

## Planes, Polygons and Objects

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## Overview

- Polygons
- Planes
- Creating an object from polygons

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## No More Spheres

- Most things in computer graphics are not described with spheres!
- Polygonal meshes are the most common representation
- Look at how polygons can be described and how they can be used in ray-casting

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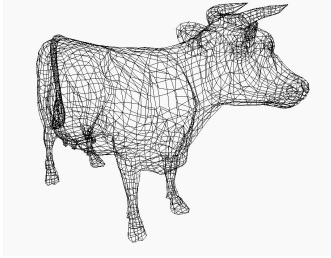
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## Polygonal Meshes



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## Polygons

- A polygon (face) Q is defined by a series of points

$$[p_0, p_1, p_2, \dots, p_{n-1}, p_n]$$
$$p_i = (x_i, y_i, z_i)$$

- The points are must be **co-planar**
- 3 points define a plane, but a 4th point need not lie on that plane

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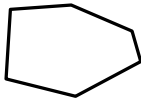
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## Convex, Concave

- Convex



- Concave



- CG people dislike concave polygons
- CG people would prefer triangles!!
  - Easy to break convex object into triangles, hard for concave

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### Equation of a Plane

$$ax + by + cz = d$$

- a,b,c and d are constants that define a unique plane and x,y and z form a vector P.

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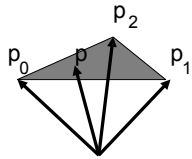
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### Deriving a,b,c & d (1)



- The cross product  $n = (p_1 - p_0) \times (p_2 - p_0)$  defines a **normal** to the plane
- There are two normals (they are opposite)
- Vectors in the plane are **all** orthogonal to the plane normal vector

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### Deriving a,b,c & d (2)

- So every  $p-p_0$  is normal to  $n$  therefore

$$n \cdot (p - p_0) = 0 \quad \Leftrightarrow$$

$$n \cdot p = n \cdot p_0$$

- But if  $n = (a,b,c)$  and  $p = (x,y,z)$  and  $d = n \cdot p_0 = n_1 \cdot x_0 + n_2 \cdot y_0 + n_3 \cdot z_0$

$$\Rightarrow ax + by + cz = d$$

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## Half-Space

- A plane cuts space into 2 **half-spaces**
- Define  $l(x, y, z) = ax + by + cz - d$
- If  $l(p) = 0$ 
  - point on plane
- If  $l(p) > 0$ 
  - point in **positive** half-space
- If  $l(p) < 0$ 
  - point in **negative** half-space

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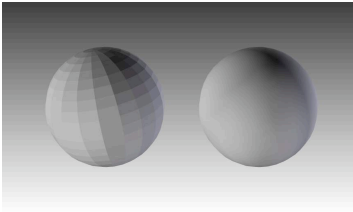
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## Vertex Normals



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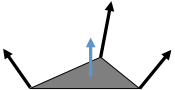
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## Vertex Normals

- Compute/store a normal at each vertex
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- Improves shading
  - Compute by averaging neighbouring "face normals" (blue)

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## Polyhedra

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## Polyhedra

- Polygons are often grouped together to form polyhedra
  - Each edge connects 2 vertices and is the join between two polygons
  - Each vertex joins 3 (or more) edges
  - No faces intersect
- #Vertices - #Edges + #Faces = 2
  - For cubes, tetrahedra, cows etc...

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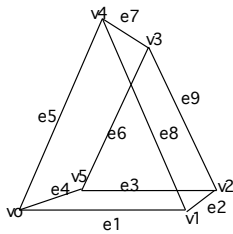
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## Example Polyhedron



- F0=v0 v1 v4
- F1=v5 v3 v2
- F2=v1 v2 v3 v4
- F3=v0 v4 v3 v5
- F4=v0 v5 v2 v1
  
- V=6, F=5, E=9
- V-E+F=2

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### Representing Polyhedron (1)

- Exhaustive (array of vertex lists)
  - faces[1] = (x0,y0,z0),(x1,y1,z1),(x3,y3,z3)
  - faces[2] = (x2,y2,z2),(x0,y0,z0),(x3,y3,z3)
  - etc ....
- Very wasteful since same vertex appears at 3 (or more) points in the list
  - Is used a lot though!

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### Representing Polyhedron (2)

- Indexed Face Set
  - Vertex array
    - vertices[0] = (x0,y0,z0)
    - vertices[1] = (x1,y1,z1)
    - etc ...
  - Face array (list of indices into vertex array)
    - faces[0] = 0,2,1
    - faces[1] = 2,3,1
    - etc ...

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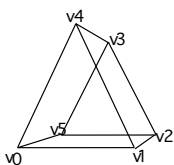
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### Vertex order matters



- Polygon v0,v1,v4 is NOT equal to v0,v4,v1
- The normal points in different directions
- Usually a polygon is only visible from points in its positive half-space
- This is known as **back-face culling**

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### Representing Polyhedron (3)

- Even Indexed face set wastes space
  - Each face edge is represented twice
- Winged edge data structure solves this
  - vertex list
  - edge list (vertex pairs)
  - face list (edge lists)

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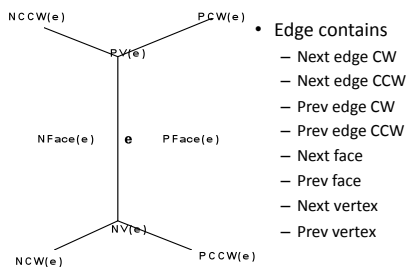
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### The Edge List Structure



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### Advantages of Winged Edge

- Simple searches are rapid
  - find all edges
  - find all faces of a vertex
  - etc...
- Complex operations
  - polygon splitting is easy (LOD)
  - silhouette finding
  - potentially efficient for hardware
  - etc...

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## Building the WE

- Build indexed face set
- Traverse each face in CCW order building edges
  - label p and n vertices, p and n faces and link previous CCW edge
    - we fill in next CCW on next edge in this face
    - we fill in next CW and prev CW when traversing the adjacent face.

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## Exercises

- Make some objects using index face set structure
- Verify that  $V-E+F=2$  for some polyhedra
- Think about testing for intersection between a ray and a polygon (or triangle)

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## Recap

- We have seen definition of planes and polygons and their use in approximating general shapes
- We have looked at data structures for shapes
  - Indexed face sets
  - Winged edge data sets
- The former is easy to implement and fast for rendering
- The latter is more complex, but makes complex queries much faster
- It is possible, though we haven't shown how, to convert between the two

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