

3004 Computational Complexity problem sheet 3.

1. Given the decision problem,

“Given a TM M does it halt on input I within 100 steps”

Consider a corresponding language L , which is

$$L = \{\text{code}(I)\text{code}(M) : \text{TM } M \text{ halts on } I \text{ within 100 steps}\}$$

Choose one of three possibilities

- (a) L is recursive.
- (b) L is recursively enumerable but not recursive.
- (c) L is not recursively enumerable.

Give a proof which justifies your answer.

2. Given the decision problem,

“Given a TM M does it halt for every input I within 100 steps”

Consider a corresponding language L , which is

$$L = \{\text{code}(M) : \text{for every input } I \text{ TM } M \text{ halts within 100 steps when run on } I\}$$

Choose one of three possibilities

- (a) L is recursive.
- (b) L is recursively enumerable but not recursive.
- (c) L is not recursively enumerable.

Give a proof which justifies your answer.

3. Given the decision problem,

“Given a TM M does there exist an input I such that M halts with within $\text{length}(I)$ steps”

Consider a corresponding language L , which is

$$L = \{\text{code}(M) : \text{there exists an input } I \text{ such that TM } M \text{ halts within } \text{length}(I) \text{ steps}\}$$

Choose one of three possibilities

- (a) L is recursive.
- (b) L is recursively enumerable but not recursive.
- (c) L is not recursively enumerable.

Give a proof which justifies your answer.

4. Given the decision problem,

“Given a TM M does the language that it semidecides contain at least one string?”

Consider a corresponding language L , which is

$$L = \{\text{code}(M) : \text{LA}(M) \text{ contains one or more strings}\}$$

Choose one of three possibilities

- (a) L is recursive.
- (b) L is recursively enumerable but not recursive.
- (c) L is not recursively enumerable.

Give a proof which justifies your answer.