3004 Computational Complexity problem sheet 1.

1. A quadratic in x over the integers has the form $q(x) = ax^2 + bx + c$, where x is a variable a, b and c are integers. Suppose our decision problem has as set of instances all the quadratics over the integers, the 'yes' instances being those for which an integer x exists with $q(x) = 0$. Design a suitable encoding system by which this problem could be tackled on a Turing machine. (It's not necessary to design the Turing machine).

2. Suppose our alphabet Σ consists of the symbols a, b and c.

   (a) Design a Turing machine T that halts and succeeds if and only if the input contains the sequence ab and halts and fails otherwise. (Assume for simplicity the input always consists of a finite string of characters from Σ followed an infinite number of blanks).

   (b) (review) What regular expression describes the language decided by T?

3. Reminder: A language is *recursive* if and only if there is a Turing machine that decides it. A language is *recursively enumerable (r.e)* if and only if there is a Turing machine that semi-decides it (halts when the input is in the language, fails to halt when the input is not in the language).

   Question: Can a language exist that is recursive but not r.e? Give reasons.

4. Suppose we define a class $\mathcal{C}$ of computational devices similar to Turing machines but without the $\leftarrow$ command, so the tape head can never move left, only right. (More formally we are demanding that $f : (Q \setminus H) \times \Sigma \to Q \times (\Sigma \cup \{\rightarrow\})$ instead of $f : (Q \setminus H) \times \Sigma \to Q \times (\Sigma \cup \{\leftarrow, \rightarrow\})$ as in the usual definition).

   (a) Give an informal argument as to why the model of computation defined by $\mathcal{C}$ is not equivalent to the one defined by the class of Turing machines with the standard definition.

   (b) (Hard) Give a formal proof of the same proposition.

   (c) Does it make any difference to the model of computation defined by $\mathcal{C}$ if instead of removing $\leftarrow$ altogether we replace it with a $-$ command which keeps the tape head in place? Give informal reasoning.

5. An *Enumerator* is a Turing machine attached to a printer subject to the following additional constraints:

   - there is no halt state.
   - there is a distinguished state 'print'.
   - the tape starts blank.
   - whenever the machine enters the 'print' state the current contents of the tape are printed by the printer. I.e, the contents of the tape immediately following "$\triangleright$" up until the first blank symbol "$\sqcup$".

   In other words an enumerator starts with a blank tape on which it writes various strings from a given alphabet, occasionally printing one off. It may print out the same string more than once. The language enumerated by an enumerator E is the collection of all the strings printed by E.

   (a) Describe an enumerator that generates the language

   $$L_{\text{even}} := \{x \in \Sigma^* : |x| \text{ is even}\}$$

   over the alphabet $\Sigma = \{1\}$.

   (b) (Hard) Prove that a language L is r.e if and only if there is some enumerator E that enumerates it.