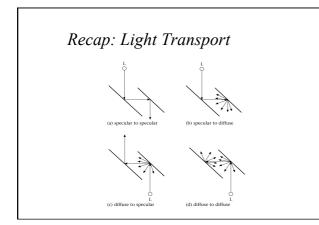


#### Overview

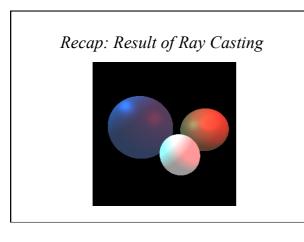
- Recursive Ray Tracing
- Shadow Feelers
- Snell's Law for Refraction
- When to stop!

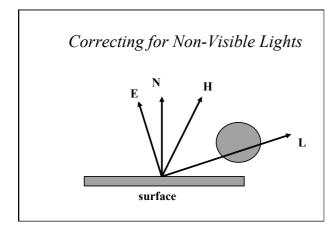


Recap: Local Illumination  

$$I_{r} = k_{d}I_{a} + \sum_{j=1}^{M} I_{i,j} \left( k_{d} (n \cdot l_{j}) + k_{s} (h_{j} \cdot n)^{m} \right)$$

• Ambient, diffuse & specular components  
• The sum is over the specular and diffuse  
components for each light

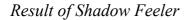






$$I_r = k_a I_a + \sum_{j=1}^{M} S_j I_{i,j} \left( k_d \left( n \cdot l_j \right) + k_s \left( h_j \cdot n \right)^n \right)$$

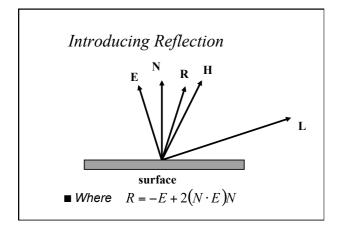
- Where Sj is the result of intersecting the ray L with the scene objects
- Note consider your intersection points along the ray L carefully
  - Hint they might be beyond the light!





# Recursive Ray-Tracing

- We can simulate specular-specular transmission elegantly by recursing and casting secondary rays from the intersection points
- We must obviously chose a termination depth to cope with multiple reflections

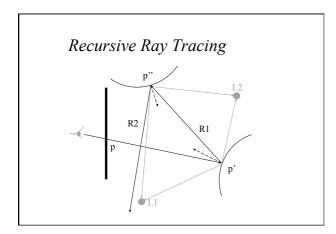




Computing Reflectance

$$I_r = I_{local} + k_r I_{r'}$$

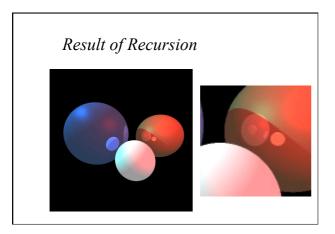
- Where I<sub>local</sub> is computed as before
- Ray r' is formed from intersection point and the direction R and is cast into the scene as before

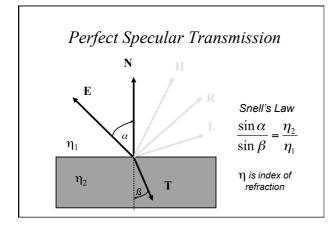


#### Pseudo Code

 $\begin{array}{l} \mbox{Color RayTrace}(Point p, Vector direction, int depth) \\ \mbox{Point pd} & /* Intersection point */ \\ \mbox{Boolean intersection} \\ \mbox{if (depth > MAX) return Black} \\ \mbox{intersect}(p, direction, \&pd, & intersection) \\ \mbox{if (lintersection) return Background} \\ \mbox{I}_{local} = k_a I_a + I_p.v.(k_d(n.l) + k_s.(h.n)^m) \\ \mbox{return } I_{local} + k_r * RayTrace(pd, R, depth+1) \\ \mbox{} \end{array}$ 

Normally  $k_r = k_s$ 





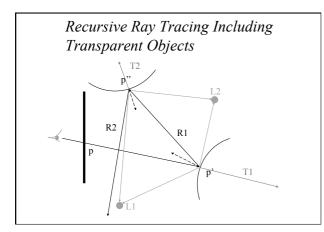


Using Snell's Law  

$$\frac{\sin \alpha}{\sin \beta} = \frac{\eta_2}{\eta_1} = \eta_{21}$$
• Using this law it is possible to show that:  

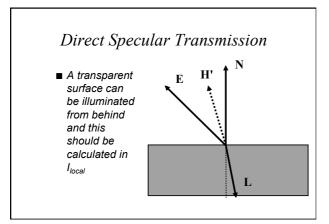
$$T = -\eta_{12}E + N\left(\eta_{12} \cdot \cos \alpha - \sqrt{1 + \eta_{12}^2 \cdot (\cos^2 \alpha - 1)}\right)$$
• Note that if the root is negative then total internal reflection has occurred and you just reflect the vector as normal

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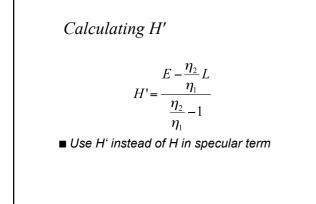


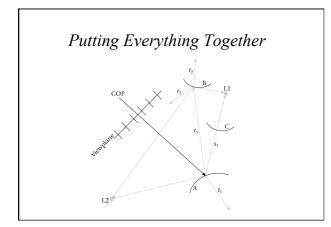
## New Pseudo Code

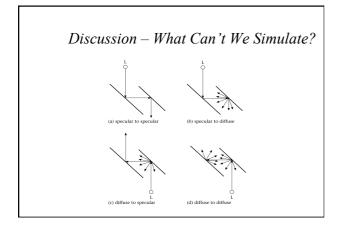
 $\begin{array}{l} Color \ RayTrace(Point \ p, \ Vector \ D, \ int \ depth) \ \{ \begin{array}{l} Point \ pd \ \ /^* \ Intersection \ point \ */ \\ Boolean \ intersection \ point \ */ \\ Boolean \ intersection \ if \ (depth > MAX) \ return \ Black \ intersect(p, \ direction, \ &pd, \ &intersection) \ if \ (lintersection) \ return \ Background \ \\ I_{local} = k_a I_a + I_p.v.(k_o(n.l) + k_s.(h.n)^m) \ return \ I_{local} + k_r \ RayTrace(pd, \ R, \ depth+1) + \\ k_t \ RayTrace(pd, \ T, \ depth+1) \ \end{array}$ 













#### Remark

- Specular and transmission only
   What should be added to consider diffuse reflection?
- Why it's expensive
- Intersection of rays with polygons (90%)
- How to reduce the cost?
  - Reduce the number of rays
  - Reduce the cost on each ray
    - First check with bounding box of the object
    - Methods to sort the scene and make it faster

## Summary

- Recursive ray tracing is a good simulation of specular reflections
- We've seen how the ray-casting can be extended to include shadows, reflections and transparent surfaces
- However this is a very slow process and still misses some types of effect!