Mathematics of the Simple Camera

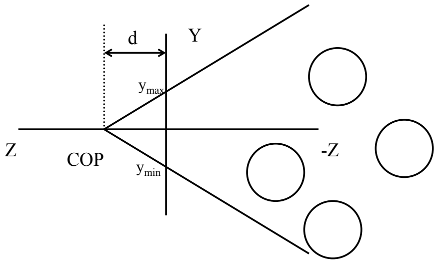
Overview

- Simple Camera
  - Scenes with spheres
  - COP on +z

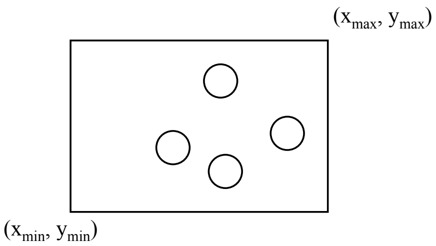
COP = Centre Of Projection

Simple Camera (Cross Section)

COP = Centre Of Projection

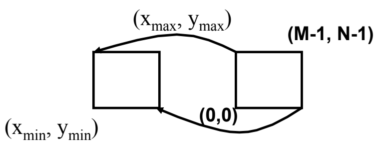


View From the Camera



Forming the Rays

- Map screen pixels (M by N window) to points in camera view plane



Forming the Rays

- Consider pixel i,j
- It corresponds to a rectangle

  width = (xmax-xmin)/M

  height = (ymax-ymin)/N
- Our ray goes through the centre of the pixel
- Thus the ray goes through the point

(xmin + width*(i+0.5), ymin + height*(j+0.5), 0.0)

Forming the Rays

- Thus the ray from the COP through pixel i,j is defined by

  p(t) = (x(t), y(t), z(t)) =
  (t*(xmin + width*(i+0.5)),
  t*(ymin + height*(j+0.5)),
  d - t*d)

Ray Casting

- Intersection of Sphere and line (sphere at origin)
- Substitute the ray equation in the sphere equation and solve!
- Get an equation in t of the form

$$At^2 + 2Bt + C = 0$$

Ray Casting

If $B^2 - AC < 0$ then the ray doesn't intersect the sphere.
If $B^2 - AC = 0$ the ray is tangent to the sphere
If $B^2 - AC > 0$ then there are two roots given by

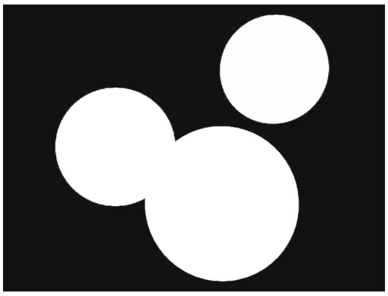
$$t = (-B \pm \sqrt{(B^2 - AC)})/A$$

chose the lowest value one (the one closest to the COP)

Ray Casting

- Intersection of Sphere and line (general case)
  - Sphere is centred at (a,b,c)
  - Translate the start of the ray by (-a,-b,-c)
  - Proceed as before

The Image - Detection



Conclusions

- We can now draw images
  - Forming rays from the camera
  - Intersecting those rays with objects (spheres) in the scene
- But
  - No colour – merely binary detection operation
  - Camera is static - at the moment we must move the objects in front of the camera to be able to see them
  - Need more interesting scenes!