

# Quality Adaptation for Congestion Controlled Video Playback over the Internet \*

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February 15, 1999

## Abstract

Streaming audio and video applications are becoming increasingly popular on the Internet, and the lack of effective congestion control in such applications is now a cause for significant concern. The problem is one of adapting the compression without requiring video-servers to re-encode the data, and fitting the resulting stream into the rapidly varying available bandwidth. At the same time, rapid fluctuations in quality will be disturbing to the users and so should be avoided.

In this paper we present a mechanism for using layered video in the context of unicast congestion control. This quality adaptation mechanism adds and drops layers of the video stream to perform long-term coarse-grain adaptation, while using a TCP-friendly congestion control mechanism to react to congestion on very short timescales. The mismatches between the two timescales are absorbed using buffering at the receiver. We present a piecewise-optimal scheme for the distribution of buffering among the active layers in order to maximize perceptual quality while minimizing rapid, disturbing changes in the quality. We discuss the issues involved in implementing and tuning such a mechanism, and present our simulation and experimental results.

**Keywords:** Quality Adaptive Video, Layered Coding

## 1 Introduction

The Internet has been experiencing explosive growth of audio and video streaming. Most current applications involve web-based audio and video playback[6, 14] where a stored video is streamed from the server to a client upon request. This growth is expected to continue, and such semi-realtime traffic will form a higher portion of the Internet load. Thus

the overall behavior of these applications have a large impact on the Internet traffic.

Since the Internet is a shared environment and does not currently micro-manage utilization of its resources, end systems are expected to be cooperative by reacting to congestion properly and promptly[5, 9]. Deploying end-to-end congestion control results in higher overall utilization of the network and improves inter-protocol fairness. A congestion control mechanism determines the available bandwidth based on the state of the network, and the application should then use this bandwidth efficiently to maximize the quality of the delivered service to the user.

Currently, many of the commercial streaming applications do not perform end-to-end congestion control. This is mainly because stored video has an intrinsic transmission rate. These rate-based applications either transmit data with a near-constant rate or loosely adjust their transmission rate on long timescales since the required rate adaptation for congestion control is not compatible with their nature. Large scale deployment of these applications could result in severe inter-protocol unfairness and possibly even congestion collapse.

This paper is not about congestion control mechanisms, but about a complementary mechanism to adapt the quality of streaming video playback while performing congestion control. However, to design an effective quality adaptation scheme, we need to know the properties of the deployed congestion control mechanism. Our main assumption is that the congestion control mechanism employs an additive increase and multiplicative decrease (AIMD) algorithm.

The simplest TCP-friendly congestion control mechanism we know of is the Rate Adaptation Protocol (RAP)[?]. RAP is a rate-based congestion control mechanism and employs an AIMD algorithm in a manner similar to TCP. There are two variants of RAP - with and without a fi ne-

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\*This work was supported by DARPA under contract No. DABT63-95-C0095 and DABT63-96-C-0054 as part of SPT and VINT projects

grain congestion avoidance mechanism. This paper only discusses the RAP variant without fine-grain adaptation because its properties are much simpler to predict. However, proposed mechanisms can be adopted to any congestion control scheme that deploys an AIMD algorithm.

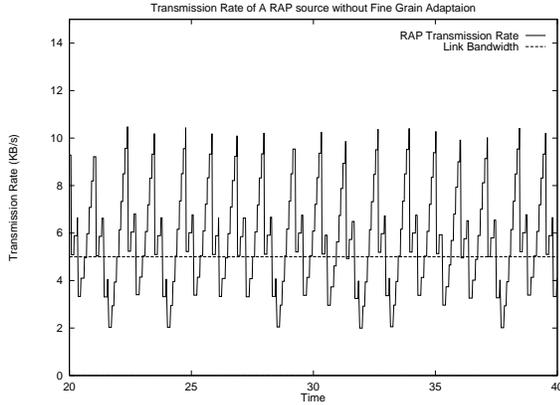


Figure 1: Transmission rate of a single RAP flow

Figure 1 shows the transmission rate of a RAP source over time. Similar to TCP, it hunts around for a fair share of the bandwidth. However but unlike TCP, RAP is not ACK-clocked and variations of transmission rate has a more regular sawtooth shape. Bandwidth increases linearly for a period of time, then a packet is lost, and an exponential backoff occurs, and the cycle repeats.

## 1.1 Target Environment

Our target environment is a video server that simultaneously plays back different video streams on demand for many heterogeneous clients. As with current Internet video streaming, we expect length of such streams to range from 30 second clips to full-length movies. The server and clients are connected through the Internet where the dominant competing traffic is TCP-based. Clients have heterogeneous network capacity and processing power. Users expect that startup playback latency to be low, especially for shorter clips played back as part of web surfing. Thus prefetching the entire stream before starting its playback is not an option. We believe that this scenario reasonably represents many of the today’s Internet streaming applications.

## 1.2 Motivation

If video for playback is stored at a single lowest-common-denominator encoding data rate on the server, high-bandwidth clients will receive poor quality despite availability of a large amount of bandwidth. However, if the video is stored at a higher quality encoding (and hence data

rate) on the server, then there will be many low-bandwidth clients that cannot play back this stream. In the past, we have often seen RealVideo streams available at 14.4 Kb/s and 28.8 Kb/s, where the user can choose their connection speed. However, with the advent of ISDN, ADSL, and cable modems to the home and faster access rates to businesses, the Internet is becoming much more heterogeneous. Customers with higher speed connections feel frustrated to be restricted to modem-speed playback. The network bottleneck is less likely to be the final hop to the end points; instead congestion in the backbone, often at provider interconnects or links to the server itself, will increasingly dominate. As the user cannot know the congestion level, congestion control mechanisms for streaming video playback will become increasingly critical.

Clearly then there is a need for the server to be able to adjust the quality of the stream it plays back so that the perceived quality is as high as the available network bandwidth will permit. We term this *quality adaptation*.

## 1.3 Quality Adaptation Mechanisms

There are several ways to adjust the quality of a pre-encoded stored stream, including adaptive encoding, switching between multiple pre-encoded versions, and hierarchical encoding.

One may re-quantizing stored encodings on-the-fly based on network feedback[2, 15, 18]. However, since encoding is CPU-intensive, servers are unlikely to be able to do this for large numbers of clients. Furthermore, once the original data has been stored compressed, the output rate of most encoders can not be changed over a wide range.

In an alternative approach, the server keeps several versions of each stream with different qualities. As available bandwidth changes, the server switches playback between streams of higher or lower quality as appropriate.

With hierarchical encoding[8, 10, 12, 19], the server maintains a layered encoded version of each stream. As more bandwidth becomes available, more layers of the encoding are delivered. If the average bandwidth decreases, the server may then drop some of the layers being transmitted. Layered approaches usually have the decoding constraint that a particular enhancement layer can only be decoded if all the lower quality layers have been received.

There is a duality between adding or dropping of layers in the layered approach and switching streams in the multiply-encoded approach. However the layered approach is more suitable for caching by a proxy for heterogeneous clients[20]. In addition, it requires less storage at the server, and it provides an opportunity for selective retransmission of the more important information. The design of a layered approach for quality adaptation primarily concerns the

design of an efficient add and drop mechanism that maximizes quality while minimizing the probability of base-layer buffer underflow.

The rest of this paper is organized as follows: first we provide an overview of the layered approach to quality adaptation and then explain coarse-grain adding and dropping mechanisms in section 2. Also we discuss fine-grain inter-layer bandwidth allocation for a single backoff scenario. Section 3 motivates the need for smoothing in the presence of real loss patterns and discusses two possible approaches. In section 4, we sketch a near-optimal filling and draining mechanism that not only achieves smoothing but is also able to cope efficiently with various patterns of losses. We evaluate our mechanism through simulation and experiments in section 5. Section 6 briefly reviews related work. Finally, section 7 concludes the paper and addresses some of our future plans.

## 2 Layered Quality Adaptation

Hierarchical encoding provides an effective way that a video playback server can coarsely adjust the quality of a video stream without transcoding the stored data. However, it does not provide fine-grained control over bandwidth, i.e. bandwidth changes with the granularity of a layer. Furthermore, there needs to be a quality adaptation mechanism to smoothly adjust the quality (i.e. number of layer) as bandwidth changes. Users will tolerate poor quality video, but rapid variations in quality are disturbing.

Hierarchical encoding allows video quality adjustment over long periods of time, whereas congestion control changes the transmission rate rapidly over short time intervals (several round-trip times, (RTTs)). The mismatch between the two timescales is made up for by buffering data at the receiver to smooth the rapid variations in available bandwidth and allow a near constant number of layers to be played.

Figure 2 graphs a simple simulation of a quality adaptation mechanism in action. The top graph shows the available network bandwidth and the consumption rate at the receiver with no layers being consumed at startup, then one layer, and finally two layers. During the simulation, two packets are dropped and cause congestion control backoffs, where the transmission rate goes below the consumption rate for a period of time. The lower graph shows the playout sequence numbers of the actual packets against time. The horizontal lines show the period between arrival time and playout time of a packet. Thus it indicates the total amount of buffering for each layer. This simulation shows more buffered data for Layer 0 (the base layer) than for Layer 1 (the enhancement layer). After the first backoff, the length of these lines decreases indicating buffered data from Layer

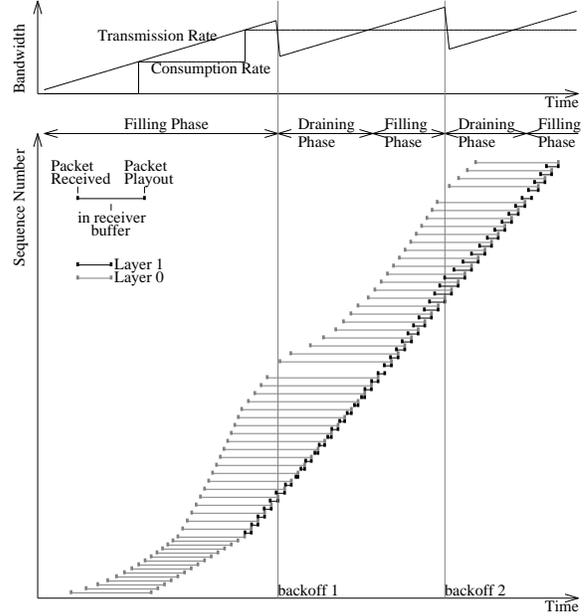


Figure 2: Layered Encoding with Receiver Buffering

0 is being used to compensate for the lack of available bandwidth. At the time of the second backoff, a little data has been buffered for Layer 1 in addition to the large amount for Layer 0. Thus data is drawn from both buffers properly to compensate for the lack of available bandwidth.

The congestion control mechanism dictates the available bandwidth<sup>1</sup>. We cannot send more than this amount, and do not wish to send less<sup>2</sup>. In a real network even the average bandwidth of a congestion controlled flow constantly changes over the session lifetime. Thus a quality adaptation mechanism must continuously evaluate the available bandwidth and adjust number of active layers accordingly.

In this analysis we assume that the layers are linearly spaced - that is each layer has the same bandwidth. This simplifies the analysis, but is not a requirement. In addition, we assume each layer has a constant consumption rate over time. In practice this is unlikely in a real codec, but to a first approximation it is reasonable. It can be ignored by slightly increasing the amount of receiver buffering for all layers to absorb variations in consumption rate.

Figure 3 shows a single cycle of the congestion control mechanism. The sawtooth waveform is the instantaneous transmission rate. There are  $n_a$  layers, each of which has a consumption rate of  $C$ . In the left hand side of the figure,

<sup>1</sup>Available bandwidth and transmission rate are used interchangeably throughout this paper.

<sup>2</sup>For simplicity we ignore flow control issues in this paper but implementations should not. However our final solutions generally require so little receiver buffering that this is not often an issue.

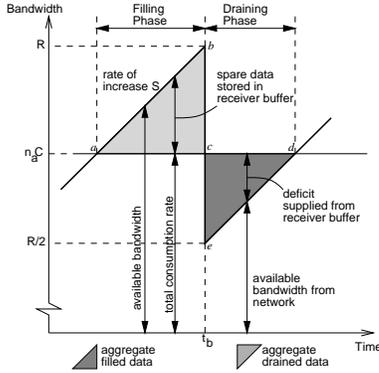


Figure 3: Filling and draining phase

the transmission rate is higher than the consumption rate, and this data will be temporarily stored in the receiver's buffer. The total amount of stored data is equal to the area of triangle  $abc$ . Such a period of time is known as a *filling phase*. Then, at time  $t_b$ , a packet is lost and the transmit rate is reduced multiplicatively. To continue playing out  $n_a$  layers when the transmission rate drops below the consumption rate, some data must be drawn from the receiver buffer until the transmission rate reaches the consumption rate again. The amount of data drawn from the buffer is shown in this figure as triangle  $cde$ . Such a period of time is known as a *draining phase*.

Note that the quality adaptation mechanism can *only* adjust the number of active layers and their bandwidth share. This paper attempts to derive near-optimal solutions for these two key mechanisms:

- A *coarse-grain* mechanism for adding and dropping layers. By changing the number of active layers, the server can perform coarse-grain adjustment on the total amount of receiver buffered data.
- A *fine-grain* inter-layer bandwidth allocation mechanism among the active layers. If there is receiver-buffered data available for a layer, we can temporarily allocate less bandwidth than is being consumed while taking the remainder from the buffer. This smooths out reductions in the available bandwidth. When spare bandwidth is available, we can send data for a layer at a rate higher than its consumption rate, and increase the data buffered for that layer at the receiver.

In the next section, we present coarse-grain adding and dropping mechanisms as well as their relations with the fine-grain bandwidth allocation. Then we discuss the fine-grain bandwidth allocation in the subsequent sections.

## 2.1 Adding a Layer

A new layer can be added as soon as the instantaneous available bandwidth exceeds the consumption rate (in the decoder) of the existing layers. The excess bandwidth could then be used to start buffering a new layer. However, this would be problematic as without knowing future available bandwidth we cannot decide when it will first be possible to start decoding the layer. As the new layer's *playout* is decided by the inter-layer timing dependency between its data and that in the base layer, this means we cannot make a reasoned decision about which data from the new layer to actually send<sup>3</sup>.

A more practical approach is to start sending a new layer when the instantaneous bandwidth exceeds the consumption rate of the existing layers plus the new layer. In this approach the layer can start to play out immediately. In this case there is some excess bandwidth from the time the available bandwidth exceeds the consumption rate of the existing layers until the new layer is added. This excess bandwidth can be used to buffer data for existing layers at the receiver.

In practice, this bandwidth constraint for adding is still not conservative enough, as it may result in several layers being added and dropped with each cycle of congestion control sawtooth. Such rapid-cycling changes in quality would be disconcerting for the viewer. One way to prevent rapid changes in quality is to add a buffering condition such that adding a new layer does not endanger existing layers. Thus, the server may add a new layer when:

1. The instantaneous available bandwidth is greater than the consumption rate of the existing layers plus the new layer, and,
2. There is sufficient total buffering at the receiver to survive an immediate backoff and continue playing all the existing layers plus the new layer.

To satisfy the second condition we assume (for now) that no additional backoff will occur during the draining phase, and the slope of linear increase can be properly estimated.

These are the minimal criteria for adding a new layer. If these conditions are held a new layer can be kept for a reasonable period of time during the normal congestion control cycles. We shall show later that we normally want to be more conservative than this. Clearly we need to have sufficient buffering at the receiver to smooth the available bandwidth signal so that number of active layers does not change due to the normal hunting behavior of the congestion control mechanism.

<sup>3</sup>Note that once the inter-layer timing for a new layer is adjusted, it is maintained as long as buffer does not dry out

Expressing the adding conditions more precisely:

$$\text{Condition 1: } R > (n_a + 1)C$$

$$\text{Condition 2: } \sum_{i=0}^{n_a} bu f_i \geq \frac{(R - (n_a + 1)C)^2}{2S}$$

where  $R$  is the current transmission rate

$n_a$  is the number of currently active layers

$bu f_i$  is the amount of buffered data for layer  $i$

$S$  is the rate of linear increase in bandwidth

(typically one packet per RTT)

\* For derivation of this equation refer to A.1.

## 2.2 Dropping a Layer

Once a backoff occurs, if the total amount of buffering at the receiver is less than the estimated required buffering for recovery, (i.e. the area of triangle  $cde$  in figure 3), the correct course of action is to immediately drop the highest layer. This reduces the consumption rate ( $n_a C$ ) and hence reduces the buffer requirement for recovery (i.e. area of triangle  $cde$ ). If the buffering is still insufficient, the server should iteratively drop the highest layer until the amount of buffering is sufficient. This rule clearly doesn't apply to the base layer which is always sent.

The dropping mechanism more precisely:

$$\text{WHILE } \left( n_a C < R + \sqrt{2S \sum_{i=0}^{n_a} bu f_i} \right)$$

$$\text{DO } n_a = n_a - 1$$

\* For derivation of this equation refer to A.2.

This mechanism provides a coarse-grain criteria for dropping a layer. However, it may be insufficient to prevent buffer underflow during the draining phase for one of several reasons:

- We may suffer a further backoff before the current draining phase completes.
- Our estimate of the slope of linear increase may be incorrect if the network RTT changes substantially.
- There may be sufficient total data buffered, but it may be allocated among the different layers in a manner that precludes its use to aid recovery.

The first two situations are due to incorrect prediction of the amount of buffered data needed to recover, and we term such an event a *critical situation*. In such events, the only appropriate course of action is to drop additional layers as soon as the critical situation is discovered.

The third situation is more problematic, and relates to the fine-grain bandwidth allocation among active layers during both filling and draining phases. We devote much of the rest of this paper to deriving and evaluating a near-optimal solution to this situation.

## 2.3 Inter-layer Buffer Allocation

Because of the decoding constraint in hierarchical coding, each additional layer depends on all the lower layers, and correspondingly is of decreasing value. Thus a buffer allocation mechanism should provide higher protection for lower layers by allocating a higher share of buffering for them.

The problem of inter-layer buffer allocation is to ensure the total amount of buffering is sufficient, and it is properly distributed among active layers to effectively absorb the short-term reductions in bandwidth that might occur. The following two examples illustrate ways that improper allocation of buffered data might fail to compensate for the lack of available bandwidth.

### Dropping layers with buffered data

A simple buffer allocation scheme might allocate an equal share of buffer to each layer. However, if the highest layer is dropped after a backoff, its buffered data is no longer able to assist the remaining layers in the recovery. The top layer's data will still be played out, but it is not providing buffering functionality. This implies that it is more beneficial to buffer data for lower layers.

### Insufficient distribution of buffered data

An equally simple buffer allocation scheme might allocate all the buffering to the base layer. Consider an example when three layers are playing, where a total consumption rate of  $3C$  must be supplied for the receiver's decoder. If the transmission rate drops to  $C$ , the base layer ( $L_0$ ) can be played from its buffer. Since neither  $L_1$  nor  $L_2$  has any buffering, they require transmission from the source. However available bandwidth is only sufficient to feed one layer. Thus  $L_2$  must be dropped *even if the total buffering were sufficient for recovery*.

### Efficiency

In these examples, although buffering is available, it cannot be used to prevent the dropping of layers. This is *inefficient* use of the buffering. In general, we are striving for a distribution of buffering that is most *efficient* in the sense that it provides maximal protection against dropping layers for any pattern of short-term reduction in available bandwidth we are likely to encounter.

These examples reveal the following tradeoffs for inter-layer buffer allocations:

- Allocating more buffering for the lower layers not

only improves their protection but it also increases *efficiency* of buffering.

- There is a minimum number of buffering layers that are required to absorb short-term reductions in available bandwidth for successful recovery. This minimum is directly determined by the reduction in bandwidth that we intend to absorb by buffering.

## 2.4 Optimal Inter-layer Buffer Allocation

Given a draining phase following a single backoff, we can derive the optimal inter-layer allocation that maximizes buffering efficiency. Figure 4 illustrates an optimal buffer allocation and its corresponding draining pattern for a draining phase. Here we assume that the total amount of buffering at the receiver at time  $t_b$  is precisely sufficient for recovery (i.e. area of triangle  $afg$ ) with no spare buffering available at the end of the draining phase.

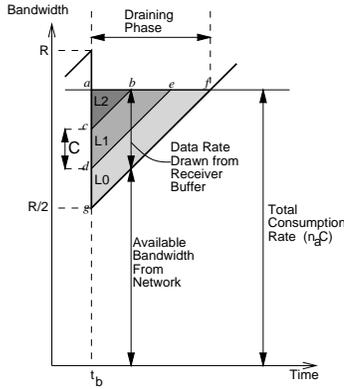


Figure 4: The optimal inter-layer buffer distribution

To justify the optimality of this buffer allocation, consider that the consumption rate of a layer must be supplied either from the network or from the buffer or a combination of the two. If it is supplied entirely from the buffer, that layer's buffer is draining at consumption rate  $C$ . The area of quadrilateral  $defg$  in figure 4 shows the maximum amount of buffer that can be drained from a single layer during this draining phase. If the draining phase ends as predicted, there is no preference on buffer distribution among layers as long as no layer has more than  $defg$  of buffered data. However, if the situation becomes critical due to further backoffs, layers must be dropped. Thus allocating area  $defg$  of buffering to the base layer would ensure that the maximum amount of the buffered data is still usable for recovery, and maximizes buffering efficiency.

By similar reasoning, the next largest amount an additional layer's buffer can contribute is quadrilateral  $bced$ , and this portion of buffered data should be allocated to

$L_1$ , the first enhancement layer, and so on. This approach minimizes the amount of buffered data allocated for higher layers that might be dropped in critical situation and consequently maximizes buffering efficiency.

Expressing this more precisely:

$$n_b = \left\lceil n_a - \frac{R}{2C} \right\rceil ; n_a > \frac{R}{2C}$$

$$n_b = 0 ; n_a \leq \frac{R}{2C}$$

where  $n_b$  is the minimum number of buffering layers

$R$  is the transmission rate (before a backoff)

The optimal amount of buffering for layer  $i$  is:

$$Buf_{i,opt} = \frac{C}{2S}(C(2n_a - 2i - 1) - R) ; i < n_b - 1$$

$$Buf_{n_b-1,opt} = \frac{C}{2S}(n_a C - \frac{R}{2} - n_b C)^2$$

\* For derivation of this equation refer to A.3.

Although we can calculate the optimal allocation of buffered data for the active layers, a backoff may occur at random any time. To tackle this problem, during the filling phase, we incrementally adjust the allocation of buffered data so that the buffer state is always as close to an optimal state as possible.

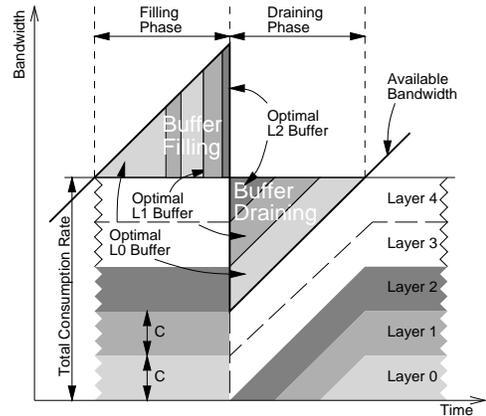


Figure 5: Optimal Buffer sharing

Toward that goal, we assume that a single backoff will occur immediately, and ask the question: "if we keep only the base layer, is there sufficient buffering to survive?". If there is not sufficient buffering, then we fill up the base layer's buffer until there is enough buffering to survive. Then we ask the question: "if we keep only two layers, is there enough buffering to survive with those buffers hav-

ing optimal allocation?”. If there is not enough base layer data, we fill the base layer’s buffer up to the optimal level. Then we start sending L1 data until both layers have the optimal amount of buffering to survive. We repeat this process and increase the number of expected surviving layers until all the buffering layers are filled up to an optimal level such that all active layers can survive from a single backoff. This approach results in a sequential filling pattern among buffering layers.

Figure 5 illustrates the optimal filling and draining scheme. If a backoff occurs exactly at time  $t_b$ , all layers can survive the backoff. Occurrence of a backoff earlier than  $t_b$  results in dropping one or more active layers. However the buffer state is always as close as possible to the optimal state without those layers. If no backoff occurs until adding conditions (section 2.1) are satisfied, a new layer is added and we repeat the sequential filling mechanism.

It is worth mentioning that the server can control the filling and draining pattern by proper fine-grain bandwidth allocation among active layers. Figure 5 illustrates that maximally efficient buffering results in the upper layers being supplied from the network during the draining phase while the lower layers are supplied from their buffers. For example, just after the backoff, layer 2 is supplied entirely from the buffer, but the amount supplied from the buffer decreases to zero as data supplied from the network takes over. Layers 1 and 0 are supplied from the buffer for longer period.

### 3 Smoothness Constraints

In the previous section, we derived an optimal filling and draining scheme based on the assumption that we only buffer to survive a single backoff with all the layers intact. However, examination of Internet traffic indicates that real networks exhibit near-random[3] loss patterns with frequent additional backoffs during a draining phase. Thus, aiming to survive only a single backoff is too aggressive and results in frequent adding and dropping of layers.

#### 3.1 Smoothing

To achieve reasonable smoothing of the add and drop rate, an obvious approach is to refine our adding conditions (in section 2.1) to be more conservative. We have considered the following two mechanisms to achieve smoothing:

- We may add a new layer if the *average* available bandwidth is greater than the consumption rate of the existing layers plus the new layer.
- We may add a new layer if we have sufficient amount of buffered data to survive  $K_{max}$  backoffs with ex-

isting layers, where  $K_{max}$  is a smoothing factor with value greater than one.

Although each one of these mechanisms results in smoothing, the latter not only allows us to directly relate the adding decision to appropriate buffer state for adding, but it can also utilize limited bandwidth links effectively. For example, if there is sufficient bandwidth across a modem link to receive 2.9 layers, the average bandwidth would never become high enough to add the third layer. In contrast, the latter mechanism would send 3 layers for 90% of the time which is more desirable. For the rest of this paper we assume that the only condition for adding a new layer is availability of optimal buffer allocation for recovery from  $K_{max}$  backoffs.

Changing  $K_{max}$  allows us to tune the balance between maximizing the short-term quality and minimizing the changes in quality. An obvious question is “What degree of smoothing is appropriate?” In the absence of a specific layered codec and user-evaluation,  $K_{max}$  can not be analytically derived. Instead it should be set based on real-world user perception experiments to determine the appropriate degree of smoothing needed to not be disturbing to the user. In practice, we probably also want to base  $K_{max}$  on the average bandwidth and RTT since these determine the duration of a draining phase.

#### 3.2 Buffering Revisited

If we delay adding a new layer to achieve smoothing, this affects the way we fill and drain the buffers. Figure 6 demonstrates this issue.

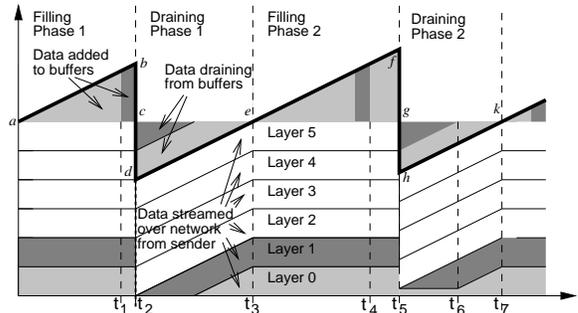


Figure 6: Revised Draining Phase Algorithm

Up until time  $t_3$ , this is the same as figure 5. The second filling phase starts at time  $t_3$ , and at  $t_4$  there is sufficient buffering to survive a backoff. However, because of smoothing purposes, a new layer is not added at this point and we continue buffering data until a backoff occurs at  $t_5$ .

Note that as the available bandwidth increases, the total amount of buffering increases but the required buffering for recovery from a single backoff decreases. At time  $t_5$ , we

have more buffering than we need to survive a single backoff, but insufficient buffering to survive a second backoff before the end of the draining phase. We need to specify how we allocate the extra buffering after time  $t_4$ , and how we drain these buffers after  $t_5$  while maintaining efficiency.

Conceptually, during the filling phase, the server sequentially examines the following steps:

- Step 1:** enough buffer for one backoff with L0 intact.
- Step 2:** enough buffer for one backoff with L0 and L1.
- ...
- Step  $n_a$ :** enough buffer for one backoff with L0 through  $L_{n_a}$  intact.
- Step  $n_a+1$ :** enough buffer for one backoff with L0 through layer  $L_{n_a}$  intact and two backoffs with L0 intact.

At any time in the filling phase we have satisfied one step and are working towards the next step.

When a backoff occurs between steps, in this case between steps  $n_a$  and  $n_a + 1$ , we essentially reverse the filling process. First we calculate between which two steps we're currently located. Then we traverse through the steps in the reverse order to determine which layers must be drained and by how much. In essence, during consecutive filling and draining phases, we traverse this sequence of steps (i.e. optimal buffer states) back and forth such that at any point of time the buffer state is as close to optimal as possible. In the next section, we describe this mechanism in more detail.

## 4 Buffer Allocation with Smoothing

To design an optimal filling and draining mechanisms in the presence of smoothing, we need to know the optimal buffer allocation among layers and the corresponding maximally efficient filling and draining patterns for multiple-backoff scenarios.

The optimal buffer allocation for a scenario with multiple backoffs is not unique because it depends on the time when the additional backoffs occur during the draining phase. If we have knowledge of future loss distribution patterns it might, in principle, be possible to calculate the optimal buffer allocation. In practice such a solution would be excessively complex for the problem it is trying to solve, and rapidly becomes intractable as number of backoffs increases. Let us first assume that only one additional backoff occurs during the draining phase. The possible scenarios are shown in figure 7. This figure illustrates that the optimal buffer allocation for each scenario depends on time of the second backoff, the value of consumption rate ( $n_a C$ ), and the transmission rate before the first backoff.

We can extend the idea of optimal buffer allocation for a single backoff (section 2.4) to each individual scenario. However, an added complexity arises from the fact that different scenarios require different buffer allocations. More

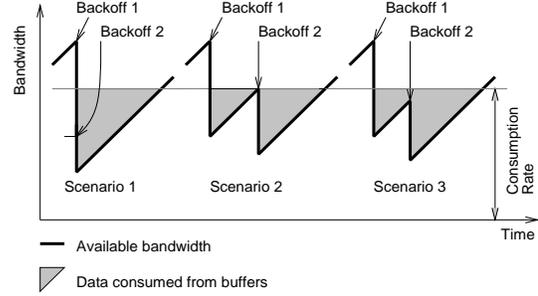


Figure 7: Possible Double-backoff Scenarios

specifically, for an equal amount of total buffering needed for recovery, scenarios 1 and 2 are two extreme cases in the sense that they need the maximum and minimum number of buffering layers respectively. Thus addressing these two extreme scenarios will cover all the intermediate scenarios (e.g. scenario 3) as well.

We need to decide which scenario to consider during the filling phase. We make a key observation here. If the total amount of buffering for scenarios 1 and 2 are equal, having the optimal buffer distribution for scenario 1 is sufficient for recovery from scenario 2, although it is not maximally efficient. However, the converse is not feasible. The higher flexibility in scenario 1 comes from the fact that buffered data for a higher layer can always compensate for lack of buffer in a lower layer, but not vice-versa.

This suggests that during the filling phase for two backoff scenarios, first we consider the optimal buffer allocation for scenario 1 and fill up the buffers in a step by step sequential fashion as described in section 3.2. Once this is achieved, then we move on to consider scenario 2.

### 4.1 Filling Phase with Smoothing

To extend this idea to scenarios of  $k$  backoffs, we need to examine the optimal buffer allocation for scenario 1 and 2 for each successive value of  $k$ . Figure 8 illustrates the optimal buffer state, including the total buffer requirement and its optimal inter-layer allocation in scenario 1 and 2, for different values of  $k$ . Ideally, we would like to fill the buffers during the filling phase such that we traverse through these buffer states in turn. Once  $k$  exceeds  $K_{max}$  (the smoothing factor), then we add a new layer and start the process again with the new sets of optimal buffer states.

Toward this goal, we order these different buffer states in increasing value of total amount of required buffering in figure 9. Thus by traversing this sequence of buffer states, we always work towards the next optimal state that requires more buffering.

Unfortunately this requires us to occasionally drain an

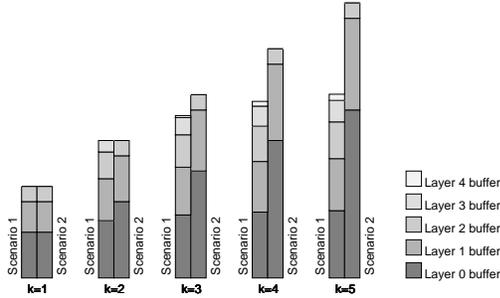


Figure 8: Buffer distributions for k backoffs

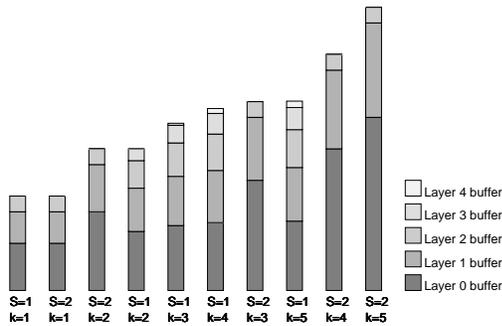


Figure 9: Distributions in increasing order of buffering

existing buffer in order to reach the next state<sup>4</sup>. Two examples of this phenomenon are visible in figure 9:

- Moving from the {scenario 2,  $k=2$ } case to the {scenario 1,  $k=2$ } case involves draining  $L_0$ 's buffer.
- Moving from the {scenario 1,  $k=4$ } case to the {scenario 2,  $k=3$ } case involves draining  $L_3$ 's buffer.

We must not drain any layer's buffer during the filling phase because that buffering provides protection for a previous scenario that we have already passed. Thus we must find the maximally efficient sequence of buffer states *that is consistent with the existing buffering*. To achieve this not only the total amount of required buffering but also per layer buffer requirement must be monotonically increasing as we go to the next buffer state.

The key observation that we mentioned earlier allows us to calculate such a sequence. We recall that higher layer buffers can substitute for lower layer buffers (they're just not as efficient) but not vice-versa. Given this flexibility, the solution is to constraint per layer buffer allocation in each scenario 2 state to be no less than the previous scenario 1 state, and no more than the next scenario 1 state in

<sup>4</sup>In order words, the order of these states based on increasing value of total required buffering is different from their order based on increasing value of per layer buffering

the sequence of states in figure 9. Figure 10 depicts a sequence of maximally efficient buffer states after applying the above constraints where each step in the filling process is numbered. By enforcing this constraint, we can traverse through the buffer states such that buffer allocation for each state satisfies buffer requirement for all the previous states. This implies that both the total amount of buffering and the amount of per layer buffering monotonically increase. Thus the per layer buffering can always be used to aid recovery. Once we have sufficient buffering for recovery from  $K_{max}$  backoffs in both scenarios, a new layer will be added.

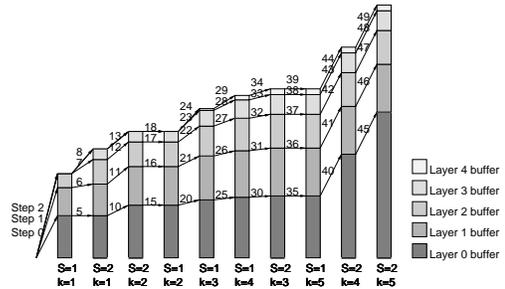


Figure 10: Step-by-step buffer filling

The following pseudocode expresses our per-packet algorithm for ensuring buffer state remains maximally efficient during the filling phase<sup>5</sup>:

```

FUNCTION SendPacket
  S1Backoffs = 0; S2Backoffs = 0
  BufReq1 = 0; BufReq2 = 0

  WHILE (BufReq1 < TotBufAvailable) AND (S1Backoffs <  $K_{max}$ )
    INCREMENT S1Backoffs
    BufReq1 = TotalBufRequired(CurrentRate, Scenario=1,
                               S1Backoffs, ActiveLayers)

  WHILE BufReq2 < TotBufAvailable
    INCREMENT S2Backoffs
    BufReq2 = TotalBufRequired(CurrentRate, Scenario=2,
                               S2Backoffs, ActiveLayers)

  FOR Layer = 1 TO ActiveLayers
    LayerBuf1 = BufRequired(CurrentRate, Scenario=1,
                            S1Backoffs, Layer, ActiveLayers)
    LayerBuf2 = BufRequired(CurrentRate, Scenario=2,
                            S2Backoffs, Layer, ActiveLayers)
    IF (BufReq1 < BufReq2) AND (S1Backoffs <  $K_{max}$ )
      #We're considering scenario 1
      IF (LayerBuf1 > BufAvailable(Layer))
        SendPacketFromLayer(Layer)
      RETURN
    ELSE
      #We're considering scenario 2
      IF (LayerBuf2 > BufAvailable(Layer)) AND
        ((S1Backoffs >  $K_{max}$ ) OR (LayerBuf1 < BufAvailable(Layer)))
        SendPacketFromLayer(Layer)
      RETURN
  
```

<sup>5</sup>The algorithm performs fine-grain bandwidth allocation by assigning the next transmitting packet to a particular layer

$K_{max}$  is the smoothing factor, giving the number of backoffs for which we buffer data before adding a new layer.

The function TotalBufRequired returns the total amount of required buffering for all layers in the scenario in question, given the current send rate, the number of active layers, and the number of backoffs being considered.

Scenario 1

$$Buf_{total} = 0 \quad ; \quad k \leq \log_2 \frac{R}{n_a C}$$

$$Buf_{total} = \frac{1}{2S} \left( n_a C - \frac{R}{2^k} \right)^2 \quad ; \quad k > \log_2 \frac{R}{n_a C}$$

where  $k$  is the number of backoffs being considered

Scenario 2

$$Buf_{total} = 0 \quad ; \quad k \leq \log_2 \frac{R}{n_a C}$$

$$Buf_{total} = \frac{1}{2S} \left( \left( n_a C - \frac{R}{2^{k_1}} \right)^2 + (k - k_1) \left( \frac{n_a C}{2} \right)^2 \right)$$

$$k_1 = \left\lceil \log_2 \frac{R}{n_a C} \right\rceil \quad ; \quad k > \log_2 \frac{R}{n_a C}$$

\* For derivation of this equation refer to A.4.

The function BufRequired returns the maximally efficient amount of required buffering for a particular layer in the scenario of the state we are currently working towards. The input parameters for this function are: the layer number, the current sending rate, the number of active layers, and the number of backoffs being considered.

Scenario 1

$$Buf_{i,opt} = 0; k \leq \log_2 \frac{R}{n_a C}$$

$$Buf_{i,opt} = \frac{C}{2S} \left( C(2n_a - 2i - 1) - \frac{R}{2^k} \right)$$

$$k > \log_2 \frac{R}{n_a C}, \quad 0 \leq i < n_b$$

Scenario 2

$$Buf_{i,opt} = 0; k \leq \log_2 \frac{R}{n_a C}$$

$$Buf_{i,opt} = \frac{C}{2S} \left( \left( C(2n_a - 2i - 1) - \frac{R}{2^{k_1}} \right) + (k - k_1) C(n_a - 2i - 1) \right)$$

$$k > \log_2 \frac{R}{n_a C}, \quad 0 \leq i < n_b$$

\* For derivation of this equation refer to A.5.

## 4.2 Draining Phase with Smoothing

As we traverse through the maximally efficient states, one or more backoffs eventually move us into a draining phase. Given that we incrementally traverse along the maximally efficient *path* through buffer state during the filling phase, we would like to traverse along the same path but in the reverse direction during the draining phase. This approach guarantees that the highest layers buffers are not drained until they are no longer required, and the lowest layers buffers are not drained too early such that the resulting buffer distribution becomes inefficient.

At the start of each step we have the optimal amount of protective buffering for one particular state, and regressively work toward the previous optimal buffer state along the maximally efficient path. However, there is an additional constraint that we cannot drain a layer's buffer faster than the layer consumption rate,  $C$ .

To achieve such a draining pattern, we periodically calculate the draining pattern for a short period of time, during which we expect to drain a certain number of packets. This number is based on the current estimate of slope of linear increase and the current value of consumption rate. We then calculate (using an algorithm similar to the above pseudocode) the previous optimal state along the maximally efficient path that we can survive with the current amount of buffering. Conceptually, then we consider draining data from each layer in turn, starting from the highest layer and

working downwards, such that each layer’s buffering does not drop below its buffer share at the previous optimal step we are draining towards. An added constraint is that we must limit the amount of drained data from a layer to the maximum amount that can be consumed during this period. If the buffer state reaches the previous optimal state being considered before we have allocated all the number of packets that must be drained in this period, then we move on to consider the previous state along the maximally efficient path and so on. We repeat this process until sufficient number of packets for draining during this period are identified. Then we allocate the bandwidth during the period such that each active layer receives the total amount of data that it must consume during this period minus those packets we just allocated to drain during the period.

## 5 Simulation

We have evaluated our quality adaptation mechanism through simulation using bandwidth traces obtained from RAP in the ns2 [ ] simulator and real Internet experiments.

Figure 11 provides a detailed overview of the mechanisms in action. It shows a 40 second trace where the quality-adaptive RAP flow co-exists with 10 Sack-TCP flows and 9 additional RAP flows through an 800 Kb/s bottleneck with 40ms RTT. The smoothing factor was set to 2 so that it provides enough receiver buffering for two backoffs before adding a new layer ( $K_{max} = 2$ ).

Figure 11 shows the following parameters:

- The total transmission rate, illustrating the saw-tooth output of RAP. We have also overlaid the consumption rate of the active layers over the transmission rate to demonstrate the add and drop mechanism.
- The transmission rate broken down into bandwidth per layer. This shows that most of the variation in available bandwidth is absorbed by changing the rate of the lowest layers (shown with the darkest shading).
- The individual bandwidth share per layer. Periods when a layer is being streamed above its consumption rate to build up receiver buffering are clearly visible as spikes in the bandwidth.
- The buffer drain rate per layer. Clearly visible are points where the buffers are used for playout because the bandwidth share is temporarily less than the layer consumption rate.
- The accumulated buffering at the receiver for each active layer.

Graphs in figure 11 clearly demonstrate that the short-term variations in bandwidth caused by congestion control mechanism can be effectively absorbed by receiver buffering. Furthermore playback quality is maximized without risking complete dropouts in the playback due to buffer underflow.

### Smoothing Factor

To examine the impact of smoothing factor on the behavior, we repeated the previous simulation with different value of  $K_{max}$ . Figure 12 shows number of active layers and buffer allocation across active layers for  $K_{max}=2$ ,  $K_{max}=3$ , and  $K_{max}=4$ . Clearly higher values of  $K_{max}$  reduce the number of changes in quality at the expense of increasing the time it takes to first observe the best short-term quality. These graphs also reveal that this manifests itself in two ways as  $K_{max}$  increases: firstly the total amount of buffering is increased, and secondly more of the buffering is allocated for higher layers to cope with the larger variations in available bandwidth as a result of successive backoffs.

### Responsiveness

We have also explored the responsiveness of the quality adaptation mechanism to large step changes in available bandwidth. Figure 13 depicts a RAP trace with the same parameters as figure 11 but a CBR source with a rate equal to half the bottleneck bandwidth is started at  $t=30s$  and stopped at  $t=60s$  and  $K_{max}=4$ . The RAP congestion control mechanism rapidly responds to these changes by reducing the average transmission rate. The quality adaptation mechanism closely follows the changes in bandwidth.  $L_3$  and then  $L_2$  are dropped when bandwidth reduces and then  $L_2$  is added when bandwidth becomes available again. Notice that every layer’s buffer is involved in this process, but the reception of the base layer is never jeopardized. Thus we have satisfied our original design goal of providing smoothing of quality while providing protection to the most critical layers.

### Efficiency

The optimality of our algorithms can be examined from the efficiency of the buffer allocation. that apply when a layer is dropped. The inter-layer buffer allocation is maximally efficient if the following conditions are both satisfied: (i) no data is buffered for a layer that is dropped, and (ii) the layer is only dropped because *total* amount of buffering is insufficient. To quantify the efficiency of our scheme, we have calculated the percentage of remaining buffer for each dropped layer as follows:

$$e = \frac{buf_{total} - buf_{drop}}{buf_{total}}$$

where  $buf_{total}$  and  $buf_{drop}$  denote the total buffering

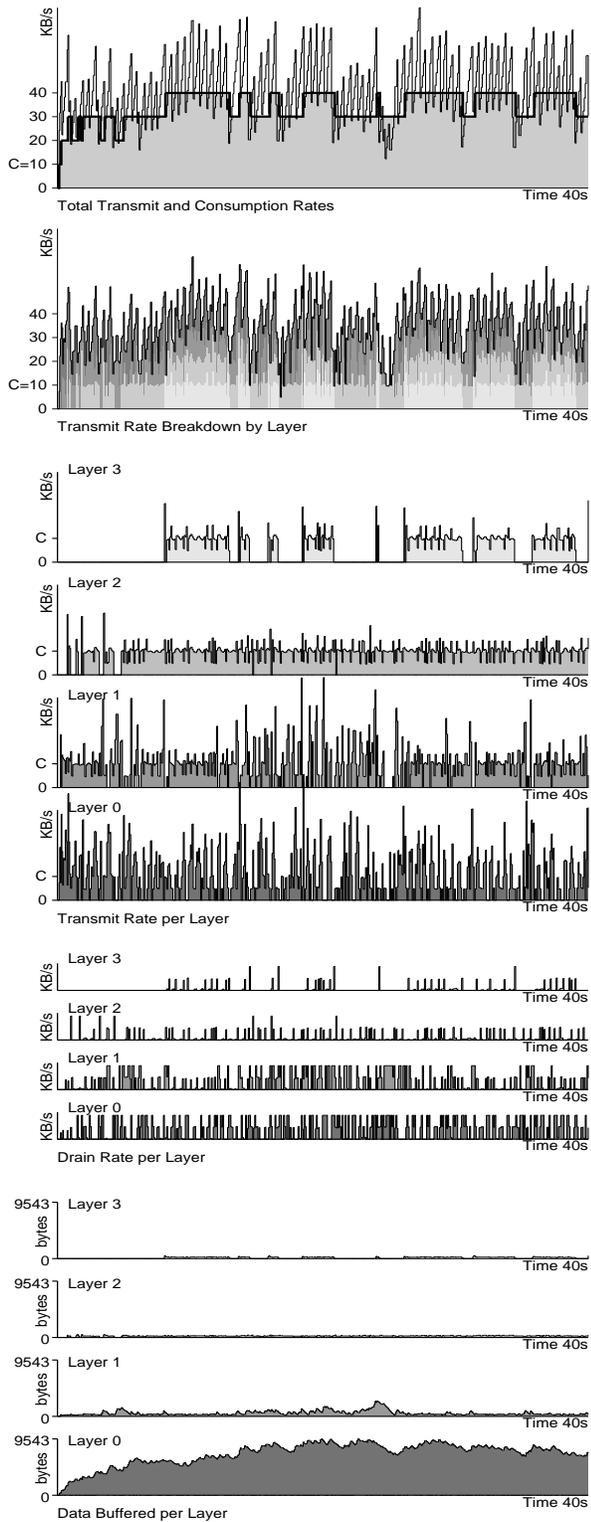


Figure 11: First 40 seconds of  $K_{max}=2$  trace

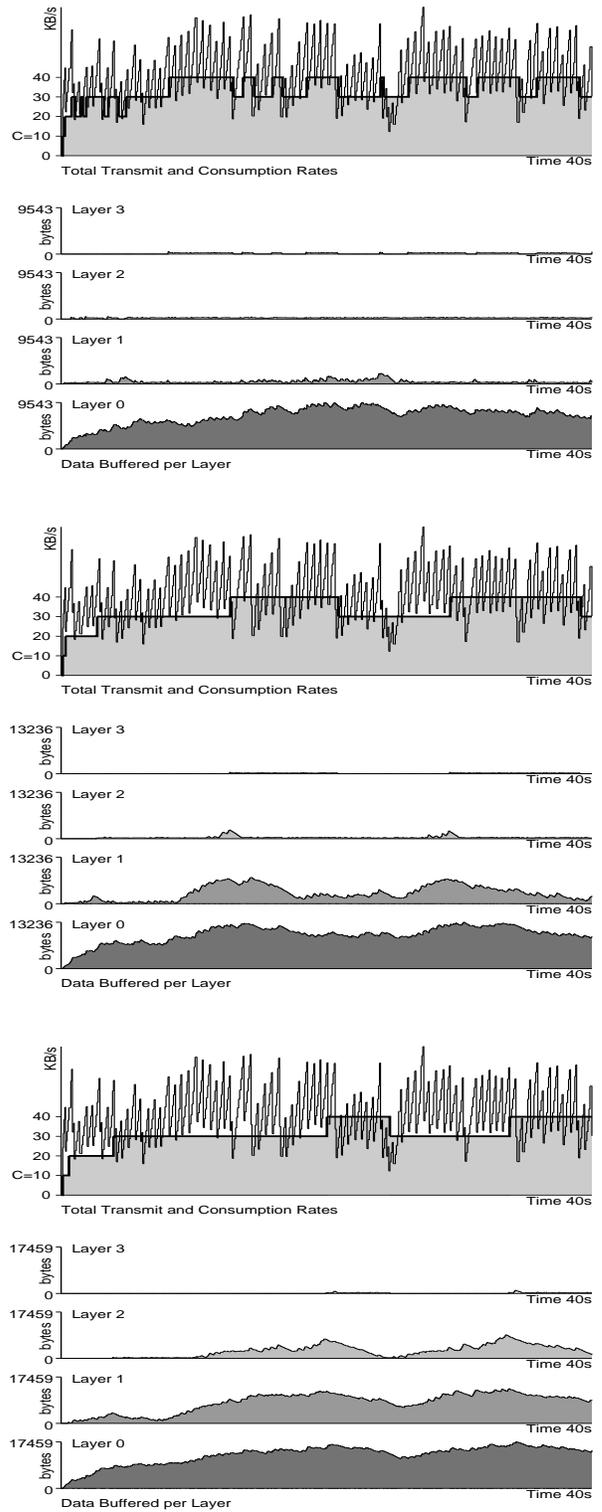


Figure 12: Effect of  $K_{max}$  on buffering and quality

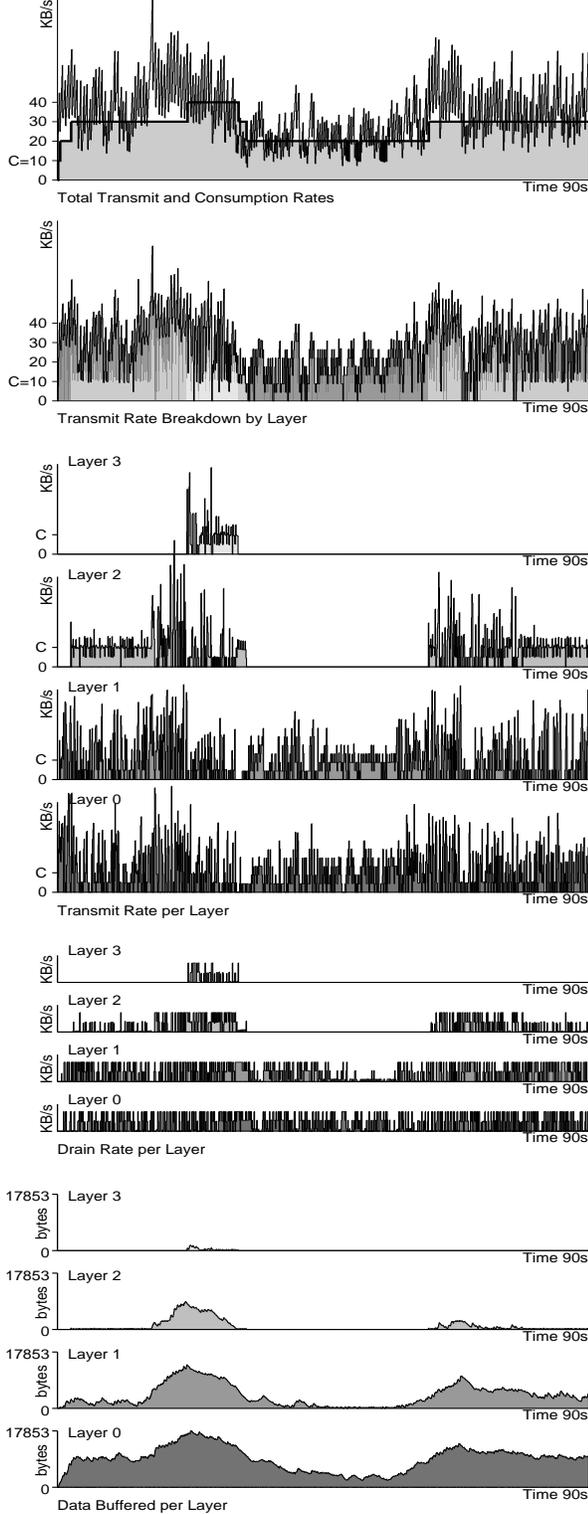


Figure 13: Effect of long-term changes in bandwidth

and the buffer share of the dropping layer. Then we averaged out the value of  $e$  across all drop events during the simulation and use that as an evaluation metric for efficiency.

Table 1 shows these efficiency values for different values of  $K_{max}$  during two test, T1 and T2. T1 is the 10 RAP, 10 TCP test depicted in figures 11 whereas T2 is the 10 RAP, 10 TCP test with a large CBR burst shown in figure 13. These results show that our scheme is very efficient - very little buffered data is still available in a layer that is dropped.

	$K_{max}=2$	$K_{max}=3$	$K_{max}=4$	$K_{max}=5$	$K_{max}=8$
T1	99.77%	99.97%	99.84%	99.85%	99.99%
T2	99.15%	99.81%	99.92%	99.80%	96.07%

Table 1

Table 2 shows the percentage of drops due to poor buffer distribution in test T1 and T2. These are drops that would not have happened if the amount of buffered data that was at the receiver had been distributed differently. Our mechanism is completely optimal in this respect for the T1 tests, and performs fairly well for the T2 case. Clearly the mechanism becomes less optimal as  $K_{max}$  increases. The higher the value of  $K_{max}$ , the more buffering is allocated for higher layers. Hence there is a higher probability of dropping the highest layer with some buffering specially after sudden drops in available bandwidth such as when CBR source appears. In essence, conservative buffering (i.e. higher  $K_{max}$ ) enables the server to cope with wider variations in bandwidth. However sudden drops of bandwidth in these situations results in lower efficiency.

	$K_{max}=2$	$K_{max}=3$	$K_{max}=4$	$K_{max}=5$	$K_{max}=8$
T1	0%	0%	0%	0%	0%
T2	2.4%	0%	4.8%	11%	-

Table 2

## 6 Related Work

Receiver-based layered transmission have been discussed in the context of multicast video[1, 11, 21] to accommodate heterogeneity while performing coarse-grain congestion control. This differs from our approach that allows fine-grain congestion control for unicast delivery with no step-function changes in transmission rate.

Merz et al. [13] present an iterative approach for sending high bandwidth video through a low bandwidth channel. They suggest segmentation methods that provide the flexibility to playback a high quality stream over several iterations, allowing the client to trade startup latency for quality.

Work in [7, 16, 17] discuss congestion control for streaming applications and focusing on rate adaptation. However, variations of transmission rate in a long-lived session could result in client buffer overflow or underflow. Quality adaptation is complementary for these scheme because it prevents buffer underflow or overflow while effectively utilizing the available bandwidth.

Feng et al. [4] propose an adaptive smoothing mechanism combining bandwidth smoothing with rate adaptation. The send rate is shaped by dropping low-priority frames based on prior knowledge of the video stream. This is meant to limit quality degradation caused by dropped frames but the quality variation cannot be predicted.

Unfortunately, technical information for evaluation of popular applications such as RealVideo G2 [14] is unavailable.

## 7 Conclusions and Future Work

We have presented a quality adaptation mechanism to bridge the gap between short-term changes in transmission rate caused by congestion control and the need for stable quality in streaming applications. We exploit the flexibility of layered encoding to adapt the quality along with long-term variation in available bandwidth. The key issue is appropriate buffer distribution among the active layers. We have described a near-optimal mechanism that dynamically adjusts the buffer distribution as the available bandwidth changes by careful allocation of the bandwidth among the active layers. Furthermore, we introduced a smoothing parameter that allows the server to trade short-term improvement for long-term smoothing of quality. The strength of our approach comes from the fact that we did not make any assumptions about loss patterns or available bandwidth. The server adaptively changes the receiver's buffer state to incrementally improve its protection against short-term drops in bandwidth in an efficient fashion. Our simulation and experimental results reveal that with a small amount of buffering the mechanism can efficiently cope with short-term changes in bandwidth due to AIMD congestion control. The mechanism can rapidly adjust the quality of the delivered stream to utilize the available bandwidth while preventing buffer overflow or underflow. Furthermore, by increasing the smoothing factor, the frequency of quality variation is effectively limited.

Given that buffer requirements for quality adaptation are not large, we believe that these mechanism can also be deployed for non-interactive *live* sessions where the client can tolerate a short delay in delivery.

We plan to extend the idea of quality adaptation to other congestion control schemes that employ AIMD algorithms and investigate the implications of the details of rate adap-

tion on our mechanism. We will also study quality adaptation with a non-linear distribution of bandwidth among layers.

Finally, quality adaptation provides a perfect opportunity for proxy caching of multimedia streams which we plan to examine. The proxy would cache each stream and missing pieces that are likely to be needed would be pre-fetched in a demand-driven fashion.

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## A Derivation of formulae

### A.1 Adding Condition 2

The left hand side of this equation is the total amount of buffering for all layers, and the right hand side is the area of triangle  $cde$  assuming that consumption rate is  $(n_a + 1)C$  (figure 3 shows a situation when the consumption rate is  $n_a C$ ). Given that  $L_{ij}$  is the length in figure 3 from  $i$  to  $j$ , then the area,  $A$ , of the triangle is:

$$A = \frac{L_{cd} * L_{ec}}{2}, \text{ Slope } S = \frac{L_{ce}}{L_{cd}} \Rightarrow A = \frac{L_{ce}^2}{2S} (1)$$

In this case  $L_{ce} = R - (n_a + 1)C$  because we are in the filling phase.

### A.2 The Dropping Mechanism

The dropping mechanism simply calculates the area of triangle  $cde$  in figure 3 based on equation (1). We solve that equation to find the value of  $n_a$  that requires buffering less or equal to the aggregate amount of buffering. Note that during the draining phase, the transmission rate is always less than the consumption rate ( $R < n_a C$ ).

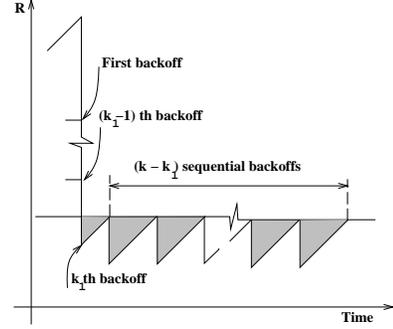


Figure 14:  $Buf_{total}$  for scenario 2

## A.3 Optimal Inter-layer Buffer Allocation

The optimal buffer share for each layer in a single backoff scenario (figure 4) can be viewed as an area between two triangles. For example, the area of quadrilateral  $bced$  is the difference of the area of two triangles  $ade$  and  $acb$ .

The area of each triangle can be calculated using equation (1). Note that  $L_{cd} = L_{dg} = C$  but  $L_{ac} \leq C$ . Thus we have an exceptional case for the optimal buffer share of the highest buffering layer ( $n_b - 1$ ).

### A.4 Derivation of $Buf_{total}$

Note that there is a minimum number of backoffs that are needed to drop the transmission rate below the consumption rate, otherwise the total required buffering is zero because we do not experience a draining phase. The value of  $Buf_{total}$  for scenario 1 can be calculated similarly to a single backoff scenario (figure 3) using equation (1), except that the transmission rate after  $k$  backoffs is  $R/2^k$ .

Figure 14 illustrates the calculation of  $Buf_{total}$  for scenario 2.  $k_1$  is the minimum number of backoffs that are needed to drop the transmission rate below the consumption rate. The remaining  $(k - k_1)$  backoffs then occur in sequential fashion.  $Buf_{total}$  is equal to the shaded area in figure 14. Using equation (1), we can calculate the area of the first triangle and one of the subsequent triangles because they have the same size.

### A.5 Derivation of $Buf_{i,opt}$

The value of  $Buf_{i,opt}$  can be calculated by extending the idea of optimal buffer allocation for a single backoff (section 2.4) to scenarios 1 and 2. For scenario 1, we only need to replace  $R$  with  $\frac{R}{2^k}$  because we simply have a bigger triangle.

Considering figure 14 for scenario 2, we can calculate the optimal buffer share of layer  $i$  for each individual triangle using the optimal buffer allocation for a single backoff

and use the cumulative buffer share as optimal allocation. Note that for the last  $(k - k_1)$  backoffs, we can calculate the optimal share for one triangle and simply multiply it by  $(k - k_1)$ .