is at most one substate that satisfies it.



Brookes's Semantic Analysis, II

- **Safety Theorem**. If $\{P\}C\{Q\}$ holds then there will be *no race condition* and *no memory fault* in any execution starting in a state satisfying P.
- **Slogan**. *Well specified processes mind their own business.*

Part V

Toy Resource Manager

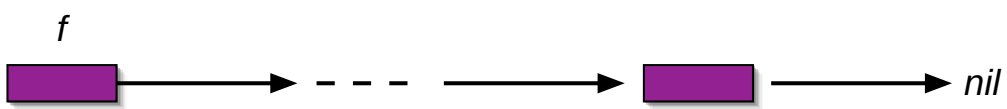
$$\texttt{resource}\ \texttt{mm}(f).$$

$$\texttt{alloc}(x, a, b) \stackrel{\text{def}}{=} \texttt{with}\ \texttt{mm}\ \texttt{when}\ \texttt{true}\ \texttt{do}\ \texttt{if}\ f = \texttt{nil}\ \texttt{then}\ x := \texttt{cons}(a, b)\ \texttt{else}\ x := f;\ f := x.2;\ z.1 := a;\ x.2 := b\ \texttt{endwith}$$

$$\texttt{dealloc}(y) \stackrel{\text{def}}{=} \texttt{with}\ \texttt{mm}\ \texttt{when}\ \texttt{true}\ \texttt{do}\ y.2 := f;\ f := y;\ \texttt{endwith}$$

Resource Invariant: *list*(f)

The manager owns the free list.



$$\mathit{list}\, x \stackrel{\text{def}}{\text{ iff }} (x = \texttt{nil} \wedge \texttt{emp}) \vee (\exists y.\, x \mapsto -, y * \mathit{list}\, y)$$

- Proof Establishing $\{y \mapsto -,-\}\texttt{dealloc}(y)\{\texttt{emp}\}$:

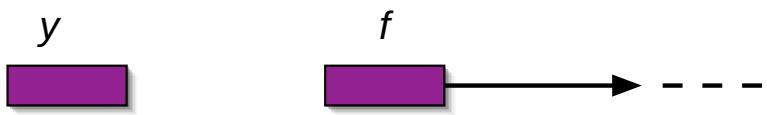
  $\{(y \mapsto -,-) * \mathit{list}\ f\}$

  $y.2 := f;$

  $f := y;$

  $\{\mathit{list}\ f\}$

  $\{\texttt{emp} * \mathit{list}\ f\}$



- Proof Establishing $\{y \mapsto -,-\}\texttt{dealloc}(y)\{\texttt{emp}\}$:

  $\{(y \mapsto -,-) * \mathit{list}\ f\}$

  $y.2 := f;$

  $f := y;$

  $\{\mathit{list}\ f\}$

  $\{\texttt{emp} * \mathit{list}\ f\}$

- Proof Establishing $\{y \mapsto -,-\}\texttt{dealloc}(y)\{\texttt{emp}\}$:

  $\{(y \mapsto -,-) * \mathit{list}\ f\}$
  $y.2 := f;$
  $f := y;$
  $\{\mathit{list}\ f\}$
  $\{\texttt{emp} * \mathit{list}\ f\}$

- $$\frac{\{(P * RI_r) \wedge B\}\ C\ \{Q * RI_r\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \texttt{endwith}\ \{Q\}}$$

- Proof Establishing $\{y \mapsto -,-\}\texttt{dealloc}(y)\{\texttt{emp}\}$:

  $\{(y \mapsto -,-) * \mathit{list}\ f\}$
  $y.2 := f;$
  $f := y;$
  $\{\mathit{list}\ f\}$
  $\{\texttt{emp} * \mathit{list}\ f\}$

- $$\frac{\{(P * RI_r) \wedge B\}\ C\ \{Q * RI_r\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \texttt{endwith}\ \{Q\}}$$

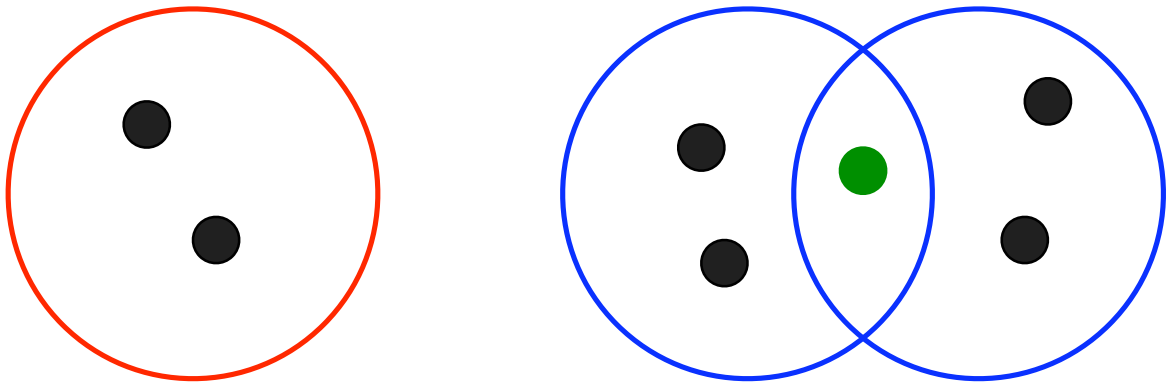
- Similarly, $\{\texttt{emp}\}\texttt{alloc}(x, a, b)\{x \mapsto a, b\}$.

A Safe Example

$$
\begin{array}{l}
\vdots \\
\{x \mapsto -, -\} \\
x.1 := 42; \\
\{x \mapsto -, -\} \\
\texttt{dealloc}(x); \\
\{\texttt{emp}\} \\
\vdots
\end{array}
\quad \| \quad
\begin{array}{l}
\vdots \\
\{\texttt{emp}\} \\
\texttt{alloc}(y, a, b); \\
\{y \mapsto a, b\} \\
y.1 := 7; \\
\{y \mapsto 7, b\} \\
\vdots
\end{array}
$$

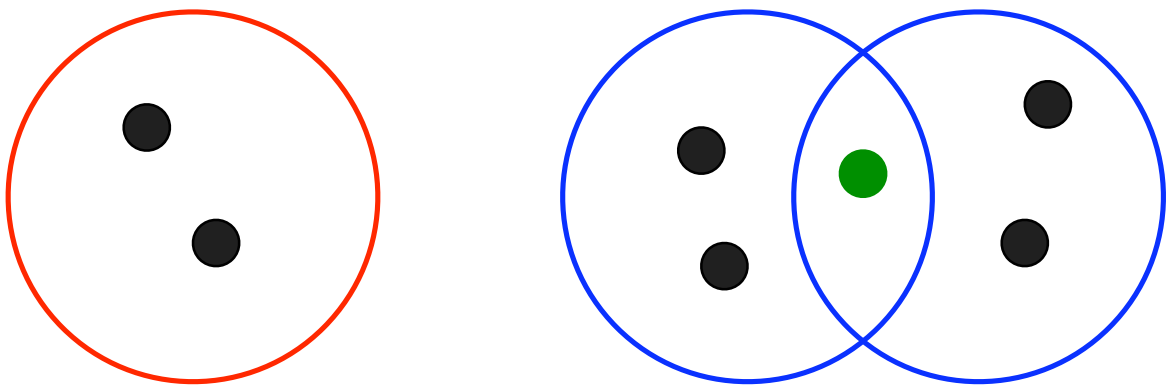
An Unsafe Example

$$
\begin{array}{l}
\vdots \\
\{x \mapsto -, -\} \\
x.1 := 42; \\
\{x \mapsto -, -\} \\
\texttt{dealloc}(x); \\
\{\texttt{emp}\}
\end{array}
\quad \| \quad
\begin{array}{l}
\vdots \\
\{\texttt{emp}\} \\
\texttt{alloc}(y, a, b); \\
\{y \mapsto a, b\} \\
y.2 := 17; \\
\{y \mapsto a, y\} \\
\vdots \\
\vdots
\end{array}
$$



An Unsafe Example

$$
\begin{array}{l}
\vdots \\
\{x \mapsto -, -\} \\
x.1 := 42; \\
\{x \mapsto -, -\} \\
\texttt{dealloc}(x); \\
\{\texttt{emp}\} \\
\\
x.2 := x \\
\\
\{???\}
\end{array}
\quad \| \quad
\begin{array}{l}
\vdots \\
\{\texttt{emp}\} \\
\texttt{alloc}(y, a, b); \\
\{y \mapsto a, b\} \\
y.2 := 17; \\
\{y \mapsto a, y\} \\
\vdots \\
\vdots
\end{array}
$$



Part VI

Modularity

Pointer-transferring Buffer

$full$:= `false`;
`resource` $buf(c, full)$

$$\texttt{put}(x) \stackrel{\text{def}}{=} \texttt{with}\ buf\ \texttt{when}\ \neg full\ \texttt{do}\ c := x;\ full := \texttt{true}\ \texttt{endwith}$$

$$\texttt{get}(y) \stackrel{\text{def}}{=} \texttt{with}\ buf\ \texttt{when}\ full\ \texttt{do}\ y := c;\ full := \texttt{false}\ \texttt{endwith};$$

Pointer-transferring Buffer

$full$:= `false`;
`resource` $buf(c, full)$

$$\texttt{put}(x) \stackrel{\text{def}}{=} \begin{array}{l} \texttt{with } buf \texttt{ when } \neg full \texttt{ do} \\ \quad c := x;\ full := \texttt{true} \\ \texttt{endwith} \end{array}$$

$$\texttt{get}(y) \stackrel{\text{def}}{=} \begin{array}{l} \texttt{with } buf \texttt{ when } full \texttt{ do} \\ \quad y := c;\ full := \texttt{false} \\ \texttt{endwith}; \end{array}$$

$\{x \mapsto -, -\}\texttt{put}(x)\{\texttt{emp}\}$ $\quad$ $\{\texttt{emp}\}\texttt{get}(y)\{y \mapsto -, -\}$

$RI_{buf} \stackrel{\text{def}}{=} (full \wedge c \mapsto -, -) \vee (\neg full \wedge \texttt{emp})$

Code Snippet

$$
\begin{array}{l}
\{\mathtt{emp} * \mathtt{emp}\} \\
\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
\{\mathtt{emp}\} \qquad\qquad\qquad\qquad\qquad\quad \{\mathtt{emp}\} \\
x := \mathtt{cons}(a, b) \qquad \| \qquad \mathtt{get}(y); \\
\{x \mapsto -, -\} \qquad\qquad\qquad\quad \{y \mapsto -, -\} \\
\mathtt{put}(x); \qquad\qquad\qquad\qquad\quad \mathit{use}(y); \\
\{\mathtt{emp}\} \qquad\qquad\qquad\qquad\quad \{y \mapsto -, -\} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \mathtt{dispose}(y); \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \{\mathtt{emp}\} \\
\{\mathtt{emp} * \mathtt{emp}\} \\
\{\mathtt{emp}\}
\end{array}
$$

$$RI_{buf} \stackrel{\text{def}}{=} (\mathit{full} \wedge c \mapsto -, -) \vee (\neg \mathit{full} \wedge \mathtt{emp})$$

Code Snippet

$$
\begin{array}{lcl}
\multicolumn{3}{c}{\{\mathtt{emp} * \mathtt{emp}\}} \\
\vdots & & \vdots \\
\{\mathtt{emp}\} & & \{\mathtt{emp}\} \\
x := \mathtt{cons}(a, b) & \| & \mathtt{get}(y); \\
\{x \mapsto -, -\} & & \{y \mapsto -, -\} \\
\mathtt{put}(x); & & \mathit{use}(y); \\
\{\mathtt{emp}\} & & \{y \mapsto -, -\} \\
\mathtt{x.1} := 44 & & \mathtt{dispose}(y); \\
\{???\} & & \{\mathtt{emp}\} \\
\multicolumn{3}{c}{\{\mathtt{emp} * \mathtt{emp}\}} \\
\multicolumn{3}{c}{\{\mathtt{emp}\}}
\end{array}
$$

$$RI_{buf} \stackrel{\text{def}}{=} (\mathit{full} \wedge c \mapsto -, -) \vee (\neg \mathit{full} \wedge \mathtt{emp})$$

Code Snippet

$$
\begin{array}{l}
\{\mathtt{emp} * \mathtt{emp}\} \\
\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
\{\mathtt{emp}\} \qquad\qquad\qquad\qquad\qquad\quad \{\mathtt{emp}\} \\
x := \mathtt{cons}(a, b) \qquad \| \qquad \mathtt{get}(y); \\
\{x \mapsto -, -\} \qquad\qquad\qquad\quad \{y \mapsto -, -\} \\
\mathtt{put}(x); \qquad\qquad\qquad\qquad\quad \mathit{use}(y); \\
\{\mathtt{emp}\} \qquad\qquad\qquad\qquad\quad \{y \mapsto -, -\} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \mathtt{dispose}(y); \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \{\mathtt{emp}\} \\
\{\mathtt{emp} * \mathtt{emp}\} \\
\{\mathtt{emp}\}
\end{array}
$$

$$RI_{buf} \stackrel{\text{def}}{=} (\mathit{full} \wedge c \mapsto -, -) \vee (\neg \mathit{full} \wedge \mathtt{emp})$$

Combining *buf* and mm

$$
\begin{array}{l}
\{\mathtt{emp}{*}\mathtt{emp}\} \\
\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
\{\mathtt{emp}\} \qquad\qquad\qquad\qquad\qquad\quad \{\mathtt{emp}\} \\
\ \mathtt{alloc}(x,a,b); \quad \| \quad \mathtt{get}(y); \\
\{x \mapsto -,-\} \qquad\qquad\qquad\quad \{y \mapsto -,-\} \\
\mathtt{put}(x); \qquad\qquad\qquad\qquad\quad \mathit{use}(y); \\
\{\mathtt{emp}\} \qquad\qquad\qquad\qquad\quad \{y \mapsto -,-\} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \ \mathtt{dealloc}(y); \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \{\mathtt{emp}\} \\
\{\mathtt{emp}{*}\mathtt{emp}\} \\
\{\mathtt{emp}\}
\end{array}
$$

$$RI_{buf} \stackrel{\text{def}}{=} (\mathit{full} \wedge c \mapsto -,-) \vee (\neg \mathit{full} \wedge \mathtt{emp})$$

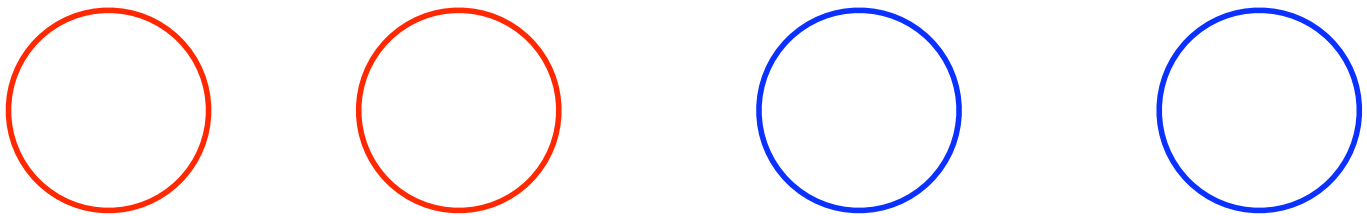
$$RI_{\mathtt{mm}} \stackrel{\text{def}}{=} \mathit{list}(f)$$

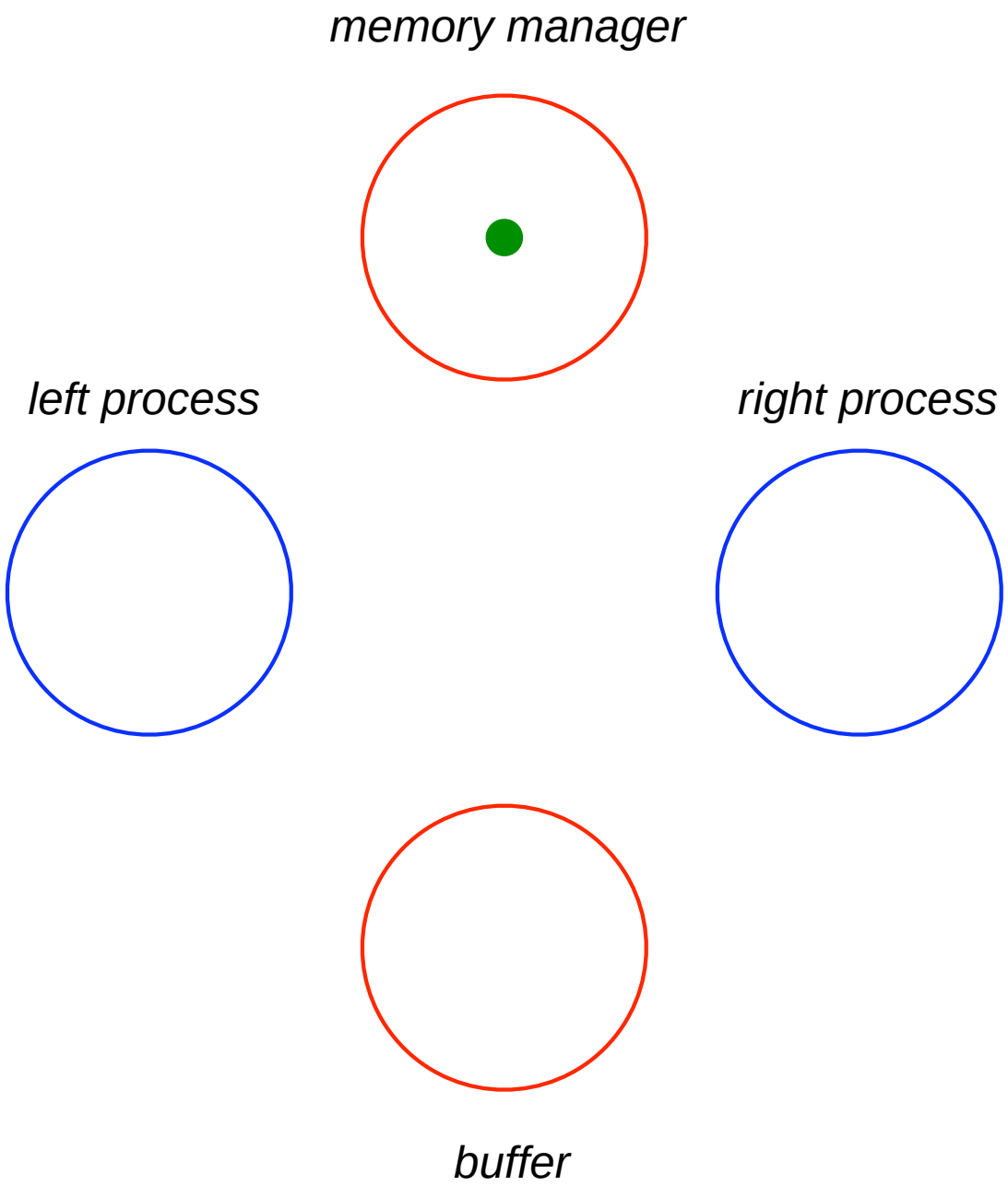
$$\frac{\{P_1\}\, C_1\, \{Q_1\} \;\cdots\; \{P_n\}\, C_n\, \{Q_n\}}{\{P_1 * \cdots * P_n\}\, C_1 \parallel \cdots \parallel C_n\, \{Q_1 * \cdots * Q_n\}}$$

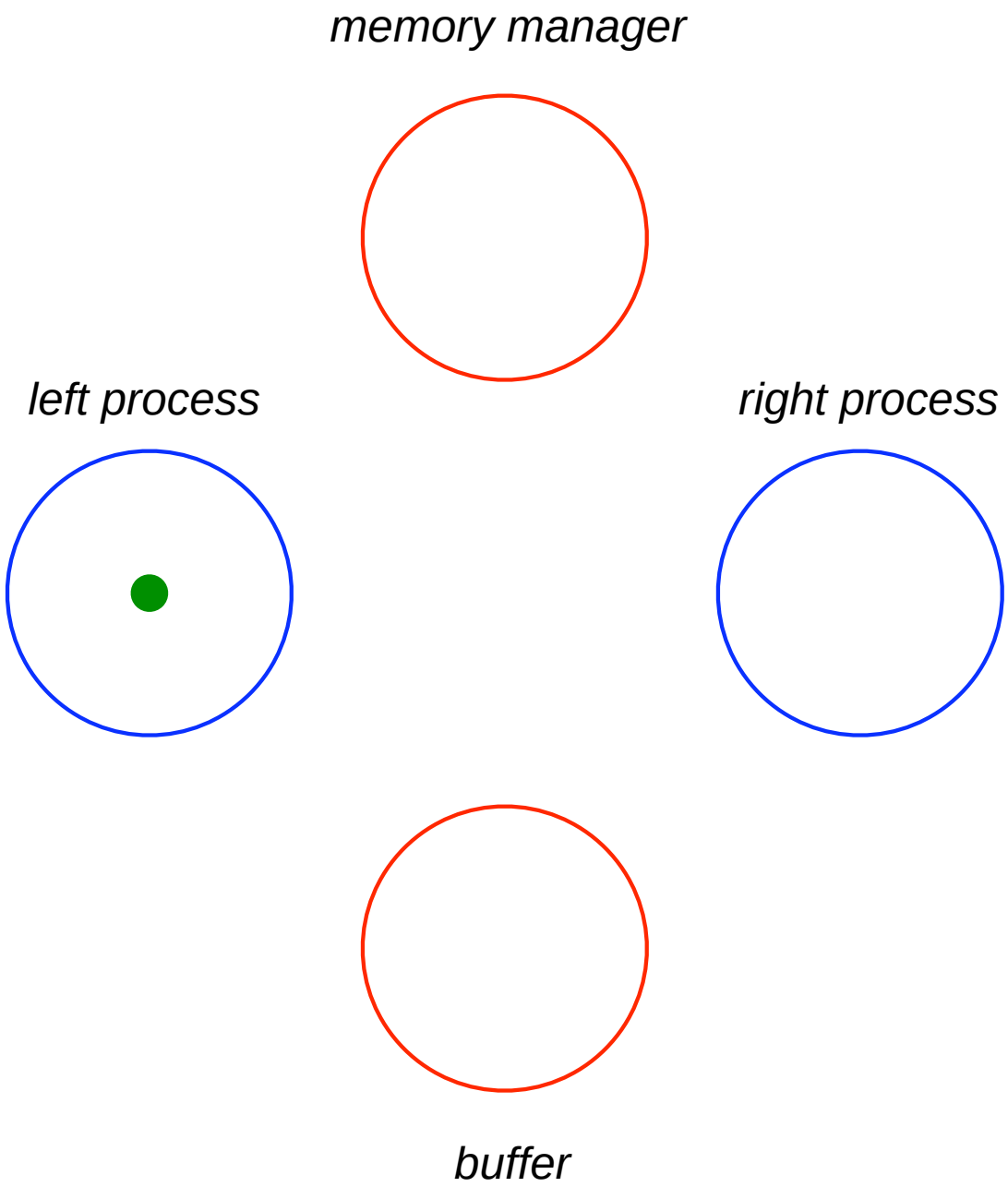
$$\frac{\{(P * RI_r) \wedge B\}\, C\, \{Q * RI_r\}}{\{P\}\, \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \texttt{endwith}\, \{Q\}}$$

RI * ··· * RI

P1 * ··· * Pn




memory manager
left process
right process
buffer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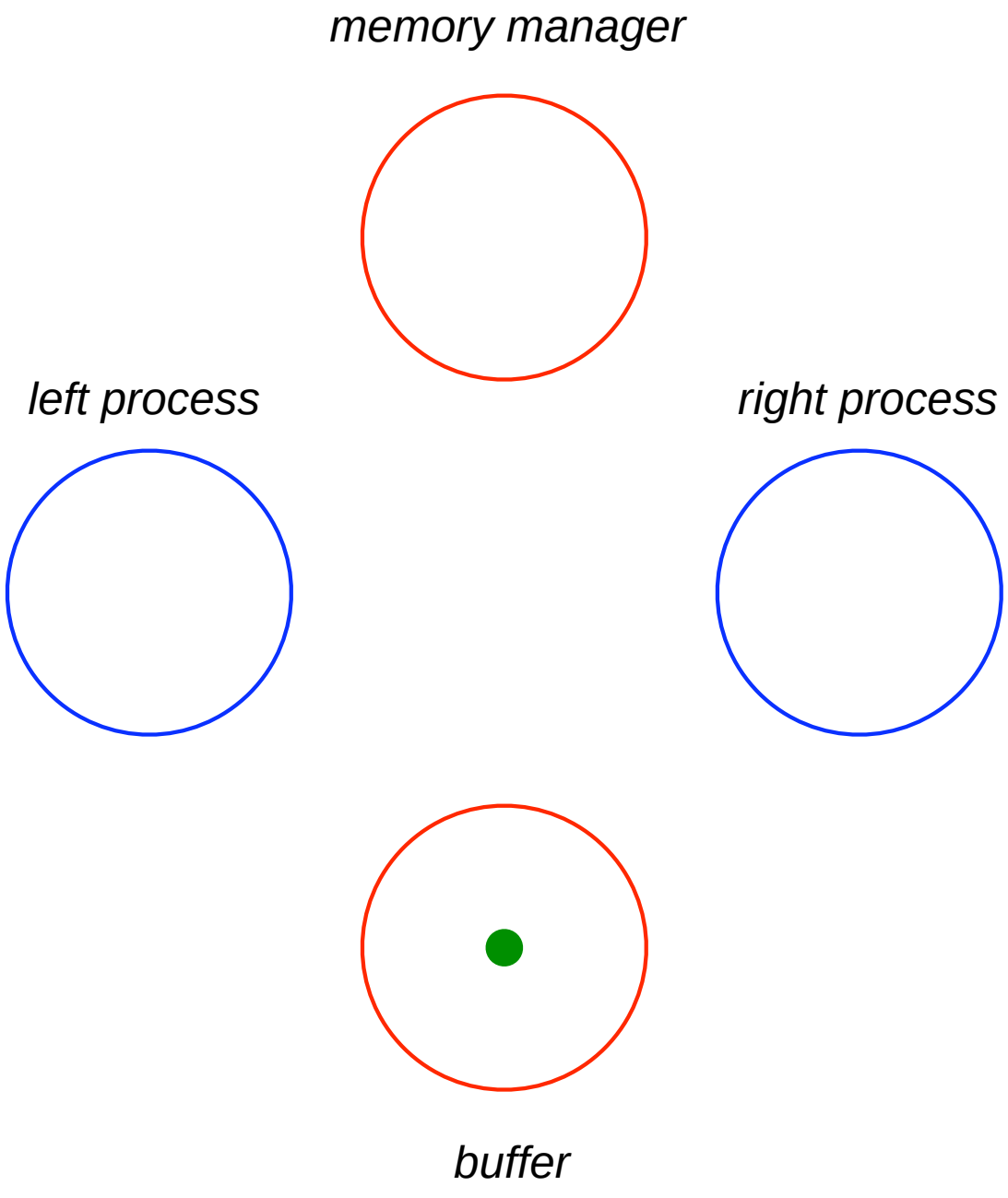
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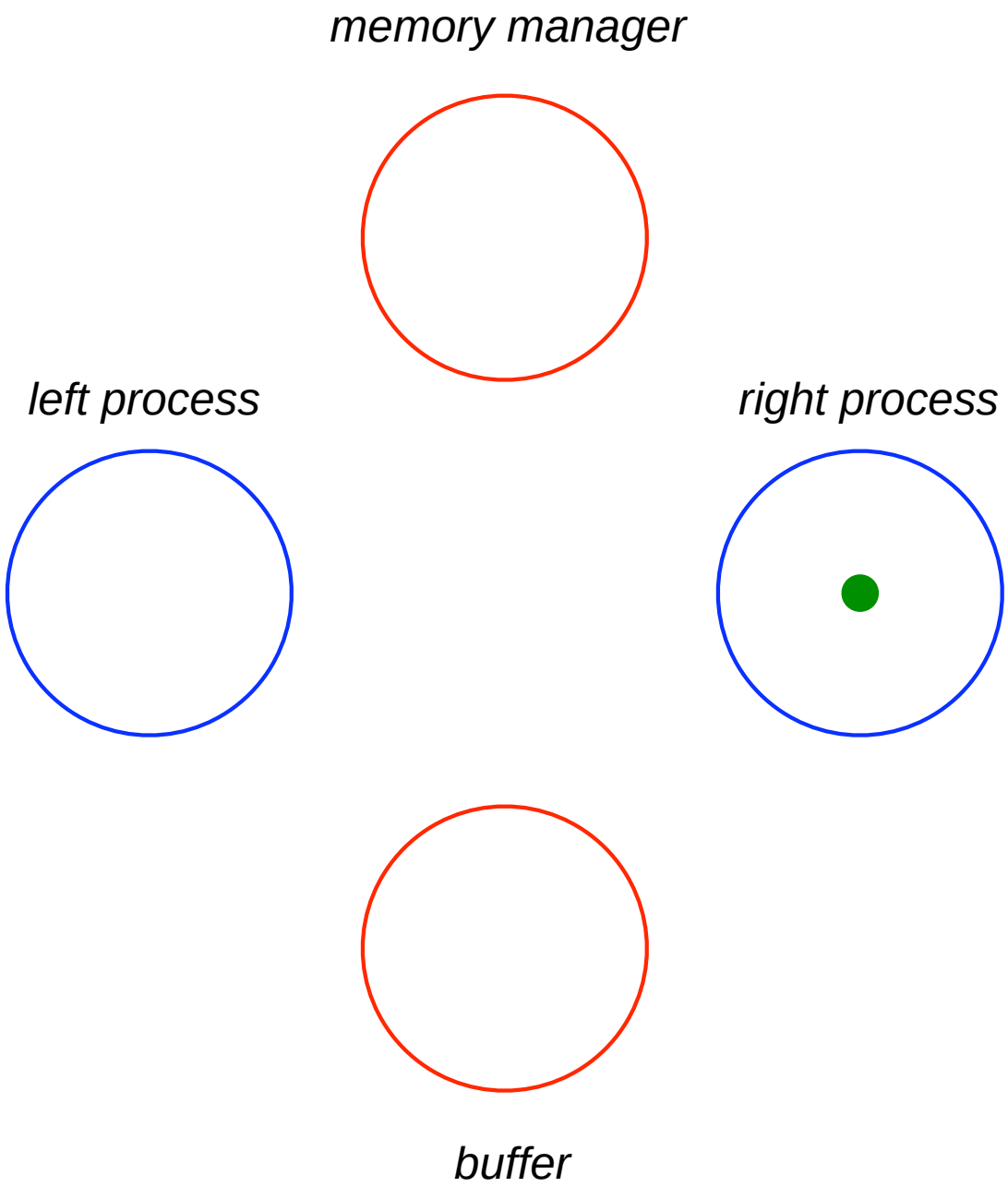
memory manager
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right process
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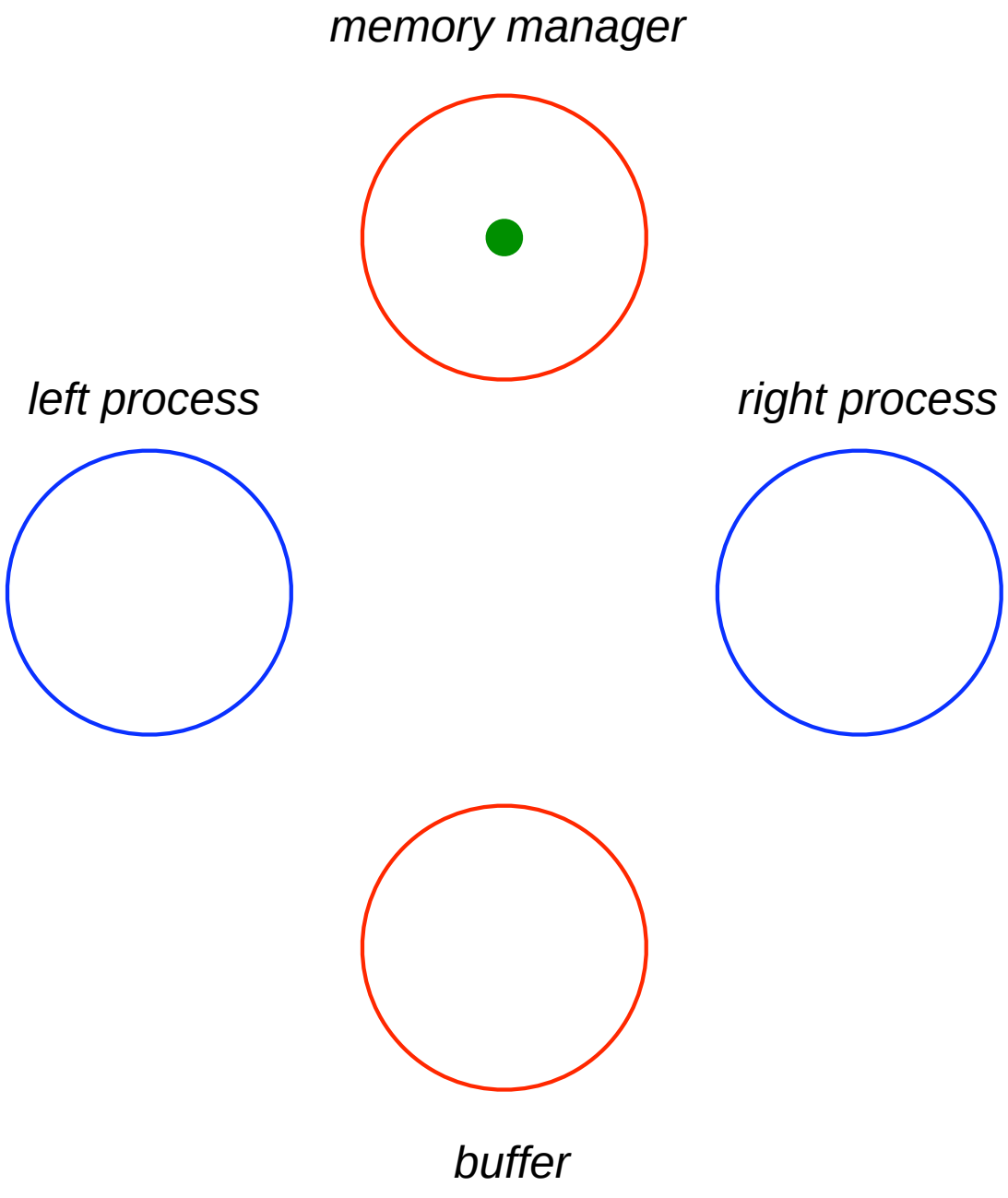

memory manager
left process
right process
buffer

Part VII

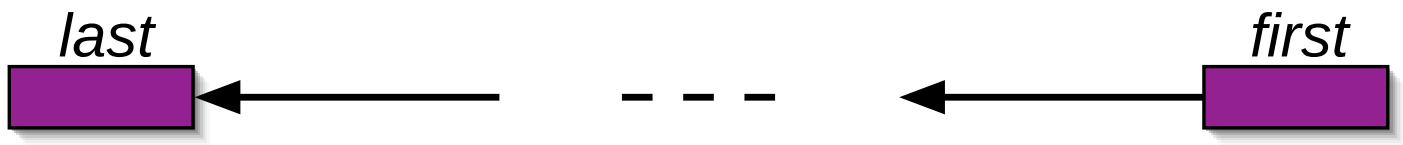
Counting Semaphore Example[5]

[5] From: Cooperating Sequential Processes, Dijkstra, 1968

```
begin integer number of queuing portions;
      number of queuing portions := 0;
      parbegin
      producer:begin
                  again 1: produce the next portion;
                           add portion to buffer;
                           V(number of queuing positions);
                           goto again 1;
                  end;
      consumer: begin
                  again 2: P(number of queuing positions);
                           take portion from buffer;
                           process portion taken;
                           goto again 2;
                  end;
      parend;
end;
```

Add and Take: No Mutexes Needed

We use a non-empty linked list.



The *last* element is a dummy node.

Producer puts message in dummy, then creates new dummy.

Consumer takes *first* element of queue.

Concurrent access is possible.

Relevant Parts of Code

$$
\begin{array}{l}
\texttt{semaphore number:= 0;} \\
\vdots \\
\mathit{produce}(m) \\
\mathit{last}.1 := m; \\
\mathit{last}.2 := \texttt{cons}(-,-); \\
\mathit{last} := \mathit{last}.2 \\
V(\texttt{number}); \\
\vdots
\end{array}
\quad \| \quad
\begin{array}{l}
\vdots \\
P(\texttt{number}); \\
n := \mathit{first}.1; \;\; t := \mathit{first} \\
\mathit{first} := \mathit{first}.2 \\
\texttt{dispose}(t); \\
\mathit{consume}(n); \\
\vdots
\end{array}
$$

Auxiliary Variables

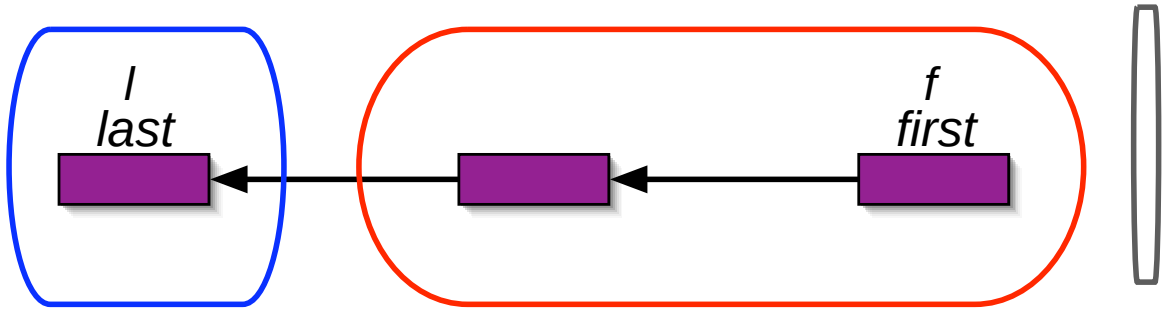
- To describe ownership we need auxiliary variables l and f.
- The semaphore will own a linked list of size `number` from f to l (but not including l).
- We insert auxiliary manipulations into CCRs simulating the semaphores.

$$P'(\texttt{number}) \stackrel{\text{def}}{=} \texttt{with number when number} > 0 \text{ do } f := \mathit{first}.2;\ \texttt{number}--$$

$$V'(\texttt{number}) \stackrel{\text{def}}{=} \texttt{with number when true do } l := \mathit{last};\ \texttt{number}{+}{+}$$

Relevant Parts of Code

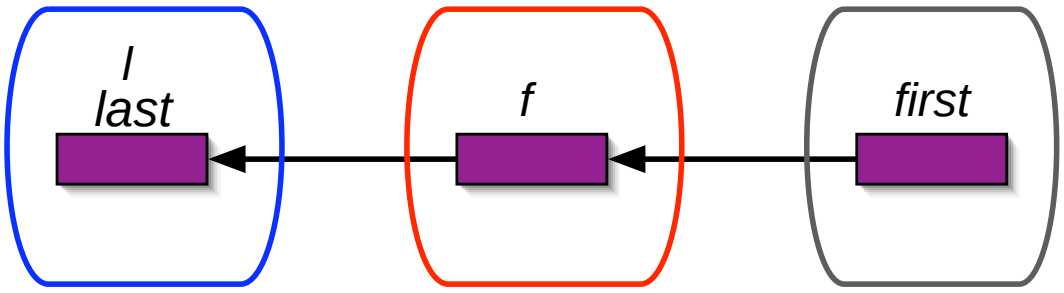
$$
\begin{array}{l}
\texttt{semaphore number} := 0; \\
\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
\mathit{produce}(m) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad P'(\texttt{number}); \\
\mathit{last}.1 := m; \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad n := \mathit{first}.1;\ \ t := \mathit{first} \\
\mathit{last}.2 := \texttt{cons}(-,-); \qquad\qquad \| \qquad\qquad\quad \mathit{first} := \mathit{first}.2 \\
\mathit{last} := \mathit{last}.2 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \texttt{dispose}(t); \\
V'(\texttt{number}); \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \mathit{consume}(n); \\
\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots
\end{array}
$$



produce(m) || *P(number)*

producer **semaphore** **consumer**

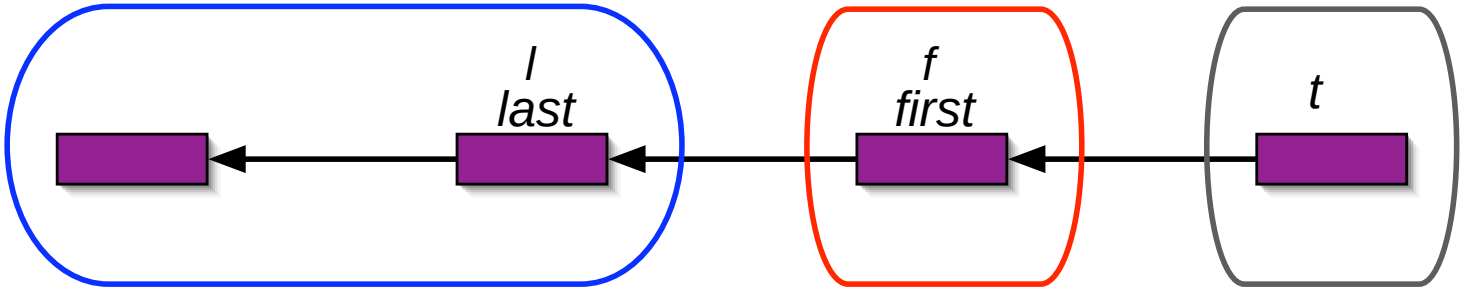
produce(m) || *P(number)*



last.1:=m
last.2:=cons(-,-) ||
n:=first.1; t:=first
first:=first.2

producer **semaphore** **consumer**

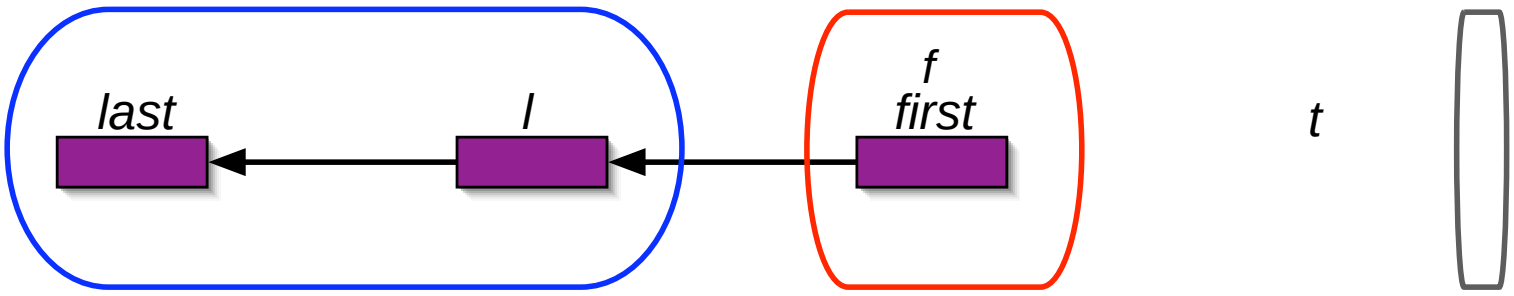
last.1:=m
last.2:=cons(-,-) || *n:=first.1; t:=first*
first:=first.2



last:=last.2 || *dispose(t)*

producer **semaphore** **consumer**

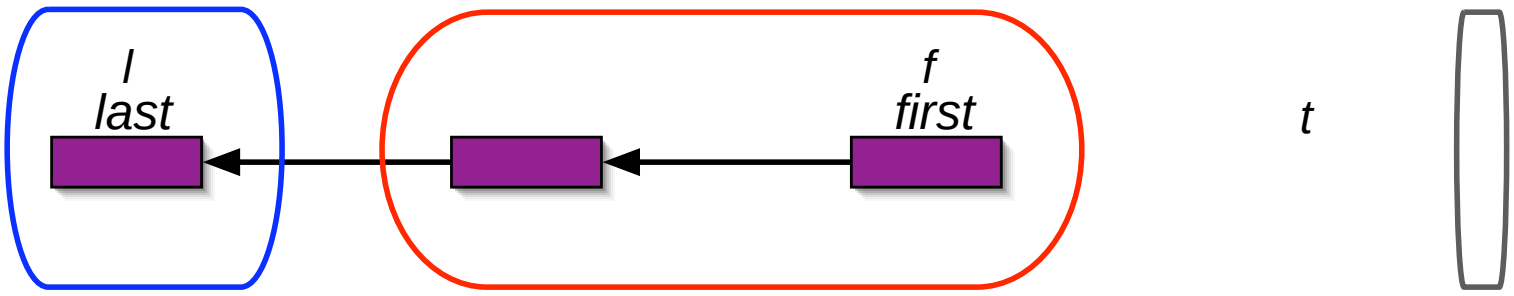
last:=last.2 || *dispose(t)*



V'(number) || *consume(m)*

producer **semaphore** **consumer**

V'(number) || *consume(m)*



producer **semaphore** **consumer**

$\{l = last \wedge last \mapsto -, -\}$
$produce(m)$
$\{l = last \wedge last \mapsto -, -\}$
$last.1 := m;$
$\{l = last \wedge last \mapsto -, -\}$
$last.2 := \mathtt{cons}(-, -);$
$\{l = last \wedge \exists g.(last \mapsto -, g * g \mapsto -, -)\}$
$last.; = last.2$
$\{l \mapsto -, last * last \mapsto -, -)\}$
$V'(\mathtt{number});$
$\{l = last \wedge last \mapsto -, -\}$

$\|$

$\{f = first \wedge \mathtt{emp}\}$
$P'(\mathtt{number});$
$\{first \mapsto -, f\}$
$n := first.1; \;\; t := first$
$\{t = first \wedge t \mapsto -, f\}$
$first := first.2$
$\{f = first \wedge t \mapsto -, f\}$
$\mathtt{dispose}(t);$
$\{f = first \wedge \mathtt{emp}\}$
$consume(n);$
$\{f = first \wedge \mathtt{emp}\}$

$$SI \;=\; \mathtt{ls}[f\ \mathtt{number}\ l]$$

where
$$\mathtt{ls}[x\,n\,z] \overset{\text{def}}{\text{iff}} (x = z \wedge n = 0 \wedge \mathtt{emp}) \vee (x \not= z \wedge n > 0 \wedge \exists y.\, x \mapsto -, y * \mathtt{ls}[y\,(n-1)\,z])$$

No Race

$\{\mathtt{emp}\}$

$\mathtt{semaphore\ number}{:=}\,0;$

$\mathtt{while\ true\ do}$		$\mathtt{while\ true\ do}$
$\quad \mathit{produce}(m)$		$\quad P(\mathtt{number});$
$\quad \mathit{last}.1{:=}\ m;$		$\quad n{:=}\ \mathit{first}.1;\ \ t{:=}\ \mathit{first}$
$\quad \mathit{last}.2{:=}\ \mathtt{cons}(-,-);$	$\parallel$	$\quad \mathit{first}{:=}\ \mathit{first}.2$
$\quad \mathit{last}{:}\ ;{=}\mathit{last}.2$		$\quad \mathtt{dispose}(t);$
$\quad V(\mathtt{number});$		$\quad \mathit{consume}(n);$

$\{\mathtt{false}\}$

Future Directions

- True concurrency model (for proven programs only!)
- Passivity (shared read access)
- Rely/guarantee formalism
- Temporal formalism
- Process calculi (CSP, pi-calculus)
- Mechanized proof
- Model checking, abstract interpretation.