The evolution of pairing-based zero-knowledge proofs

2nd ZKProof Workshop 2019

Jens Groth, DFINITY

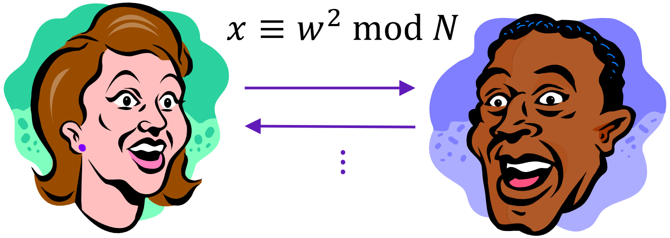
Recurring motifs

- Language
  - Types of statements we can prove
- Security
  - Underpinning assumptions
  - Unconditional soundness vs unconditional zero-knowledge
- Efficiency
  - Prover computation, verifier computation, interaction, setup size, succinctness

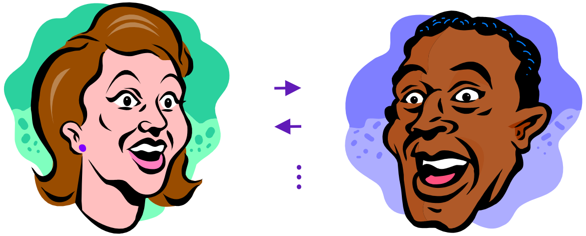
Pre-pairing

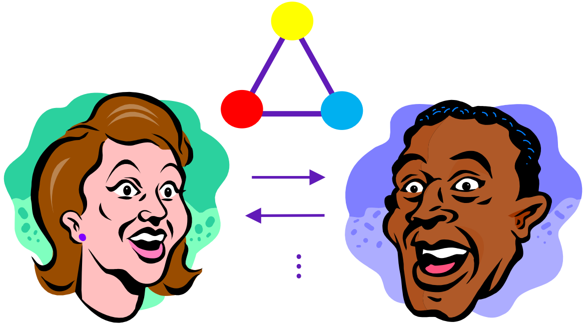
Abiogenesis

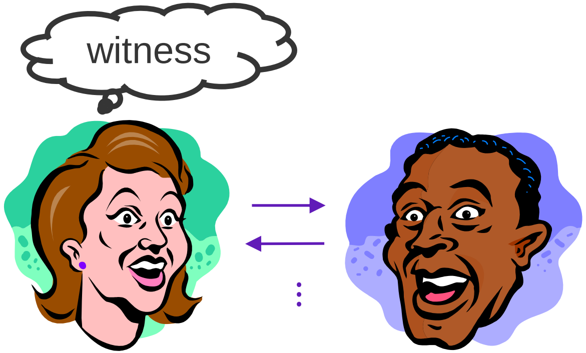
- Goldwasser-Micali-Rackoff 85/89
  - Defined zero-knowledge proofs: complete, sound, zero knowledge
- Constructed interactive zero-knowledge proof systems
  - Quadratic residuosity as well as quadratic non-residuosity

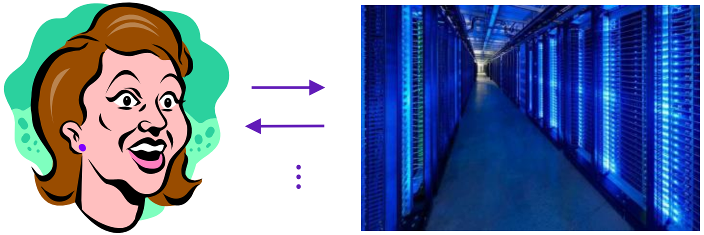


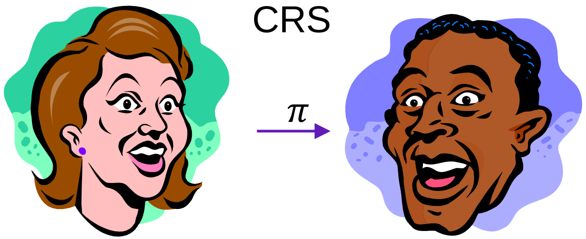
Cambrian explosion









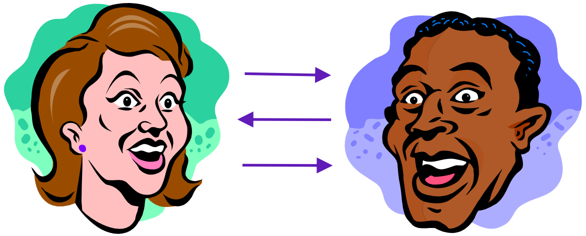


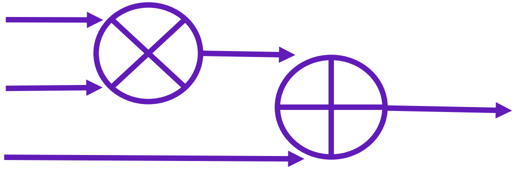
Biodiversity

- Language
  - [GMW86/91] All of NP (Graph 3-colorability)
  - [FLS90/99] Hamiltonicity, [BCC86] Boolean circuit SAT, …
- Security
  - [GMW86/91] One-way functions suffice for computational zero-knowledge
  - [(BC86,C86)/BCC88] Zero-knowledge against unbounded adversaries
- Efficiency
  - [BFM88] Non-interactive zero-knowledge proofs in CRS model
  - [Kil92] Succinct interactive proofs
- Properties
  - [TW87,FFS88,FS90,BG92] Proof of knowledge

Devonian explosion

- [Sch90/91] Discrete log based signatures
- [CDS94,Cra96] Σ-protocols
- [CD98] Arithmetic circuit satisfiability





Proof system	Communication
[CD98]	$O(N)$ elements
[Gro09]	$O(\sqrt{N})$ elements
[BCCGP16]	$O(\log N)$ elements

[BBBPWM18] Bulletproofs

- [CS98] Non-interactive designated verifier proofs for linear relations

Pairing-based NIZK proofs

Adaptive radiation?

ZK proof for NP-complete languages	Computational zero knowledge	Unconditional zero knowledge
Interactive	Goldreich-Wigderson-Micali 1986	Brassard-Crépeau 1986
Non-interactive	Blum-Feldman-Micali 1988	?

- Are there non-interactive zero-knowledge proofs with everlasting privacy?
    - Fiat-Shamir suggested answer could be yes. But based on standard assumptions?

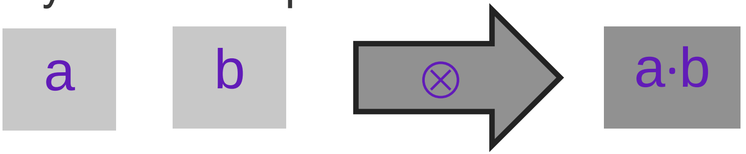
Exogenous genetic material

[BGN05] Pairing-based double-homomorphic encryption scheme

- Symmetric composite order groups of order $N = pq$ with $e: G \times G \to G_T$
- Public key (g, h) where $\mathrm{ord}(g) = N$ and $\mathrm{ord}(h) = q$
  - Assumption: Hard to decide whether h has full order N or lives in subgroup of order q
- Encrypt small integer m under public key (g, h) as $g^m h^r$
- Additively homomorphic

$$g^a h^r \cdot g^b h^s = g^{a+b} h^{r+s}$$

- Multiplicatively homomorphic



$$e\left(g^a h^r, g^b h^s\right)$$
$$= e(g,g)^{ab} e(g^{(a+b)} h^{rs}, h)$$

Transformation

- [BGN05] + NIZK $\Longrightarrow$ NIZK with perfect and everlasting zero knowledge

ZK proof for NP-complete languages	Computational zero knowledge	Unconditional zero knowledge
Interactive	Goldreich-Wigderson-Micali 1986	Brassard-Crépeau 1986
Non-interactive	Blum-Feldman-Micali 1988	Groth-Ostrovsky-Sahai 2006

- Main idea to prove circuit satisfiability
  - Commit to wire values in circuit as $g^a h^r, g^b h^s, g^c h^t, \ldots$
  - Verifier can easily add committed elements $g^a h^r \cdot g^b h^s = g^{a+b} h^{r+s}$
  - Prover can demonstrate multiplicative relation $e\left(g^a h^r, g^b h^s\right) = e\left(g^c h^t, g\right) e(\pi, h)$
  - Perfectly sound if $\mathrm{ord}(h) = q$ and perfectly zero knowledge if $\mathrm{ord}(h) = N$

Clade

- [GOSb] NIZK proofs based on prime-order groups with pairings
  - Also setup-free non-interactive witness-indistinguishable proofs
- [Gro06] NIZK proofs for practical language (pairing-product equations)
  - [BW06,BW07] NIZK proofs in pairing-based group signatures
- [GS08] Efficient proofs for practical languages
  - Practical language captures correctness of generic computation (here for symmetric pairing)
  - Statement: a public set of equations using generic operations and public values
    - Pairing-product equations $e(A, X) \cdot e(X, Y)^{\gamma} = 1$
    - Multi-exponentiation equations $X^{a} \cdot B^{y} \cdot X^{z} = T$
    - Quadratic equations $1 \cdot y + z \cdot z = t$
  - Witness: Secret field elements y, z and group elements X, Y satisfying all equations

Fitness

Statement: Here is a ciphertext and a document. The ciphertext contains a digital signature on the document.

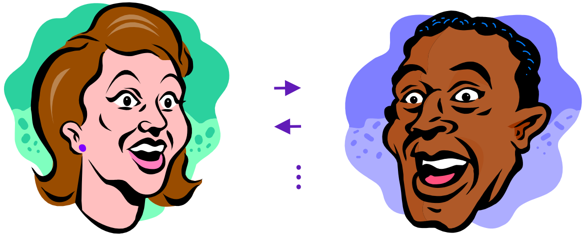
TB	Circuit SAT	Practical language
Inefficient	Kilian-Petrank 1994	Groth 2006
Efficient	Groth-Ostrovsky-Sahai 2006	Groth-Sahai 2008

KB

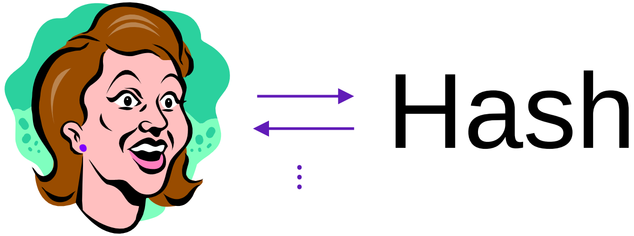
Pairing-based SNARKs

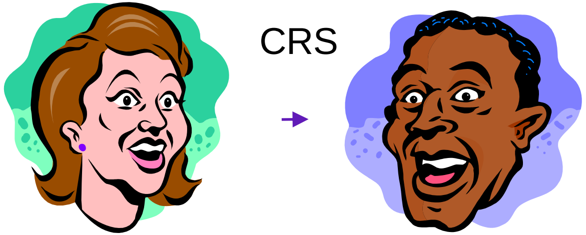
Pterygota

[Kil92] Probabilistically checkable proofs
→ succinct zero-knowledge arguments



[FS86] Hash + public-coin proofs
→ non-interactive arguments





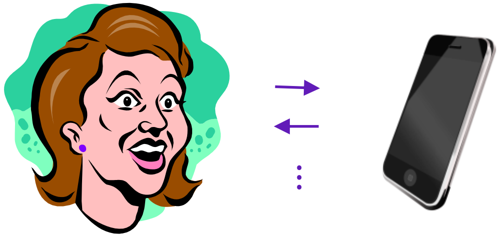
[Mic00(94)] Succinct non-interactive arguments

Succinctness

- [GW11] SNARG = Succinct non-interactive argument
  - Need non-falsifiable assumptions to build SNARGs
- [BCCT12] SNARK = Succinct non-interactive argument of knowledge
  - …and zk-SNARK = zero-knowledge SNARK
  - Added ease of verification requirement - verifier time polynomial in statement size

Succinct = almost zero-knowledge

Succinct = easy verification



Pairing-based SNARKs

- [Gro10] Short pairing-based non-interactive zero-knowledge arguments
  - Preprocessing zk-SNARK, but the term had not been coined yet ☺

Common reference string	Proof size
$O(N^2)$ elements	$O(1)$ elements
$O(N^{\frac{2}{3}})$ elements	$O(N^{\frac{2}{3}})$ elements

And $O(N^2)$ computation for prover and verifier, where N is size of Boolean circuit or arithmetic circuit

- Power knowledge of exponent assumption
  - CRS includes group elements $g, g^x, \dots, g^{x^q}, g^\alpha, g^{\alpha x}, \dots, g^{\alpha x^q}$
  - Can only compute $c, \hat{c} = c^\alpha$ by using *known* $a_0, \dots, a_q$ in $c = \prod \left(g^{x^i}\right)^{a_i}$ and $\hat{c} = \prod \left(g^{\alpha x^i}\right)^{a_i}$
  - Symmetric pairing, so can verify $\hat{c} = c^\alpha$ by checking $e(c, g^\alpha) = e(\hat{c}, g)$

Balancing polynomials in the exponent

- Core idea
  - For multiplication gates, suppose we have commitments $A = \prod_{i=0}^{n-1} \left(g^{x^i}\right)^{a_i}, B = \prod_{j=0}^{n-1} \left(g^{x^{jn}}\right)^{b_j}$

$$e(A,B) = e\left(g, \prod_{i,j} \left(g^{x^{i+jn}}\right)^{a_i b_j}\right) = e(g,g)^{\sum_{i=0}^{n-1} a_i b_i x^{i(1+n)} + \sum_{i \neq j} a_i b_j x^{i+jn}}$$

  - So given commitment $C = \prod_{i=0}^{n-1} \left(g^{x^i}\right)^{c_i}$ and constant commitment $D = \prod_{i=0}^{n-1} \left(g^{x^{jn}}\right)^{-1}$

$$e(A,B)e(C,D) = e(g,g)^{\sum_{i=0}^{n-1}(a_i b_i - c_i) x^{i(1+n)}} \cdot e(g,g)^{\sum_{i \neq j} \text{something} \cdot x^{i+jn}} = e(g,\pi)$$

    where the left-hand side disappears if and only if $c_i = a_i b_i$
  - The main thrust of the proof system is to design the CRS so the proof π cannot have $i = j$ coefficients, but can eliminate any $i \neq j$ coefficients of the in-the-exponent polynomials

Intelligent design

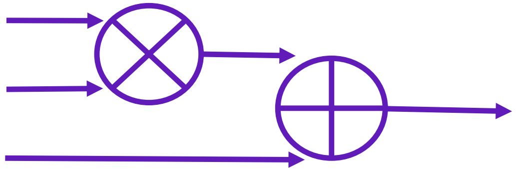
	Idea	CRS	Proof size
[Lip13]	Sum-free sets	$N^{1+o(1)}$ elements	$O(1)$ elements
[GGPR13]	QSP and QAP	$O(N)$ elements	$O(1)$ elements

- Quadratic span programs and quadratic arithmetic programs
  - Instead of coefficients of polynomials, look at distinct evaluation points $r_1, \ldots, r_n$
  - Lagrange polynomials $\ell_1(x), \ldots, \ell_n(x)$ such that $l_i(r_k) = 1$ if $i = k$ and otherwise $\ell_i(r_k) = 0$
  - Given commitments $A = g^{\sum a_i \ell_{i(x)}}, B = g^{\sum b_j \ell_j(x)}, C = g^{\sum c_i \ell_i(x)}$ and constant $D = g^{-\sum \ell_i(x)}$

    $$e(A,B)e(C,D\ ) = e(g,g)^{\sum (a_i b_i - c_i)\ell_i^2(x)} \cdot e(g,g)^{\sum_{i \neq j} \text{something} \cdot \ell_i(x)\ell_j(x)} = e\left(\pi, g^{\prod (x - r_i)}\right)$$

    if and only if $a_i b_i = c_i$, since $\ell_i^2(r_k) = 1$ if and only if $i = k$, and $\ell_i(r_k)\ell_j(r_k) = 0$ in all r_k

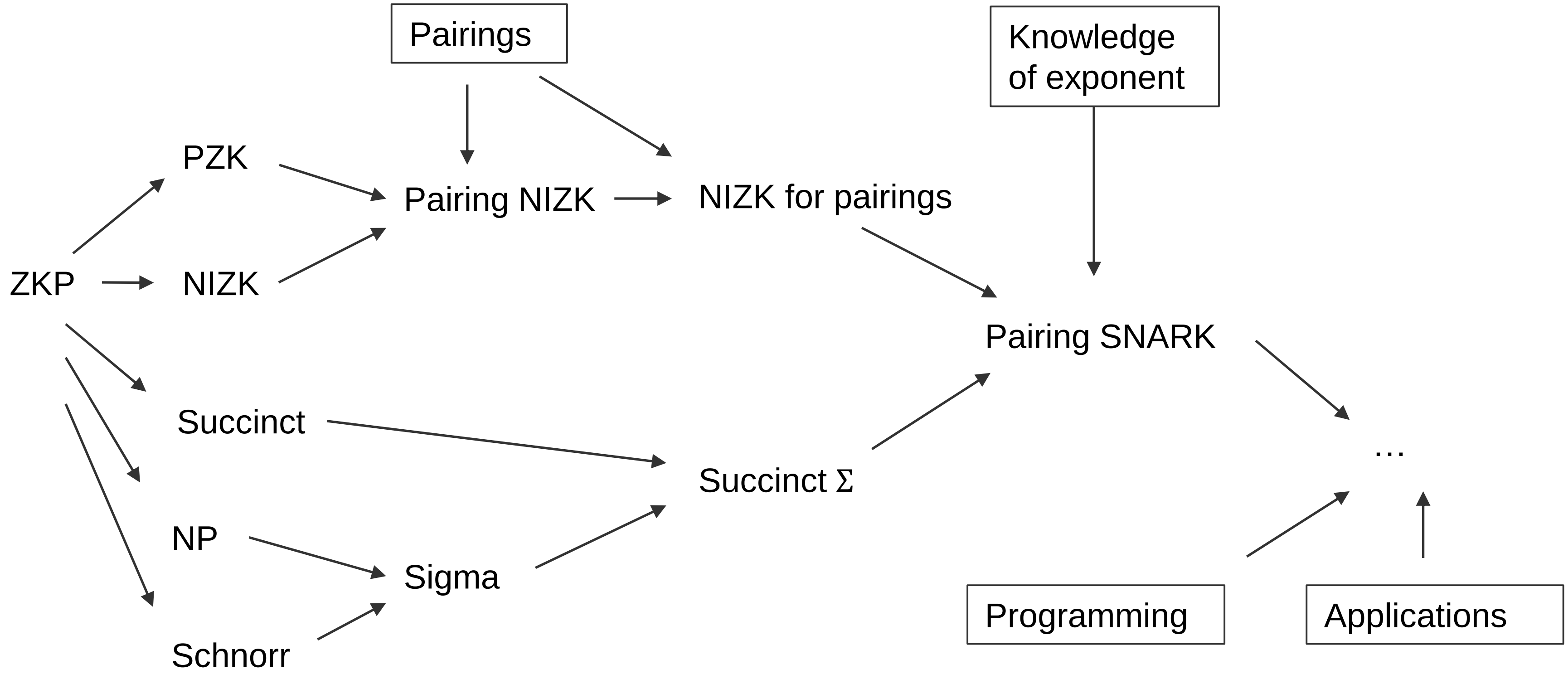
Arithmetic circuit satisfiability



- Universal CRS, works for any circuit [Gro10,Lip13]
  - Statement consists of circuit description, public inputs, and witness of private inputs
- Specialized CRS, tailored to specific circuit [GGPR13]
  - Circuit is fixed, statement consists of public inputs, and witness of private inputs
  - With logarithmic overhead use universal circuit, adaptive with CRS size $O(N \log N)$

Past, present and future

Phylogenetic web

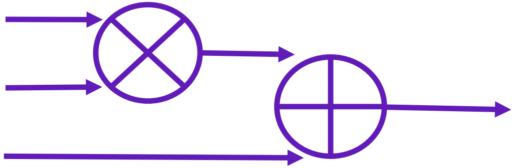


Sexual reproduction

Theory ❤ Practice

- [PHGR13] Pinocchio implements pairing-based zk-SNARKs for simple C
- [BFR+13, WSR+15,BCG+13] Pantry, Buffet, Libsnark
- Language

R1CS



Compilers, general computation

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Adaptation

- Language
  - Pairing-friendly languages, R1CS, general computation ⇨ Applicable
  - Programming languages, interoperability
- Security
  - Setup: multi-crs, mpc-generated, updatable ⇨ Trustworthy
  - Formal verification, usable security, convincing people
- Efficiency
  - Asymmetric pairings, nesting of proofs, commit-and-prove ⇨ Cost effective
  - Research & development

Invasive Species