# Syntax and semantics of the weak consistency model specification language cat

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## Abstract

We provide the syntax and semantics of the cat language, a domain specific language to describe consistency properties of parallel/distributed programs. The language is implemented in the herd7 tool Alglave and Maranget (2015).

# 1 Introduction

The cat language Alglave et al. (2015b) is a domain specific language to describe consistency properties succinctly by constraining an abstraction of parallel program executions into a candidate execution and possibly extending this candidate execution with additional constraints on the execution environment. The *analytic semantics* of a program is defined by its *anarchic semantics* that is a set of executions describing computations and a cat specification cat describing a weak memory model. An example of anarchic semantics semantics for LISA is given in Alglave and Cousot (2016). An anarchic semantics is a truly parallel semantics, with no global time, describing all possible computations with all possible communications. The cat language operates on abstractions of the anarchic executions called *candidate executions*. The cat specification *cat* checks a candidate execution for the consistency specification (including, maybe, by defining constraints on the program execution environments, such as the final writes or the coherence order).

The abstraction of an anarchic execution into a candidate execution is overview in Section 2 while the cat language is introduced is Section 3. Its formal semantics is defined in Section 4. Examples can be found in Alglave [2015].

# 2 Abstraction to candidate executions

The anarchic semantics is a set of executions. Each execution is abstracted to a candidate execution  $\langle evts, po, rf, IW, sr \rangle$  providing

- events evts, giving a semantics to instructions; for example in LISA Alglave and Cousot (2016), a write instruction w[] x v yields a write event of variable x with value v. Events can be (for brevity this is not an exhaustive list):
  - *writes*, gathered in the set W, including the set IW of *initial writes* coming from the prelude of the program;
  - reads, gathered in the set R;
  - branch events, gathered in the set B;
  - fences, gathered in the set F.
- the program order **po**, relating accesses written in program order in the original LISA program;
- the read-from **rf** describing a communication between a write and a read event;
- the scope relation **sr** relating events that come from threads which reside within the same scope;

A cat specification *cat* may add other components to the candidate execution (*e.g.* to specify constraints on the execution environment) and then checks that this extended candidate execution satisfies the consistency specification, that is, essentially, that the communication relation rf satisfies the consistency specification (under hypotheses on the execution environment).

# 3 The cat language

A weak consistency specification written in the cat language defines constraints to be satisfied by the communication relation rf of any candidate execution. A typical cat specification defines new objects depending on the sets and relations of the candidate execution (*e.g.* the program order **po** or the initial writes IW) and then imposes constraints on these objects that ultimately restrict the allowed communications rf.

## 3.1 Objects and expressions

## 3.1.1 Types.

The objects defined in a cat specification may be of the following types (see Appendix 4.9 and Figure 5 for the formal details): evt (event), tag (tag), rel (relation between events), set (set), tuple (tuple), enum (enumeration of tags), fun (unary function type), proc (unary procedure type).

#### **3.1.2** Definitions in binding statements

(see Appendix 4.12.5 and Figure 17 for their formal semantics) can bind an expression to a name, which can be used in place of that expression. For example

let rfe = rf & ext

defines the relation **rfe** as the restriction of the communications **rf** to events coming from different processes. Formally, **rfe** is built as the intersection (denoted by & in cat) of the read-from relation **rf** and the predefined relation **ext** which links events coming from different processes (Figure 11).

A set, relation, function or procedure can be given a name by binding (see Figure 7). Bindings (see Appendix 4.12.5 and Figure 17) can be (mutually) recursive (using let rec ... and ...).

## 3.1.3 Functions

(see Appendix 4.12.3 and Figure 15 for their formal semantics) define an object as a function of a unique formal parameter (which may be an empty tuple () in absence of parameter or a non-empty tuple for multiple parameters). For example

let extof r = r & ext
let rfe = extof rf

defines a function extof of a parameter r which intersects the relation r with the relation ext between events belonging to different processes. We then define the relation rfe as the function extof applied to the read-from relation rf.

We note that our definition of extof above is an abbreviation for the binding of an anonymous function

let extof = fun r -> r & ext

Functions can be recursive (using let rec) and get their actual parameters in a call by tuplematching their actual argument.

#### 3.1.4 Events.

All events come out of the candidate execution and there is no way in **cat** to generate any other event.

## 3.1.5 Sets

(see Appendix 4.12.4 and Figure 16 for their formal semantics) are either empty  $\{\}$  or a homogeneous set  $\{o_1, \ldots, o_n, \ldots\}$ . We do not allow sets of functions or procedures. Predefined sets of events are denoted by the following identifiers (see Appendix 4.11.1 and Figure 10 for their formal semantics):

- the set of all write events W, including the initial writes IW;
- the set of all read events R;
- the set of all branch events B;

- the set of all fence events F;
- the universe containing all events of the candidate execution, which is denoted "\_".

New sets can be defined from existing ones using the following operations (see Appendix 4.12.6, Figures 18, 19 and 20 for the formal semantics of these operations):

- the  $\sim S$  is the complement of a set S;
- the union of two sets  $S_1$  and  $S_2$  is  $S_1 | S_2$ ;
- the intersection of two sets  $S_1$  and  $S_2$  is  $S_1 \& S_2$ ;
- the difference of two sets  $S_1$  and  $S_2$  is  $S_1 \setminus S_2$ ;
- the addition of an element e to a set S is  $e^{++S}$ ;

Matching over sets (see Appendix 4.12.4 and Figure 16 for the formal semantics) can be used for (recursive) set definitions. Match is against the empty set {} or, for a non-empty set, a partition e ++ es into a singleton {e} and the rest of the set es. For example, given a function f, a set  $S = \{e_1, e_2, \ldots, e_n\}$  and an element y, the call fold f(S, y) returns the value  $f(e_{i_1}, f(e_{i_2}, \ldots, f(e_{i_n}, y)))$ , where  $i_1, i_2, \ldots, i_n$  is some permutation of  $1, 2, \ldots, n$ :

```
let fold f =
   let rec fold_rec (es,y) = match es with
   || {} -> y
   || e ++ es -> fold_rec (es, f(e,y))
   end
in fold_rec
```

#### 3.1.6 Relations between events

(see Appendix 4.11.1 for their formal semantics) can be the empty relation 0, the identity relation id, or the relations defined from the candidate execution:

- the program order **po**;
- the read-from **rf**,

or predefined relations on events (see Figure 11):

- the relation loc between events accessing the same memory location;
- the relation ext between events coming from different threads.

New relations (see Appendix 4.12.6) can be defined from sets of events (see Figure 20):

• the cartesian product of two sets of events  $S_1$  and  $S_2$  is  $S_1 * S_2$ 

or using unary operators on relations (see Figure 19):

- the identity closure of a relation **r** is **r**?
- its reflexive-transitive closure is r\*

- its transitive closure is r+
- its complement is ~r
- its inverse is r^-1

or using binary operators on relations (see Figure 20):

- the union of two relations r1 and r2 is r1 | r2
- the intersection of two relations r1 and r2 is r1 & r2
- the difference of two relations r1 and r2 is r1  $\ r2$
- the sequence of two relations r1 and r2 is r1;r2 (*i.e.* the set of pairs (x, y) such that there exists an intervening z, such that  $(x, z) \in r1$  and  $(z, y) \in r2$ ).

Moreover the following primitives can be used to manipulate sets and relations over events (see Figure 12 for their formal semantics):

- classes takes a relation r; if r is an equivalence relation, then we return the equivalence classes of r, otherwise an error is raised;
- linearisations takes a set S and a relation r and returns a set of relations; viz., if the relation r is acyclic, we return all the possible linearisations (topological sorts in the finite case) of r over S, otherwise we return the empty set.

#### 3.1.7 Tuples

(see Appendix 4.12.4 and Figure 16 for their formal semantics) include the empty tuple (), and constructed tuples  $(o_1, \ldots, o_n)$ . Tuples can be heterogeneous. Tuples are essentially used to pass parameters to functions and procedures. Tuples can be destructured by pattern matching; for example in the example of fold above, we match the argument of fold\_rec into the pair (es,y).

#### 3.1.8 Tags.

Events can be tagged (using the annotations on the program instruction generating this event) and these tags can be used to build relations. The tags must be declared (see Appendix 4.12.1 and Figure 14) using the enum construct. For example

```
enum memory-order = 'rlx || 'acq || 'rel
```

defines an enumeration type memory-order, which contains three tags: 'rlx (relaxed), 'acq (acquire), 'rel (release).

LISA instructions can be annotated with such tags. In cat, tags have a quote ' to not be confused with identiers. This confusion is impossible in LISA so quotes ' are omitted. The tags that can be worn by instructions must be declared (see Figure 14 in Appendix 4.12.1), as follows:

```
instructions W[{'rlx,'rel}]
instructions R[{'rlx,'acq}]
```

Events generated by an annotated LISA instruction will bear the same tags as the instruction. The set of events bearing a given tag t is provided by tag2events (t) (see Figure 13 in the Appendix 4.12.1). For example

```
let Release = tag2events('rel)
let Acquire = tag2events('acq)
```

define the set Release (resp. Acquire) of events bearing the tag 'rel (resp. 'acq).

Tags can be matched against their names as defined in an enum, and with the wildcard \_ (see Figure 13); examples are provided in the next section.

#### 3.1.9 Scopes.

The organisation of a parallel system is not always flat. Often, threads (and physical processors or cores alike) are organised in a hierarchical fashion, threads being members of a hierarchy of nested levels, or *scopes*. Examples include: the eponymous scope notion in GPU models (*e.g.* Cooperative Thread Array, or cta, in Nvidia PTX), or the notion of shareability domain in ARM (*e.g.* ish in ARMv8).

Scopes (see Appendix 4.12.2 for their formal semantics) are special tags which must be declared with the reserved identifier scopes:

enum scopes = 'cta || 'gpu || 'system

The hierarchy of scopes is described in a **cat** file by the functions **narrower** and **wider** (which are reserved identifiers but user-defined, as **scopes** is). In the most simple and frequent case, levels are totally ordered. Then, the **wider** function takes a scope tag as argument and returns the immediately wider scope tag, while the **narrower** function returns the immediately narrower scope tag:<sup>1</sup>

```
let wider(s) = match s with 'gpu -> 'system || 'cta -> 'gpu end
let narrower(s) = match s with 'system -> 'gpu || 'gpu -> 'cta end
```

The above definitions specify that scopes are ordered from narrowest to widest as: 'cta < 'gpu < 'system. In other words, a system contains one or more GPUs, and each GPU contains one or more CTAs.

All LISA litmus tests specify how many threads P0, P1, *etc.* are involved. Additionally a scoped litmus test specifies how threads are distributed along the scope hierarchy, by means of a *scope tree* such as

```
scopes: (system (gpu (cta P0 P1) (cta P2 P3)) (gpu (cta P4 P5) (cta P6 P7)))
```

which describes the scope hierarchy



 $<sup>^{1}</sup>$ One may also consider heterogeneous systems such as coupled CPUs and GPUs. In that case, the hierarchical is no longer total and the function **narrower** returns a set of tags.

The herd7 tool checks that the wider function does define a hierarchy, in the sense that each scope has an unique immediately wider scope except one, the root of the hierarchy, which has none. It also checks the compatibility of the narrower function and of scope trees with the defined hierarchy.

The events in a given scope s are gathered as an equivalence relation tag2scope (s) (see Figure 13 in the Appendix 4.12.2). More precisely two events are related by tag2scope (s) when they are generated by threads that are contained in the same scope instance of level s.

Consider for instance the hierarchy depicted above, and two events  $e_0$ , and  $e_2$ , generated by PO and P2 respectively. Then  $e_0$  and  $e_2$  are related by tag2scope ('gpu) and unrelated by tag2scope ('cta) since PO and P2 belong to the same GPU but to different CTAs.

## 3.2 Constraint statements

After defining sets and relations depending on the candidate execution, we can impose constraints on them (see Figure 22 for the formal semantics).

## 3.2.1 Checks

(see Appendix 4.13.1 and Figure 22 for their formal semantics) can have the following syntax: [~][acyclic|irreflexive|empty] x.

The checks [~][acyclic|irreflexive|]r check if the relation r on events is be acyclic or irreflexive. The check or [~][empty]S checks if the set S is empty. The check acyclicr is a shorthand for irreflexiver+. The symbol ~ denotes the negation. Failed checks reject the candidate execution which is therefore forbidden. For example

acyclic po | rf

checks whether the union of **po** and **rf** is acyclic in all the candidate executions of a given program.

Users have the option to not enforce the checks, but rather to use them to report properties of the candidate execution. To do so, users must prefix the check they are interested in with the keyword flag, and name the flagged test with an identifier *name* (by using the postfix qualifier as *name*). A failed flagged check has no consequence over the acceptance or rejection of the candidate execution. It is simply reported (*viz.*, flagged with the name *name*) for the user's information. For example

flag ~(acyclic po | rf) as cycle-found

will flag, using the name cycle-found, all the candidate executions in which there is a cycle in the union of po and rf.

We often use flag in models that involve *data races*, *e.g.* C++ or HSA. In such models, executions that have data races are typically deemed undefined. We handle this in cat by flagging candidate executions that exhibit data races with the name undefined.

#### 3.2.2 Procedures. (see Appendix 4.13.2 and Figure 23 for their formal semantics)

Definitions of sets and relations and their checks in constraint statements can be gathered and parameterised using procedures and checked by procedure calls. Procedures are not recursive and return no result. They have one formal parameter (but that can be a tuple, including the empty one). Their body is a non-empty list of statements.

For example the following procedure sc implements Sequential Consistency Lamport (1979), given the relation com as parameter:

```
procedure sc(com) =
   let sc-order = (po | com)+
   acyclic sc-order
end
```

The procedure may have local definitions (like sc-order). The scope of the formal parameter and local definitions is limited to the procedure body. Global definitions (like the relation po) can be used in the procedure body.

A call (e.g. call sc(rf)) passes the actual parameter (a tuple, here rf, matching the formal parameter com) and the procedure body is evaluated with the actual parameter.

## 3.2.3 Iteration.

A universally quantified check (*i.e.* a finite conjunction of checks) can be done for all values **e** chosen in a set **S** by a **forall** iterator (see Appendix 4.13.3 and Figure 24 for the formal semantics), as follows:

```
forall e in S do
    call check_contraint(e)
end
```

## 3.2.4 Candidate execution extension (with ... from ...)

The construct with o from S requirements introduces an additional constituant o of the semantics, not already part of the candidate execution. This constituant o of the semantics is introduced in the cat file rather than in the anarchic semantics because it only depends on the program execution events (e.g. the coherence order).

The with construct enumerates all possible objects o in S and checks the *requirements*. Typically S is a set of relations on events and o a relation between events which must satisfy the *requirements* appearing in the remainder of the cat file.

For example total orders over certain accesses can be built using with:

- the coherence order between writes to a given memory location ;
- SC accesses in C++ or HSA (we use this in our modelisation of HSA, see HSA Foundation (2015)).

## 3.3 Evaluation of a cat file on a candidate execution

When evaluated on a candidate execution, a cat file returns an error if the cat file is syntactically incorrect. Otherwise the binding definitions, constraint statements, and with requirements are evaluated in sequence (see Figures 3 and 25). If some (unflagged *i.e.* mandatory) constraint fails, we return forbidden to stipulate that the candidate execution does not satisfy the weak consistency model specified by the cat file. Otherwise the candidate execution is accepted, *i.e.* 

we return allowed. In both cases we return a possibly empty set of flags for conditions that are not enforceable (see Appendix 4.7), as well as the objects introduced by with constructs.

## 4 Syntax and formal semantics of the cat language

## 4.1 Analytic semantics

The analytic semantics S[P] of a parallel program P with a given cat consistency specification (or weak consistency model) *cat* is a set of execution behaviors  $\pi$  conforming to this consistency specification. Each such execution behavior  $\pi = \langle \Xi, \mathbf{rf}, \Gamma \rangle$  is described in two parts, the computations  $\langle \Xi, \mathbf{rf} \rangle$  and the communications  $\langle \mathbf{rf}, \Gamma \rangle$  where the *read-from relation*  $\mathbf{rf}$  is their common interface.

The possible computations (\(\alpha\), rf\) are described by the anarchic semantics \$\(S\_a[P]\)]\$ of the program P. The read-from relation rf records the correspondance between the reads, the matching writes, and the communicated values on the computation \$\(\alpha\).

The anarchic semantics  $S_a[\![P]\!]$  places only the following restrictions on the communications of P so all possible computations with all possible read-from relations rf are considered.

- Satisfaction: a read event has at least one corresponding communication in rf;
- Singleness: a read event must have at most one corresponding communication in rf;
- Match: if a read reads from a write, then the variables read and written and communicated value must be the same;
- Inception: no communication is possible without the occurrence of both the read and (maybe initial) write it involves (this does not prevent a read to read from a future write).

Otherwise stated the consistency specification/weak consistency model is not taken into account at all by the anarchic semantics  $S_a[P]$ .

 The possible communications are described by communications (rf, Γ) between communication *i.e.* read and/or write events.

The cat file *cat* generates all possible communication relations  $c \in \Gamma$  (using the with construct). The communication relations  $c \in \Gamma$  include the coherence order co, *etc*. More generally, they specify requirements on the execution environment of the program P.

The cat file semantics sorts out the executions  $\pi = \langle \Xi, \mathbf{rf}, \Gamma \rangle$  that are feasible for weak consistency model, one by one.

## 4.2 Consistent semantics specification by cat files

A cat file  $cat \in Cat$  defines a check that an execution  $\pi = \langle \Xi, \mathbf{rf}, \Gamma \rangle$  satisfies a consistency specification.

• First the computation  $\langle \Xi, \mathbf{rf} \rangle$  of the anarchic semantics is abstracted to a candidate execution  $X = \alpha_{\Xi}(\langle \Xi, \mathbf{rf} \rangle) = \langle evts, \mathbf{po}, \mathbf{rf}, \mathbf{IW}, \mathbf{sr} \rangle$  (collecting read, write, branch, fence and rmw events in *evts*, the program order **po**, the read-from relation **rf**, initial IW writes, and the program scope tree **sr**) but where *e.g.* events on local registers or communicated values are abstracted away.

- The cat file *cat* is then evaluated on X. Thanks to with  $c_i$  from  $C_i$  constructs, the cat file generates all necessary communication relations  $\Gamma = c_1, \ldots, c_n$  between communication events (including  $co \in allCo, etc.$ ) which are necessary to express the consistency specification.
- In absence of error in *cat*, the final result
  - can be (allowed, f,  $\Gamma$ ) meaning that the computation  $\langle \Xi, \mathbf{rf} \rangle$  (with abstraction  $X = \alpha_{\Xi}(\langle \Xi, \mathbf{rf} \rangle)$ ) together with the communication specification  $\Gamma$  satisfies the consistency specification, or
  - can also be  $\langle \texttt{forbidden}, f, \Gamma \rangle$  meaning that the  $\pi = \langle \Xi, \texttt{rf}, \Gamma \rangle$  does not satisfy the consistency specification.

In both cases  $f \in \mathcal{F}$  is the set of flagged constraints in *cat* satisfied by the execution  $\pi = \langle \Xi, \mathbf{rf}, \Gamma \rangle$  (without any influence on the allowed/forbidden result).

# 4.3 Analytic semantics specified by an anarchic semantics and a cat specification

We define below, in Figure 2, the semantics  $\textcircled{ob}[\![cat]\!]X$  of a candidate execution X which returns a set of answers of the form  $\langle j, f, \Gamma \rangle$  where  $j = \{\texttt{allowed}, \texttt{forbidden}\}, f$  is the set of flags that have been set up on X and  $\Gamma$ , and  $\Gamma$  defines the communication relation for the execution to be allowed/forbidden.

The analytic semantics of a program P with consistency specification *cat* is therefore

$$\begin{split} S[\![\mathsf{P}, cat]\!] &\triangleq \{ \langle \Xi, \mathtt{rf}, \Gamma \rangle \mid \langle \Xi, \mathtt{rf} \rangle \in S_{\mathfrak{a}}[\![\mathsf{P}]\!] \land \\ &\exists f \in \mathcal{F} . \langle \mathtt{allowed}, f, \Gamma \rangle \in \widecheck{\bullet} [\![cat]\!] (\alpha_{\Xi}(\langle \Xi, \mathtt{rf} \rangle)) \} \end{split}$$

This analytic semantics S[P] of a program P for a cat specification *cat* is the composition  $S[P] = \alpha_{\text{int}}[cat] \circ \alpha_{\Xi}(S_{a}[P])$  of two abstractions of the anarchic semantics *viz*.

## 4.4 Candidate executions

Candidate executions are tuples:

which gather the events, the program order po on each thread, the read-from relation rf, modeling who reads from where, the initial writes IW, and a scope relation sr.

#### 4.4.1 Events

 $e \in Evt$  are abstractions of the events generated by a program execution. Events  $e \in evts$  carry the unique program instruction (and its unique program label) which execution generated this event e. However, this information is not directly available to cat. Auxiliaries to extract components of an event e are as follows:

$$\mathsf{loc-of}(e) \triangleq \mathsf{location} \text{ of } e \quad \mathsf{kind-of}(e) \triangleq \mathsf{kind} \text{ of } e$$
  
 $\mathsf{pid-of}(e) \triangleq \mathsf{process} \text{ identifier of } e \quad \mathsf{annot-of}(e) \triangleq \mathsf{annotations} \text{ of } e$   
 $\mathsf{from-to-of}(e) \triangleq \text{ events separated by fence event } e$ 

The set *evts* of events belong to  $Evts \triangleq \wp(Evt)$ .

- The process identifier pid-of(e) refers to the identifier of the unique process at the origin of the event e;
- The location loc-of(e) can be a memory location or a register;
- kind-of(e) is the kind of event e: write (W), read (R), branch (B), fence (F), begin or end of a rmw. We define the following sets of events by kind:

$$\begin{array}{lll} W(X) & \triangleq & \{e \in \mathsf{evts-of}(X) \mid \mathsf{kind-of}(e) = \mathbb{W}\} \\ F(X) & \triangleq & \{e \in \mathsf{evts-of}(X) \mid \mathsf{kind-of}(e) = \mathbb{F}\} \\ \end{array}$$

- the annotations annot-of(e) of e is a possibly empty set of tags and scopes carried by the action at the origin of e;
- let e<sub>F</sub> ∈ F(X) be a fence event generated by a localised fence instruction f[ts] {L<sub>1</sub>,...,L<sub>n</sub>} {L'<sub>1</sub>,...,L'<sub>m</sub>} where this instruction and all L<sub>1</sub>, ..., L<sub>n</sub>, L'<sub>1</sub>, ..., L'<sub>m</sub> belong to the same process of pid-of(e<sub>F</sub>).

Then from-to-of  $(e_F)$  is the set of pairs  $\langle e_f, e_t \rangle$  such that  $e_f$  is an event generated by the execution of a program instruction labelled  $\mathbf{L}_i$ ,  $i \in [1, n]$  and  $e_t$  is an event generated by the execution of a program instruction labelled  $\mathbf{L}'_j$ ,  $j \in [1, m]$ . Additionally, we require  $\langle e_f, e_F \rangle \in \mathsf{po-of}(X)$  and  $\langle e_F, e_t \rangle \in \mathsf{po-of}(X)$ , viz. the fence does separate the two events  $e_f$  and  $e_t$ . If the fence carries an empty set of labels this is from-to-of  $(e_F) \triangleq \emptyset$ . If the fence carries no sets of labels, we set from-to-of  $(e_F) \triangleq \{\langle e_f, e_t \rangle \mid \langle e_f, e_F \rangle \in \mathsf{po-of}(X) \land \langle e_F, e_t \rangle \in \mathsf{po-of}(X)\}$ .

#### 4.4.2 Program order,

abbreviated  $\mathbf{po} \in Program-order$ , abstracts the order of the events of a process in the execution hence lifts the order in which instructions have been executed to the level of events. For each candidate execution, it is a total order over events within the same thread, hence irreflexive and transitive, and cannot relate events from different threads.

#### 4.4.3 Read-from,

abbreviated  $\mathbf{rf} \in Read$ -from  $\triangleq \wp(Write \times Read)$ , relates a read event of a certain shared variable  $\mathbf{x}$  to a unique write event of the same variable. The read-from relation essentially indicates which events read from where.

#### 4.4.4 Initial writes

are gathered in the set  $IW \in Writes \triangleq \wp(Write)$ . The initial writes IW simply are the writes in the prelude of the program.

#### 4.4.5 Scope relation,

abbreviated  $sr \in Scope-rel$ , relates events that come from threads which reside within the same scope; this is a notion that is mostly used for scoped models such as GPUs (see *e.g.*, Alglave et al. (2015a) and Sections 3.1.9).

Auxiliaries to extract components of a candidate execution  $X \triangleq \langle evts, po, rf, IW, sr \rangle$  are as follows:

 $\begin{array}{rcl} \operatorname{evts-of}(X) & \triangleq & \operatorname{evts} & \operatorname{po-of}(X) & \triangleq & \operatorname{po} & \operatorname{rf-of}(X) & \triangleq & \operatorname{rf} \\ & & \operatorname{sr-of}(X) & \triangleq & \operatorname{sr} & \operatorname{init-of}(X) & \triangleq & \operatorname{IW} \end{array}$ 

# 4.5 Program scope relation defined by a scope tree and cat scope hierarchy

A program scope tree specifies a scope relation. The syntax of program scope trees and their semantics, that is the scope relation that they define are defined in Figure 1. Program scope trees must match the scope hierarchy defined by the **cat** file through a scope tag declaration (see Figure 14) and the user specified functions with reserved names **narrower** and **wider**, as checked in Figure 13.

## 4.6 Values

The cat language is much inspired by OCaml Leroy et al. (2014), featuring for example types, immutable bindings, first-class functions and pattern matching. However, cat is a domain specific language, with important differences from OCaml:

- base values are specialised; they are: sets of events, relations over events, first class functions; there are also tags, including scope tags, akin to C enumerations or OCaml constant constructors. There are two structured values: sets of values and tuples of values, see Figure 5.
- there is a distinction between expressions in Figure 7 that evaluate to some value, statements in Figure 3, which introduce new definitions or constraints, and requirements in Figure 25 which introduce new communication relations on the execution environment and constraints on them.

We use the following notations: square brackets [...] denote optional components, parentheses (...) denote grouping,  $(...)^*$  (resp.  $(...)^+$ ) denotes zero, one or several (resp. one or several) repetitions of the enclosed components. Scope trees —

$$\begin{array}{rcl} st & ::= & (s \text{-} tag \ \mathsf{P}_0 \ \dots \ \mathsf{P}_n) \\ & & | & (s \text{-} tag \ st_0 \ \dots \ st_n) \end{array}$$

program scope trees

where  $\{P_0, \ldots, P_n\} \subseteq \{\mathsf{pid-of}(e) \mid e \in \mathsf{evts-of}(X)\}.$ 

Given a scope tree st, a set E of events and a scope-tag st, define srel(st) E s-tag to be the relation between different events that come from threads which reside in the scope s-tag, as follows

$$\begin{aligned} \operatorname{srel}((s \operatorname{-tag} \operatorname{P}_0 \ \dots \ \operatorname{P}_n)) E s \operatorname{-tag}' & \triangleq \emptyset & (\text{when } s \operatorname{-tag} \neq s \operatorname{-tag}') \\ \operatorname{srel}((s \operatorname{-tag} \operatorname{P}_0 \ \dots \ \operatorname{P}_n)) E s \operatorname{-tag} & \triangleq \{\langle e, e' \rangle \mid e, e' \in E \land \exists i, j \in [0, n] . \\ & \operatorname{pid-of}(e) = \operatorname{P}_i \land \operatorname{pid-of}(e') = \operatorname{P}_j \} \\ \operatorname{srel}((s \operatorname{-tag} st_0 \ \dots \ st_n)) E s \operatorname{-tag}' & \triangleq \bigcup_{i=0}^n \operatorname{srel}(st_i) E s \operatorname{-tag}' & (\text{when } s \operatorname{-tag} \neq s \operatorname{-tag}') \\ \operatorname{srel}((s \operatorname{-tag} st_0 \ \dots \ st_n)) E s \operatorname{-tag}' & \triangleq \{\langle e, e' \rangle \mid e, e' \in E \land \\ & \exists \operatorname{P}_i, \operatorname{P}_j \in \bigcup_{i=0}^n \operatorname{processes}(st_0 \ \dots \ st_n) . \\ & \operatorname{pid-of}(e) = \operatorname{P}_i \land \operatorname{pid-of}(e') = \operatorname{P}_j \} \end{aligned}$$

$$\begin{aligned} \operatorname{processes}((s \operatorname{-tag} st_0 \ \dots \ P_n)) & \triangleq \{\operatorname{P}_0, \dots, \operatorname{P}_n\} \\ \operatorname{processes}((s \operatorname{-tag} st_0 \ \dots \ st_n)) & \triangleq \bigcup_{i=0}^n \operatorname{processes}(st_i) \\ \operatorname{tags-of}((s \operatorname{-tag} st_0 \ \dots \ st_n)) & \triangleq \{s \operatorname{-tag}\} \\ \operatorname{tags-of}((s \operatorname{-tag} st_0 \ \dots \ st_n)) & \triangleq \{s \operatorname{-tag}\} \cup \bigcup_{i=0}^n \operatorname{tags-of}(st_i) \end{aligned}$$

If the program has a scope-tree st then the candidate execution X must have its scope relation component sr = sr-of(X) be such that for all s-tag  $\in tags-of(st)$ , sr(s-tag) = srel(st) (evts-of(X)) s-tag.

Figure 1: Semantics of program scope trees

## 4.7 Consistency specifications

Consistency specifications (or cat files/specifications) cat filter candidate executions and extend them with communication relations. In other words, the semantics  $\Im [cat] X$  of a cat specification cat is defined with respect to a candidate execution X and its result extend it to specify requirements on the execution environment.

### 4.7.1 Evaluating a *cat* specification

means allowing or forbidding that candidate execution. More precisely, evaluating a cat file makes a result object  $\langle j, f, \rho, \omega \rangle$  evolve, where:

- Judgements  $j \in \mathcal{J} \triangleq \{\texttt{allowed}, \texttt{forbidden}\}\$  can be of two kinds: allowed when a candidate execution passes all the checks imposed by the cat specification, or forbidden when a candidate execution fails on one of the checks of the cat specification *cat*.
- Flagged checks  $f \in \mathcal{F} \triangleq \wp(Identifier)$  collect identifiers of checks that have been flagged and are recorded to signal certain executions (e.g., the ones with data races).
- Environments  $\rho \in \mathcal{E}$  associate identifiers (which belong to the set *Identifier*) to typed values; more precisely environments are partial functions from identifiers to values:

 $\mathcal{E} \triangleq Identifier \not\rightarrow \mathcal{V}$ .

During the evaluation of the cat file *cat*, the environment  $\rho \in \mathcal{E}$  gets augmented with new definitions as evaluation progresses. It evolves also locally when evaluating functions and procedures, according to the static scoping or block-structured visibility rule.

• Sets of communication relation identifiers  $\omega$  record the identifiers of communication relations introduced by a with requirement.

 $\omega \in \mathcal{W} \triangleq \wp(Communication-relation-identifier)$ 

During the evaluation of the cat file cat, the set  $\omega \in \mathcal{W}$  of communication relation identifiers gets augmented with new identifiers introduced by with *id* from ... requirements, see Figure 25. The relation  $\rho(id)$  which is the value of such communication relation identifiers *id* is found in the environment  $\rho$ . The final verdict in Figure 2 collects this information in the final result of the *cat* evaluation.

• *Results* collect judgements, flagged checks, environments, and communication relation identifiers or raise error if needed.

 $r = \langle j, f, \rho, \omega \rangle \in \mathcal{R} \triangleq (\mathcal{J} \times \mathcal{F} \times \mathcal{E} \times \mathcal{W}) \cup \{\text{error}\}$ 

A result may be undefined *e.g.* when an implementation might not terminate, for example, when evaluating a non-terminating function. The result can also be error when the cat file is incorrect. The difference is that an implementation of cat is assumed to signal error but is not required to report undefined results. The final result is collected in the final verdict  $\langle j, f, \prod_{id \in \omega} \rho(id) \rangle$ , see Figure 2.

Initially, the judgement is allowed, the set of flags is empty, predefined identifiers are implicitly bound to event sets and relations over events as described in Section 4.11.1 and Figures 10 and 11, and the set of communication relation identifiers is empty, see Figure 2.

#### 4.7.2 Specifications

(or cat files) are lists of *requirements* preceded by an identifier, used for documentation purposes. We give the syntax and semantics of specifications in Figure 2.

The requirements constitutive of the specification are evaluated in sequence, until one requirement raises error or forbidden, or until the end of the requirement list. In that latter case, the specification accepts the candidate execution, hence raises allowed.

## 4.7.3 The final verdict

in Figure 2 is given at the top-level, gets rid of the environment, and returns the communication relations in S obtained by finding the value of the communication relation identifiers *id* in the environment. If S is empty, we return forbidden (with unmodified flags). If S contains error, the error is returned.

 $\begin{array}{rcl} cat & \in & Cat \\ cat & ::= & identifier \\ & & | & identifier \ requirements \end{array}$ 

 $\textcircled{\tiny (identifier]} X \triangleq \{ \langle \texttt{allowed}, \emptyset, \emptyset \rangle \}$ 

 $\textcircled{oo} \llbracket identifier \ requirements \rrbracket X \triangleq \mathsf{verdict}(\textcircled{oo} \llbracket requirements \rrbracket X \langle \mathsf{allowed}, \emptyset, \emptyset, \emptyset \rangle)$ 

$verdict \emptyset \hspace{.1in} \triangleq \hspace{.1in} \{ \langle \texttt{forbidden}, \hspace{.05in} \emptyset, \hspace{.05in} \emptyset \rangle \}$	
verdict $S \triangleq$ error	when $error \in S$
$verdict\;\{\langle j_i,\;f_i,\;\rho_i,\;\omega_i\rangle\; \;i\in\Delta\}\;\;\triangleq\;\;\{\langle j_i,\;f_i,\;\;\prod\;\;\rho_i(id)\rangle\; \;i\in\Delta\}$	otherwise
$id \in \omega_i$	

Figure 2: Semantics of specifications

## 4.8 Statements

Requirements can be *statements* introducing new binding definitions and checking constraints, or the with *id* from S requirement introducing a new communication relation identified by *id*.

Statements are evaluated for their effect: adding new *definitions* or checking *constraints*. We give their syntax and semantics in Figure 3. Note that once an error has been raised, we stay in that state. Moreover statements have no with requirement so cannot introduce new

communication identifiers. Therefore the set of communication relation identifiers is unchanged by the evaluation of a statement,  $\omega' = \omega$  in Figure 3.

 $[ \bullet \bullet \bullet ] [statement] X error \triangleq error$ 

Figure 3: Semantics of statements

## 4.9 Typed values and semantic domains

**Typed values,** (gathered in the set  $\mathcal{V}$ ) are given in Figure 5. Events (of type evt) belong to the set *Evt*. There are no operation on events so the type evt can only be used to type elements of relations or sets. Typed values include (see Figure 5):

- the error symbol;
- tags (of type tag), which belong to *Tag*;
- relations over events (of type rel), which belong to  $\wp(Evt \times Evt)$ ;
- sets (of type set) of values, which belong to φ(V); sets have to be homogeneous, and cannot be sets of functions or procedures, as reflected by the predicate well-formed;
- tuples (of type tuple) of values, which belong to  $\bigcup_{n \in \mathbb{N}} \prod_{i=1}^{n} \mathcal{V}$ ;
- enumerations of tags (of type enum), which belong to  $\wp(Tag)$ ;
- functions (of type fun);
- non-recursive procedures (of type proc).

The value of functions and procedures are closures memorising their parameter (which belongs to Pat), their body (which in the case of functions belongs to Expr, and in the case of procedures can be a list of elements of *Statement*), and declaration *environment* (which belongs to  $\mathcal{E}$ ). On a call, the actual parameters are evaluated in the calling environment and the body in the declaration environment enriched by the value of the formal parameters and the local bindings. After the call, evaluation goes on in the calling environment. This is therefore static scoping.

	e ∈	type
	e ::=	type
events		
$\operatorname{tag}$		
relation between events		
$\operatorname{set}$		
tuple		
enumeration		
unary function type		
unary procedure type		

## Figure 4: Typed values

## 4.10 Auxiliaries

To define the semantics of operators over sets and relations in particular we need to define a certain number of auxiliaries (summarised in Figure 6).

## 4.11 Expressions

Expressions let the user build new sets or relations over tags and events. Figure 7 summarises the syntax of expressions.

Several constructs are non-deterministic: the set matching of Section 4.12.4, the iteration over sets of Section 4.13.3. In the semantics, only one result is nondeterministically picked out of all possible ones. This is different from the with requirement of Section 4.14.2 where all possibilities for choosing the communication relation are enumerated.

The semantics of an expression is error whenever the semantics of any one of its subexpressions is error. To leave this check implicit, we assume that the mathematical construct let  $\mathsf{type}_i: v_i = \bigotimes [\![expr_i]\!] X \rho$ ,  $i \in [1, \ell]$  in ... equals error whenever there exists i in  $[1, \ell]$  such that  $\bigotimes [\![expr_i]\!] = \mathsf{error}$ .

well-formed  $(S) \triangleq \forall$  type:  $v \in S$ . type  $\notin$  {fun, proc}  $\land \forall$  type':  $v' \in S$ . type = type'

check sets Semantic domains —  $\mathcal{V} \triangleq$ typed values {error}  $\cup$  tag: Tag $\cup$  rel:  $\wp(Evt \times Evt)$  $\cup \mathsf{set}: \{S \in \wp(\mathcal{V}) \mid \mathsf{well}\text{-}\mathsf{formed}(S)\}$  $\cup \operatorname{\mathsf{tuple}}: (\{()\} \cup \bigcup_{n \in \mathbb{N}, n > 1} \prod_{i=2}^{n} \mathcal{V})$  $\cup$  enum:  $\wp(Tag)$  $\cup$  fun: ((*Pat*  $\rightarrow$  *Expr*)  $\times \mathcal{E}$ )  $\cup$  proc: ((*Pat*  $\rightarrow$  {*Statement*}<sup>+</sup>)  $\times \mathcal{E}$ )  $\rho \in \mathcal{E} \triangleq Identifier \not\rightarrow \mathcal{V}$ environments  $j \in \mathcal{J} \triangleq \{\texttt{allowed}, \texttt{forbidden}\}$ judgements  $f \in \mathcal{F} \triangleq \wp(Identifier)$ flagged checks  $\in \mathcal{W} \triangleq \wp(Communication-relation-identifier)$ ω set of com. identifiers  $\in \mathcal{R} \triangleq (\mathcal{J} \times \mathcal{F} \times \mathcal{E} \times \mathcal{W}) \cup \{\mathsf{error}\}$ results

Figure 5: Semantic domains

$\mathbb{I}_{\mathcal{X}} \triangleq \{ \langle e, e \rangle \mid e \in \mathcal{X} \}$	identity relation on set $\mathcal{X}$
$r  {\rm i}  r' \ \triangleq \ \{ \langle e,  e' \rangle \mid \exists e''  .  \langle e,  e'' \rangle \in r \land \langle e'',  e'' \rangle \in r \land \langle e'',  e'' \rangle \}$	$\langle r'  angle \in r'  brace$ sequence of relations
$dom(r) \hspace{.1in} \triangleq \hspace{.1in} \{x \mid \exists y \; . \; \langle x, \; y  angle \in r \}$	domain of relation $r$
$range(r) \hspace{.1in} \triangleq \hspace{.1in} \{y \mid \exists x \; . \; \langle x, \; y  angle \in r \}$	range of relation $r$
$fld(r) \hspace{.1in} \triangleq \hspace{.1in} dom(r) \cup range(r)$	field of relation $r$
$\mathbf{lfp}^{\subseteq} F = \bigcap \{ X \in \wp(S) \mid F(X) \subseteq X \}$	the least fixpoint of the $\subseteq$ -increasing operator $F$ on the powerset $\wp(S)$ Tarski (1955)

Figure 6: Auxiliaries for defining operators' semantics

## 4.11.1 Identifiers

are either predefined or defined by the user through *definition* statements. We list the reserved identifiers in Figure 8. User-defined identifiers cannot be reserved identifiers and are bound in the environment  $\rho$  (see Figure 9).

simple	$\in$	Simples	
simple	::=		
		id	identifiers
		tag	$\operatorname{tags}$
		function	anonymous functions
		procedure	procedures
		set	sets
		tuple	tuples
clause	E	Clauses	
clause	::=		
		$[  ] tag \rightarrow expr \{   tag \rightarrow expr \}^* [\_\_ \rightarrow$	expr]
		$[II] \{\} \rightarrow expr II id ++ id \rightarrow expr$	
expr	e	Expr	
expr	::=		
-		simple	simples
	İ	expr expr	function application
	Ì	(expr)   begin expr end	grouping
	ĺ	<pre>let [rec] binding {and binding}* in expr</pre>	binding expressions
	Ì	match expr with clause end	matching
	İ	op	operators on sets and relations
definiti	on	$\in$ Definition	
definiti	on	::= decl	
		<pre>let [rec] binding {and binding}*</pre>	

Figure 7: Simple expressions, expressions and definitions

**Predefined identifiers denoting sets of events** appear in Figure 10. We have: the universal sets, the set of all write, read, memory, branch and fences events, as well as the set of initial writes. The semantics of these identifiers, given in Figure 10 is straightforward; they denote the eponymous sets of events.

**Predefined identifiers denoting relations on events** appear in Figure 11. We have: the empty and identity relations, the relation over events accessing the same memory location, the relation over events with different pids, the program order, and the read-from relation.

Keywords ≜
{acyclic, and, as, begin, call, do, empty, end, enum, flag, forall, from, fun, in,
instructions, irreflexive, let, match, procedure, rec, scopes, with }
Primitives ≜
{classes, fromto, linearisations, tag2events, tag2scopes}
Names ≜
{\_\_\_, 0, B, ext, F, id, IW, loc, M, narrower, po, R, rf, rmw, W, wider }
Reserved ≜ Keywords ∪ Primitives ∪ Names

Figure 8: List of reserved identifiers

(for  $id \in Names$ , see Figures 10 or 11)

Figure 9: Semantics of identifiers

aevt	::=			annotable events
		W		write events
		R		read events
		В		branch events
		F		fence events
predefi	ined-e	vents ::=		
				all events
			IW	initial writes
			М	memory events, $M = W \cup R$
			aevt	annotable events
(iii) [[]] X	$\rho \triangleq$	set: {evt: $e$	$  e \in evts-of(X) \}$	events
$\mathrm{\widetilde{OO}}[\![W]\!]\; X$	$\rho \triangleq$	set: {evt: $e$	$\mid e \in evts-of(X) \cap \mathit{W}(X) \}$	write events
$\mathop{\scriptstyle{}}\nolimits^{\scriptstyle{}}}_{\scriptstyle{}} [\![ \mathtt{R} ]\!] X$	$\rho \triangleq$	set: {evt: $e$	$  \ e \in evts-of(X) \cap R(X) \}$	read events
$\textcircled{\tiny{\textcircled{\tiny 0}}}{\textcircled{\tiny 0}}[\![{\tt B}]\!] X$	$\rho \triangleq$	set: {evt: $e$	$  \ e \in evts-of(X) \cap B(X) \}$	branch events
$\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny$	$\rho \triangleq$	set: {evt: $e$	$  e \in evts-of(X) \cap F(X) \}$	fence events
$\mathop{\scriptstyle\frown}{}_{\scriptstyle\frown}{}_{\scriptstyle\bullet}[\![\mathtt{M}]\!]\;X$	$\rho \triangleq$	set: {evt: $e$	$\mid e \in evts-of(X) \cap (\mathit{R}(X) \cup \mathit{W}(X)) \}$	memory events
•••[[W]] X	$\rho \triangleq$	set: {evt: $e$	$  e \in init-of(X) \}$	initial write events

where  $\textcircled{\tiny{\tiny{\tiny{OO}}}}{[\![}{\tt IW}{]\!]} X \, \rho \subseteq \textcircled{\tiny{\tiny{OO}}}{[\![}{\tt W}{]\!]} X \, \rho$ 

Figure 10: Predefined sets and their semantics

The semantics of these predefined identifiers, given in Figure 11 is relatively straightforward again: 0 is the empty relation, id is the identity relation, loc the relation between events accessing the same variable, and ext the relation between events from different threads. It is the eponymous relation for po and rf.

– Predefined relations over events —

empty relation	0	::=	$predefined\mbox{-}relations$
identity	id		
same location	loc		
external (different pids)	ext		
program order	ро		
read-from	rf		
read-modify-write	rmw		

– Semantics of predefined relations –

Figure 11: Predefined relations over events and their semantics

## 4.12 Primitives to manipulate sets and relations over events

appear in Figure 12. We have five primitives (Primitives  $\triangleq$  {classes, fromto, linearisations, tag2events, tag2scope}). We will detail the primitives tag2events and tag2scope in Section 4.12.1. For the other three primitives:

- classes takes as argument an expression *expr*, which should evaluate as a relation *r*; if *r* is an equivalence relation, then we return the equivalence classes of *r*, otherwise we raise an error;
- linearisations takes as argument a pair of two expressions  $expr_1$ , which should evaluate to a set S, and  $expr_2$ , which should evaluate to a relation r; if this relation is acyclic, then we return all the possible linearisations (topological sorts) of r over S, otherwise we return the empty set.
- from to takes as argument a expression *expr*, which should evaluate to a set S of tags, the events tagged with these tags should be fence events, and the result is the union of all their sets of pairs of events separated by these fence events.

```
Semantics of primitive functions —
if (type = rel ) \wedge (\mathfrak{l}_{\mathsf{fld}}(r) \subseteq r \wedge (r)^{-1} \subseteq r \wedge r\mathfrak{g}r \subseteq r) then
        set: {set: {evt: e \in \mathsf{fld}(r) \mid \langle e, e' \rangle \in r} \mid e' \in \mathsf{fld}(r)}
     else error
[\bullet\bullet] [linearisations expr] X \rho \triangleq let type: v = [\bullet\bullet] [expr] X \rho in
    if (type: v = tuple: \langle set: s, rel: r \rangle) \land (\forall type_v: v \in s . type_v = evt) then
        if r^+ \cap i_{\bullet} = \emptyset then
            \mathsf{set:} \left\{ \mathsf{rel:} r' \in \wp(s \times s) \mid r \cap (s \times s) \subseteq r' \wedge r' \mathfrak{g} r' \subseteq r' \wedge \right.
                                                   (\forall e \neq e' \in s : \langle e, e' \rangle \in r' \lor \langle e', e \rangle \in r')\}
        else set: \emptyset
     else error
(\bullet) [fromto expr] X \rho \triangleq let type: S = (\bullet) [expr] X \rho in
    if (\mathsf{type} = \mathsf{set}) \land (\forall \, \mathsf{type}_e \colon e \in S \ . \ \mathsf{type}_e = \mathsf{evt} \ \land e \in F(X)) then
        rel: \bigcup from-to-of (e)
              e \in S
    else error
```

Figure 12: Semantics of primitives

4.12.1 Tags

**Tags** essentially are identifiers preceded by a quote ' (to distinguish them form identifiers in bindings); and we gather them in sets, as shown in Figure 13. We first define an auxiliary over a tag tag:

• is-tag-declared checks that tag has been defined in an environment  $\rho$ , i.e. belongs to an enumeration tag-set in  $\rho$ .

Now, the value of a tag '*id* is the corresponding typed value if the tag has been declared in the environment  $\rho$ , or an error if not.

Finally, the primitive tag2events gathers all events bearing the tag tag, provided that the tag tag is declared in the environment  $\rho$ .

**Declarations.** One can declare enumerations of tags named by an identifier with the construct enum. One can use these tags to *annotate* LISA instructions, using the eponymous instructions construct.

Declarations (see Figure 14) augment the environment. The effect of an enum declaration is to extend the environment with the corresponding set of tags. In other terms, the semantics of enum  $id = [||] tag_1 \ldots || tag_n$  is to augment the environment  $\rho$  with the set of typed tags  $tag_1, \ldots tag_n$ , under the name id.

The semantics of an instruction declaration is as follows: if there is a tag not in the environment, we raise an error; and if there is an event whose  $i^{\text{th}}$  tag is not in the  $i^{\text{th}}$  tag set, we raise an error.

### 4.12.2 Scopes.

Semantically, we distinguish scope tags s-tag from other tags, as shown in Figure 13. Thus for enum declarations, the identifier scopes is reserved to declare scopes. If an enum scopes declaration is provided then two functions narrower and wider must be declared on scope tags, to define the set of all possible scope hierarchies. Finally, the primitive tag2scope builds the relation between events coming from instructions that belong to the same scope (viz., the scope instances of that scope) — relatively to a scope tree appearing in the original program. We give its semantics in Figure 13.

Matching over tags is as follows:

```
match expr with

|| tag_1 \rightarrow expr_1

|| \dots

|| tag_n \rightarrow expr_n

|| \_ \rightarrow expr_d
```

```
end
```

The value of the match expression is computed as follow: first evaluate expr to some value v, which must be a tag t. Then v is compared with the tags  $tag_1, \ldots, tag_n$ , in that order. If some tag pattern  $tag_i$  equals t, then the value of the match is the value of the corresponding expression  $expr_i$ . Otherwise, the value of the match is the value of the default expression  $expr_d$ . As the default clause \_\_\_\_->  $expr_d$  is optional, the match construct may fail in error. We give the semantics of matching over tags in Figure 13.

```
Tags
     tag
                        Taq
                       'id
     tag
   s-taq
                       taq
                                                                                                                     scope tags
 Auxiliaries over tags 'id —
  is-tag-declared [i, id] \rho \triangleq \exists tag-set \in dom(\rho) . \rho(tag-set) = enum: T \land tag: 'id \in T
  tag-set-of [\![id]\!] \rho \triangleq \{id' \mid \exists tag-set \in dom(\rho) . \rho(tag-set) = enum: T \land
                                              tag: 'id \in T \land tag: 'id' \in T}
Value of a tag 'id —
  [\bullet \bullet] [\bullet id ] X \rho \triangleq
      if is-tag-declared [ ' id ] \rho then tag: ' id else error
Gathering all events bearing the same tag 'id —
  (\bullet) [tag2events('id)] X \rho \triangleq
     if is-tag-declared [\![id]\!] \rho then \{e \in \mathsf{evts-of}(X) \mid id \in \mathsf{annot-of}(e)\}
      else error
Building the scope instance of level s-tag —
  (\bullet \bullet) [tag2scope(s-tag)] X \rho \triangleq
     let type<sub>w</sub>: t = \langle \bullet \bullet \rangle wider s-tag X \rho
      and type<sub>n</sub>: n = \langle \bullet \bullet \rangle [\![ narrower s-tag ]\!] X \rho in
          if type_w = tag: \land type_n \in \{tag, set\} \land is-tag-declared[s-tag]] \rho \land \forall id \in tag-set-of[s-tag]] \rho.
              t = id \Rightarrow \operatorname{sr-of}(X) \ s\text{-}tag \subseteq \operatorname{sr-of}(X) \ id \land
             \mathsf{type}_n = \mathsf{tag} \land n = \mathsf{tag}: id \Rightarrow \mathsf{sr-of}(X) id \subseteq \mathsf{sr-of}(X) s - tag \land
              \mathsf{type}_n = \mathsf{set} \land \mathsf{tag}: id \in n \Rightarrow \mathsf{sr-of}(X) id \subseteq \mathsf{sr-of}(X) s-tag
          then
                sr-of(X) s-tag
          else
                error
Tag matching —
  [match expr_0 with [||] tag_1 \rightarrow expr_1 \dots || tag_n \rightarrow expr_n otherwise end ] X \rho \triangleq
     let type: t = \bigotimes [\![expr_0]\!] X \rho in
         if type \neq tag then error else let i = \min\{k \mid t = tag_k\} in
            if i \neq +\infty then (\bullet \bullet) [expr_i] X \rho
            else if otherwise = || \_ \rightarrow expr_{n+1} then (\bullet) [expr_{n+1}] X \rho
            else error
```

Figure 13: Tags and their semantics

Declarations annotation $\in$ Annotation annotationset(annotations are sets of tags) ::=declDecl $\in$ declenum  $id = [[1]tag \{1|tag\}^*]$  $(when id \neq \texttt{scopes})$ ::= enum scopes =  $[[||]s-tag \{||s-tag\}^*]$ instructions ainstr [{annotation}<sup>+</sup>] 

- Semantics of enum —

- Semantics of instructions -

$$\begin{split} & \overleftarrow{\circ} & \texttt{[instructions a evt [annotation_1, \ldots, annotation_n]]} X \langle j, f, \rho, \omega \rangle \\ & \texttt{let type}_i: T_i = \overleftarrow{\circ} & \texttt{[annotation_i]} X \langle j, f, \rho, \omega \rangle, \quad i \in [1,n] \text{ in} \\ & \texttt{if } \forall i \in [1,n] \text{ . type}_i = \texttt{set } \land \forall \texttt{type}': t \in T_i \text{ . type}' = \texttt{tag } \land \\ & \forall e \in \texttt{evts-of}(X) \text{ . (kind-of}(e) = aevt) \Rightarrow (\forall i \in [1,n] \text{ . annot-of}(e)_i \in T_i) \\ & \texttt{then } \langle j, f, \rho, \omega \rangle \text{ else error} \end{split}$$

Figure 14: Declarations

## 4.12.3 Functions

**Functions** are first class values, as reflected by the anonymous function construct fun pat  $\rightarrow$  expr. We call the expression expr the body of the function. A function takes one argument pat only. When this argument is a tuple, it may be destructured by a tuple pattern  $(id_1, \ldots, id_n)$ .

The value of a function, given in Figure 15, is its *closure*; we write  $\langle \lambda i d \cdot body, \rho' \rangle$  for a closure with parameter *id*, body *body* and declaration environment  $\rho'$  (this closure can also be understood as a triple  $\langle id, body, \rho' \rangle$ ).

**Function calls** are written  $expr_1 expr_2$ . That is, functions are of arity one and the application operator is left implicit. Notice that function application binds tighter than all binary operators (see Section 4.12.6) and looser that postfix operators (see Section 4.12.6). Furthermore the implicit application operator is left-associative.

The cat language has call-by-value semantics. That is, the effective parameter  $expr_2$  is evaluated before being bound to the function formal parameter, see Figure 15.

#### 4.12.4 Sets and tuples.

**Sets** are written as follows: {  $expr_1$ ,  $expr_2$ , ...,  $expr_n$ } with n greater than 0. As events are not values, one cannot build a set of events using explicit set expressions. Sets are homogeneous, *i.e.* contain elements of the same type. We give their semantics in Figure 16. The value of {} is the empty set, and the value of { $expr_1$ , ...,  $expr_n$ } is the set of values { $v_1 \ldots v_n$ } where the  $v_i$  are the values of  $expr_i$ .

Matching over sets is as follows:

match expr with || {} ->  $expr_1$ ||  $id_1$  ++  $id_2$  ->  $expr_2$ end

We compute the value of the match as follow: first evaluate expr to some value v, which must be a set. If v is the empty set  $\{\}$ , then the value of the match is the value of  $expr_1$ . Otherwise, if v is a non-empty set S, then let e be some element in S and S' be the set S minus the element e. The value of the match is the value of  $expr_2$  in a context where  $id_1$  is bound to e and  $id_2$  is bound to S'. We give the semantics of matching over sets in Figure 16, where the non-deterministic choice  $e \in s$  is arbitrary (and unknown). So the semantics in Figure 16 returns one possible match (as opposed to all possibilities).

**Tuples** include the empty tuple (), and constructed tuples  $(expr_1, expr_2, \ldots, expr_n)$ , with n greater than 2. In other words there is no tuple of size one (which avoids ambiguity with grouping between parentheses).

We give their semantics in Figure 16. The value of () is the empty tuple  $\langle \rangle$ , and the value of  $\langle expr_1, \ldots, expr_n \rangle$  is the tuple of values  $\langle v_1, \ldots, v_n \rangle$  where the  $v_i$  are the values of  $expr_i$ ; we do not impose that these values  $\langle v_1, \ldots, v_n \rangle$  have the same type.

Patterns – pat $\in$ Pat $id \mid$  ()  $\mid$  (id {, id}\*) pat ::= Anonymous functions function Function  $\in$ function fun pat -> expr ::=Values of functions —  $\begin{array}{l} & \overbrace{\bullet\bullet}^{\bullet\bullet} \llbracket \texttt{fun } id \ \text{->} \ expr \rrbracket X \ \rho' & \triangleq \ \texttt{fun:} \langle \boldsymbol{\lambda} \ id \ \cdot \ expr, \ \rho' \rangle \\ & \overbrace{\bullet\bullet}^{\bullet\bullet} \llbracket \texttt{fun } () \ \text{->} \ expr \rrbracket X \ \rho' & \triangleq \ \texttt{fun:} \langle \boldsymbol{\lambda} \ () \ \cdot \ expr, \ \rho' \rangle \\ & \overbrace{\bullet\bullet}^{\bullet\bullet} \llbracket \texttt{fun } (id_1, \ \dots, \ id_\ell) \ \text{->} \ expr \rrbracket X \ \rho' & \triangleq \end{array}$  $\ell \ge 1$ fun:  $\langle \boldsymbol{\lambda} \langle id_1, \ldots, id_\ell \rangle \cdot expr, \rho' \rangle$ Semantics of function calls —  $\langle \bullet \bullet \rangle \llbracket expr_1 expr_2 \rrbracket X \rho \triangleq$ (when  $expr_1 \notin \mathsf{Primitives}$ ) let type<sub>i</sub>:  $v_i = \bigoplus expr_i X \rho$ ,  $i \in [1, 2]$  in match type<sub>1</sub>:  $v_1$  with  $|\operatorname{fun:} \langle \boldsymbol{\lambda} \, id \cdot expr, \, \rho' \rangle \rightarrow \langle \widetilde{\bullet \bullet} \rangle [\![ expr ]\!] X \, \rho' [id := type_2: v_2]$ | fun:  $\langle \boldsymbol{\lambda} (\boldsymbol{)} \cdot expr, \rho' \rangle \Rightarrow$  if type<sub>2</sub>:  $v_2 =$ tuple:  $\langle \rangle$  then  $(\widetilde{\bullet})$   $[expr] X \rho'$ else error | fun:  $\langle \lambda \langle id_1, \ldots, id_n \rangle \cdot expr, \rho' \rangle \rightarrow \text{ if type}_2: v_2 = \text{tuple: } \langle e_1, \ldots, e_\ell \rangle \land \ell = n \text{ then}$  $(\widetilde{\bullet}) \llbracket expr \rrbracket X \rho' [id_1 := e_1] \dots [id_n := e_n]$ else error \_\_\_ → error



**Grouping** is straightforward, as shown in Figure 16: the semantics of a parenthesised expression (*expr*) is the semantics of *expr*, idem for begin *expr* end.

## 4.12.5 Bindings

are of the form pat = expr or  $id \ pat = expr$ , where  $id \ pat = expr$  is syntactic sugar for id = funpat -> expr. As shown in Figure 17, bindings simply update the environment  $\rho$ . The bindings

Sets and tuples  $set \in Set$   $set ::= {} | {expr {, expr}}^*$   $tuple \in Tuple$   $tuple ::= () | (expr, expr {, expr}^*)$ sets tuples Semantics of sets —  ${\scriptstyle \fbox{\tiny{(0)}}} \llbracket \{\} \rrbracket X \ \rho \quad \triangleq \quad \mathsf{set:} \ \emptyset$  $\underset{\textcircled{0}}{\overset{}_{\textcircled{0}}} \llbracket \{expr_1, \dots, expr_n\} \rrbracket X \rho \triangleq$  $n \ge 1$ let type<sub>i</sub>:  $v_i = \bigotimes [expr_i] X \rho, \quad i \in [0, n]$  in let  $S = set: \{type_1: v_1, \ldots, type_n: v_n\}$  in if well-formed(S) then S else error – Set matching — let type:  $s = \bigotimes [expr_0] X \rho$  in if type  $\neq$  set then error else if  $s = \emptyset$  then  $(\bullet) [expr_1] X \rho$ else let  $e \in s$  in  $(\widetilde{\bullet})[expr_2] X \rho[id_2 := \mathsf{set}: (s \setminus \{e\})][id_1 := e]$ – Semantics of tuples –  $\begin{array}{l} & \overbrace{\textcircled{oo}} \llbracket () \rrbracket X \rho & \triangleq & \mathsf{tuple:} \langle \rangle \\ & \overbrace{\textcircled{oo}} \llbracket (expr_1, \dots, expr_n) \rrbracket X \rho & \triangleq \\ & \mathsf{tuple:} \langle \overbrace{\textcircled{oo}} \llbracket expr_1 \rrbracket X \rho, \dots, \langle \overbrace{\textcircled{oo}} \llbracket expr_n \rrbracket X \rho \rangle \end{array}$  $n \ge 2$ - Semantics of grouping —  $\underbrace{\textcircled{\baselineskip}{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} X \rho \triangleq \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} X \rho \triangleq \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} \underbrace{\textcircled{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} X \rho = \underbrace{\textcircled{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} \underbrace{\textcircled{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} X \rho = \underbrace{\textcircled{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} X \rho = \underbrace{\textcircled{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} X \rho = \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \textcircled{\baselineskip}} \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \hline \hline \hline \baselineskip} \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \hline \hline \baselineskip}} X \rho = \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \hline \hline \baselineskip} \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \hline \hline \baselineskip} \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \hline \baselineskip} X \rho = \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \hline \hline \baselineskip} \underbrace{\rule{\baselineskip}{\baselineskip}}_{\scriptsize \hline \baselineskip} \underbrace{\rule{\bas$ 



for pat = expr are as follows: if pat is (), then expr must evaluate to the empty tuple; if pat is id or (id), then id is bound to the value of expr; if pat is a proper tuple pattern  $(id_1, \ldots, id_n)$  with n greater than 2, then expr must evaluate to a tuple value of size n ( $v_1, \ldots, v_n$ ) and the names  $id_1, \ldots, id_n$  are bound to the values  $v_1, \ldots, v_n$ .

– Bindings **—** 

Value of a binding -

Binding definitions —

$$\underbrace{\textcircled{\tiny[let binding_1 and ... and binding_n]}}_{(\textcircled{\tiny[wow]}[binding_n]]} X \rho \triangleq n \ge 1$$

Recursive function binding:

$$\stackrel{\text{\tiny (io)}}{=} \llbracket \texttt{let rec } id_1 \ pat_1 \ = \ expr_1 \ \texttt{and} \ \dots \ \texttt{and} \ id_n \ pat_n \ = \ expr_n \rrbracket X \ \rho \ \triangleq \\ \texttt{let } cl^{\infty} = \prod_{j=1}^n \stackrel{\text{\tiny (io)}}{=} \llbracket \texttt{fun } pat_i \ -> \ expr_i \rrbracket X \ \rho[id_1 := cl^{\infty}_1] \ \dots \ [id_n := cl^{\infty}_n] \ \texttt{in} \\ \rho[id_1 := cl^{\infty}_1] \ \dots \ [id_n := cl^{\infty}_n]$$

Recursive set/relation binding:

$$\begin{split} & \underbrace{\textcircled{oo}}_{\bullet}\llbracket \texttt{let rec } id_1 \ = \ expr_1 \ \texttt{and} \ \dots \ \texttt{and} \ id_n \ = \ expr_n \rrbracket X \ \rho \ \triangleq \\ & \mathsf{let} \ F_i(\prod_{j=1}^n x_j) \triangleq \mathsf{let} \ \mathsf{type}_i: s_i = \underbrace{\textcircled{oo}}_{\bullet}\llbracket expr_i \rrbracket X \ \rho[\prod_{j=1}^n id_j := \prod_{j=1}^n x_j] \ \texttt{in} \\ & \quad \mathsf{if} \ (\mathsf{type}_i \in \{\mathsf{set}, \mathsf{rel}\,\}) \ \mathsf{then} \ \mathsf{type}_i: s_i \ \mathsf{else} \ \mathsf{error}, \quad i \in [1, n] \\ & \quad \mathsf{in} \ \mathsf{let} \ \prod_{i=1}^n e_i = \mathsf{lfp}^{\subseteq} \ \pmb{\lambda} \ \boldsymbol{\cdot} \prod_{i=1}^n x_i \prod_{i=1}^n F_i(\prod_{i=1}^n x_i) \ \mathsf{in} \\ & \quad \mathsf{if} \ (\exists i \in [1, n] \ . \ e_i = \mathsf{error}) \ \mathsf{then} \ \mathsf{error} \ \mathsf{else} \ \rho[id_1 := e_1] \dots [id_n := e_n] \end{split}$$

- Binding expressions —

```
 \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\textcircled{\baselineskip}{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\textcircled{\baselineskip}{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\textcircled{\baselineskip}{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\textcircled{\baselineskip}{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\textcircled{\baselineskip}{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\textcircled{\baselineskip}{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\textcircled{\baselineskip}{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\textcircled{\baselineskip}{\baselineskip}}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\begin{smallmatrix} x \ baselineskip}{\baselineskip}} \underbrace{\textcircled{\baselineskip}{\baselineskip}}_{$\begin{smallmatrix} x \ baselineskip}{\baselineskip}} \underbrace{ \begin{smallmatrix} x \ baselineskip}{\baselineskip} \underbrace{ \begin{smallmatrix} x \ baselineskip} \underbrace{
```

Figure 17: Bindings and their semantics

**Binding definitions** happen through the let and let rec constructs, which bind value names for the rest of a specification evaluation. We give the semantics of binding definitions in Figure 17.

First, the construct let  $binding_1$  and ... and  $binding_n$ , that is, let  $pat_1 = expr_1$  and ... and  $pat_n = expr_n$ , evaluates  $expr_1, \ldots, expr_n$ , and binds the names in the patterns  $pat_1, \ldots, pat_n$  to the resulting values.

Second, for recursive function bindings let rec  $id_1 pat_1 = expr_1$  and ... and  $id_e pat_n = expr_n$ , we follow Milner and Tofte (1991) where the proof of existence and unicity of the infinite closure  $cl^{\infty}$  is based on Aczel (1988).

Third, for recursive set or relation bindings let rec  $id_1 = expr_1$  and ... and  $id_n = expr_n$ , we compute the least solution of the equations  $id_1 = expr_1, \ldots, id_n = expr_n$  on sets or relations using inclusion for ordering. These fixpoint equations must satisfy the  $\subseteq$ -monotony (increasingness) hypotheses of Tarski (1955) fixpoint theorem or else the result is undefined.

The recursive bindings may be mutually recursive. We suppose these recursive definitions well-formed, *i.e.* terminating. The result of ill-formed definitions is undefined (*i.e.* an implementation might return an error or never terminate).

**Binding expressions** happen through the construct let [rec] *bindings* in *expr*, which locally binds the names defined by *bindings* to evaluate *expr*. Both non-recursive and recursive bindings are allowed.

#### 4.12.6 Operators on sets and relations

Operators can be unary or binary. We list them in Figure 18, and detail their semantics below.

– Operators on sets and relations –					
	Operators	$\in$	op		
set addition	expr ++ expr	::=	op		
cartesian product	expr * expr				
union	$expr \mid expr$				
intersection	expr & expr				
relation composition	expr ; expr				
transitive closure	expr +				
reflexive closure	expr ?				
reflexive and transitive closure	expr *				
inverse	expr^-1				
subtraction	$expr \setminus expr$				
complement	$\sim expr$				

Figure 18: List of both unary and binary operators

**Unary operators.** Given an expression denoting a relation, we can build its identity closure with the operator ?, its reflexive-transitive closure with the operator \*, its transitive closure with +, its complement with ~ and its inverse with ^-1. These operators are postfix, and are defined on relations only, except for the complement, which can apply to sets of events or tags as well. Figure 19 gathers them all. We recall that the value of identifier 0 is the empty relation.

postfixed ${\tt op}$	cat-operation-of $[\![ \texttt{op} ]\!] \mathrel{X} r$	relation operation
*	$\mathbf{lfp}^{\subseteq} \boldsymbol{\lambda} \boldsymbol{\cdot} x \mathfrak{i}_{evts-of(X)} \cup x \mathfrak{g} r$	reflexive closure
+	$\mathbf{lfp}^{\subseteq} \boldsymbol{\lambda} \boldsymbol{\cdot} xr \ \cup \ x\mathfrak{g}r$	irreflexive closure
?	$\mathring{l}$ evts-of $(X) \cup r$	identity closure
^-1	$\{\langle e', \ e \rangle \mid \langle e, \ e' \rangle \in r\}$	inverse

```
    Unary operators on relations —
```

```
 \begin{split} & \underbrace{\textcircled{oo}}{[\!\![expr op]\!]} X \rho & \triangleq \\ & \texttt{let type:} r = \underbrace{\textcircled{oo}}{[\!\![expr]\!]} X \rho \quad \texttt{in} \\ & \texttt{if type} = \texttt{rel then} \\ & \texttt{rel: cat-operation-of} [\!\![op]\!] X r \\ & \texttt{else error} \end{split}
```

Complement of a set or relation —

```
\begin{split} & \underbrace{\textcircled{\scalestic}}_{\bullet\bullet\bullet} \llbracket \sim expr \rrbracket X \rho & \triangleq \\ & \mathsf{let type:} s = \underbrace{\textcircled{\scalestic}}_{\bullet\bullet\bullet} \llbracket expr \rrbracket X \rho & \mathsf{in} \\ & \mathsf{if type = set } \land \forall \mathsf{type:} e \in s \ \mathsf{.type = evt then} \\ & \mathsf{set:} \left( \{\mathsf{evt:} e \mid e \in \mathsf{evts-of}(X) \} \setminus s \right) \\ & \mathsf{else if type = set } \land \exists id \in \mathsf{dom}(\rho) \ \mathsf{.} \ \rho(id) = \mathsf{enum:} T \land s \subseteq T \ \mathsf{then} \\ & \mathsf{set:} \left( T \setminus s \right) \\ & \mathsf{else if type = rel then} \\ & \mathsf{rel:} \left( (\mathsf{evts-of}(X) \times \mathsf{evts-of}(X)) \setminus s \right) \\ & \mathsf{else error} \end{split}
```

Figure 19: Semantics of unary operators

**Binary operators.** We can build the sequence (or composition in the sense of Figure 6) of two expressions with the operator ;, defined on relations only. We can add an element to a set: the addition operator  $expr_1 + expr_2$  operates on sets. The value of  $expr_2$  must be a set of values S and the operator returns the set S augmented with the value of  $expr_1$ . We can build

a new relation out of the cartesian product of two sets of events, with the infix operator **\***.

We can build the union, intersection, and difference of sets and relations as summarised in Figure 20. The semantics of  $expr_1$  op  $expr_2$  is the operator op applied to the sets (resp. relations)  $s_1$  and  $s_2$ , viz., the values of  $expr_1$  and  $expr_2$ .

```
- Adding an element to a set —
```

Cartesian product of two sets of events —

```
\begin{split} & \underbrace{\textcircled{\sc opt}}_{\bullet}\llbracket expr_1 \ast expr_2 \rrbracket X \ \rho & \triangleq \\ & \mathsf{let} \ \mathsf{type}_i : s_i \ = \ \operatornamewithlimits{\textcircled{\sc opt}}_{\bullet}\llbracket expr_i \rrbracket X \ \rho, \quad i \in [1,2] \ \mathsf{in} \\ & \mathsf{if} \ \mathsf{type}_1 \ = \mathsf{type}_2 \ = \mathsf{set} \ \land \ \forall \ \mathsf{type} : v \in s_1 \cup s_2 \ . \ \mathsf{type} \ = \mathsf{evt} \ \mathsf{then} \\ & \mathsf{rel} : s_1 \times s_2 \\ & \mathsf{else} \ \mathsf{error} \end{split}
```

op	$cat-operation-of\llbracket op \rrbracket$	
Ι	U	union
&	$\cap$	intersection
\	\	difference

Binary operators relative to both sets and relations —  $\langle \widetilde{\bullet \phi} \| expr_1 \text{ op } expr_2 \| X \rho \triangleq$ 

```
\begin{split} & \left[ \exp r_1 \operatorname{op} \operatorname{sup} r_2 \right] \cong \rho \\ & \text{let type}_i : s_i = \bigotimes_{i=0}^{\infty} \left[ expr_i \right] X \rho, \quad i \in [1,2] \text{ in} \\ & \text{if type}_1 = \text{type}_2 = \text{set } \wedge \text{well-formed}(s_1 \cup s_2) \text{ then} \\ & \text{set: } s_1 \text{ (cat-operation-of } [\![ \operatorname{op} ]\!]) s_2 \\ & \text{else if type}_1 = \text{type}_2 = \text{rel then} \\ & \text{rel: } s_1 \text{ (cat-operation-of } [\![ \operatorname{op} ]\!]) s_2 \\ & \text{else error} \end{split}
```

Figure 20: Semantics of binary operators

## 4.13 Constraints

Constraints (see Figure 21) can be checks, procedure calls, or iteration over sets.

Constraints —

 constraint ∈ Constraint

 constraint ::=

 |
 flagoption check expr as id

 |
 call id expr

 |
 forall id in expr do statements end

 flagoption
 ::=

Figure 21: Constraints

## 4.13.1 Checks

happen through the construct *check expr*, which evaluates *expr* and applies the check *check*. There are six checks: acyclicity (keyword acyclic), irreflexivity (keyword irreflexive) and emptyness (keyword empty); and their negations. If the check succeeds, the candidate execution is allowed so far. Otherwise, the candidate execution is forbidden.

A check can optionally be named id, using the keyword **as**. A check can also be flagged, by prefixing it with the **flag** keyword. Flagged checks must be named with the **as** construct. Failed flagged checks do not stop evaluation; instead failed flagged checks are recorded under their name in the component f of the semantics of the **cat** specification, for example to handle flagged candidate executions later within our herd7 tool. We give the semantics of checks in Figure 22.

Flagged checks are useful for specifications with statements that impact the semantics of an entire program, e.g., in the case of specifications phrased in terms of data races, such as C++ Batty et al. (2016) or HSA HSA Foundation (2015).

## 4.13.2 Procedures

**Procedures** have no result and cannot be recursive: the body of a procedure is a list of statements and the procedure will be invoked to apply the constraints within its body. Intended usage of procedures is to define constraints that are checked later. Figure 23 gives the semantics of procedures: just like functions, procedure declarations simply augment the environment  $\rho$  with their closure.

**Procedure calls** are written call id expr, where id is the name of a previously defined procedure. The bindings performed during the call of a procedure are discarded when the procedure returns, all other effects (*e.g.* checks or flags, see Section 4.13.1) performed are retained. Procedures cannot be recursive.

```
Checks —

      check
      ::=
      checkname
      ~ checkname

      checkname
      ::=
      acyclic | irreflexive | empty
```

```
check-condition-on-relation [check] R \triangleq
```

```
match check with
```

```
\texttt{cat-check-relation} \llbracket \textit{flagoption check } R \texttt{ as } \textit{id} \rrbracket \langle j, \ f, \ \rho, \ \omega \rangle \ \triangleq
```

```
 \begin{array}{l} \text{if } (\text{check-condition-on-relation}\llbracket check \rrbracket R) \text{ then } \\ \langle j, \ f, \ \rho, \ \omega \rangle \\ \text{else if } flagoption = \texttt{flag then} \\ \langle j, \ f \cup \{id\}, \ \rho, \ \omega \rangle \\ \text{else } \langle \texttt{forbidden}, \ f, \ \rho, \ \omega \rangle \\ \end{array}
```

Value of checks —

$$\begin{split} & \underbrace{\textcircled{\mbox{\tiny bolymbox{\tiny bolymby} bolymbox{\tiny bolymbox{\tiny bolymby} bolymbox{\tiny bolymbox{\tiny bolymbox{\tiny bolymbox{\tiny bolymbox{\tiny bolymby} bolymbox{\tiny bolymbox{\tiny bolymby} bolymbox{\tiny bolymbox{\tiny bolymby} bolymbox{\tiny bolymbox{\tiny bolymby} bolymbox{\tiny bolymbox{\tiny bolymby} bolymbox{\tiny bolymbox{\tiny bolymbox{\tiny bolymbox{\tiny bolymbox{\tiny bolymby} bolymbox{\tiny bolymbox{\tiny bolymbox{\tiny bolymbox bolymbox bolymbox{\tiny bolymbox{\tiny bolymby} bolymbox{\tiny bolymby} bolymbox{\tiny bolymbox bolymby} bolymbox{\tiny bolymby} bolymbox{\tiny bolymby} bolymbox{\tiny bolymby} bolymbox{\tiny bolymby} bolymbox{\tiny bolymby} bolymby} bolymbox{\tiny bolymby} bolymby} bolymbox{\tiny bolymby} bolymby} bolymbox{\tiny bolymby} bolymbox{\tiny bolymby} bolymby} bolymby} bolymby} bilbel bolymby} bolymby} bolymby} blac}}}}}}}}}}}}$$

Figure 22: Checks and their semantics

Procedures  $procedure \in Procedure$  $procedure ::= procedure id pat = {statement}^+ end$ 

```
Declarations of procedures —

 \begin{split} & \overleftarrow{\circ} \\ & \texttt{[procedure } id_1 \ id_2 = \{statement\}^+ \ \texttt{end} \\ & X \ \langle j, \ f, \ \rho, \ \omega \rangle \\ & \triangleq \\ & \langle j, \ f, \ \rho[id_1 := \texttt{proc: } \langle \boldsymbol{\lambda} \ id_2 \cdot \{statement\}^+, \ \rho \rangle], \ \omega \rangle \\ & \overleftarrow{\circ} \\ & \texttt{[procedure } id \ () = \{statement\}^+ \ \texttt{end} \\ & X \ \langle j, \ f, \ \rho, \ \omega \rangle \\ & \triangleq \\ & \langle j, \ f, \ \rho[id := \texttt{proc: } \langle \boldsymbol{\lambda} \ () \cdot \{statement\}^+, \ \rho \rangle], \ \omega \rangle \\ & \overleftarrow{\circ} \\ & \texttt{[procedure } id \ (id_1, \ \dots, \ id_\ell) = \{statement\}^+ \ \texttt{end} \\ & X \ \langle j, \ f, \ \rho, \ \omega \rangle \\ & \triangleq \\ & \langle j, \ f, \ \rho[id := \texttt{proc: } \langle \boldsymbol{\lambda} \ \langle id_1, \ \dots, \ id_\ell \rangle \cdot \{statement\}^+, \ \rho \rangle], \ \omega \rangle \end{split}
```

– Procedure calls –

```
(\bullet \bullet) [call id expr] X \langle j, f, \rho, \omega \rangle \triangleq
     if id \not\in \operatorname{dom}(\rho) then error else
     match \rho(id) with
          |\langle \boldsymbol{\lambda} id_1 \cdot \{statement\}^+, \rho' \rangle \rightarrow
                     \textup{I}{} \{\texttt{statement}\}^+ \texttt{I} X \ \rho'[\texttt{id}_1 := (\textup{I}{} (\texttt{I}_0) \texttt{I} \texttt{expr} \texttt{I} X \ \rho)]
          |\langle \boldsymbol{\lambda} () \cdot \{statement\}^+, \rho' \rangle \rightarrow
                     let type: v = \langle \widehat{\bullet \bullet} \rangle \llbracket expr \rrbracket X \langle j, f, \rho, \omega \rangle in
                          if type: v = tuple: \langle \rangle then
                                (\bullet \bullet) [{statement}<sup>+</sup>] X \langle j, f, \rho', \omega \rangle
                           else error
          |\langle \boldsymbol{\lambda} \langle id_1, \ldots, id_n \rangle \cdot \{statement\}^+, \rho' \rangle \Rightarrow
                     let type: v = \langle \bullet \bullet \rangle \llbracket expr \rrbracket X \langle j, f, \rho, \omega \rangle in
                          if type: v = tuple: \langle e_1, \ldots, e_\ell \rangle \land (n = \ell) then
                                (\bullet\bullet) [ \{statement\}^+ ] X \langle j, f, \rho'[id_1 := e_1] \dots [id_n := e_n], \omega \rangle
                          else error
                 __ → error
```

Figure 23: Semantics of procedures

#### 4.13.3 Iteration over sets

We can iterate checks over sets with the forall construct:

forall id in expr do
 statements
end

The expression *expr* must evaluate to a set S. Then, the list of statements *statements* is evaluated for all bindings of the name *id* to some element e of S. In practice, as failed checks forbid the candidate execution, this amounts to checking the conjunction of the checks within *statements* for all the elements of S. Similarly to procedure calls, the bindings performed during an iteration are discarded when iteration ends, all other cumulated effects (e.g. checks) being retained. We give the semantics of iteration in Figure 24; the iteration is non-deterministic since the choice e in S is arbitrary and unknown.

```
 \begin{array}{ll} \mbox{cat-iterate } id \ S \ \{statement\}^+ \ X \ \langle j, \ f, \ \rho, \ \omega \rangle & \triangleq \\ \mbox{if } S = \emptyset \ \mbox{then } \langle j, \ f, \ \rho, \ \omega \rangle \\ \mbox{else } \ \mbox{let } e \in S \ \mbox{in } \\ \mbox{let } r = & \fbox{o} \left[ \{statement\}^+ \right] X \ \langle j, \ f, \ \rho[id := e], \ \omega \rangle \ \mbox{in } \\ \mbox{if } r = \mbox{error then error else } \\ \mbox{let } \langle j', \ f', \ \rho', \ \omega \rangle = \ r \ \mbox{in } \\ \mbox{if } \ j' = \mbox{allowed then } \\ \mbox{cat-iterate } id \ (S \setminus \{e\}) \ \{statement\}^+ \ X \ \langle j', \ f', \ \rho', \ \omega \rangle \\ \mbox{else } \langle j', \ f', \ \rho', \ \omega \rangle \end{array}
```

Iteration over sets —

$$\begin{split} & \underbrace{\textcircled{\basel{eq:constraint}}}_{\textcircled{\basel{eq:constraint}}} \left[ \texttt{forall} \ id \ \texttt{in } expr \ \texttt{do} \ \{statement\}^+ \ \texttt{end} \ \end{bmatrix} X \ \langle j, \ f, \ \rho, \ \omega \rangle \\ & \texttt{let type:} \ S \ = \ \underbrace{\textcircled{\basel{eq:constraint}}}_{\textcircled{\basel{eq:constraint}}} \left[ expr \ \end{bmatrix} X \ \langle j, \ f, \ \rho, \ \omega \rangle \\ & \texttt{in } \\ & \texttt{if type } \neq \texttt{set then error} \\ & \texttt{else cat-iterate } id \ S \ \{statement\}^+ \ X \ \langle j, \ f, \ \rho, \ \omega \rangle \\ \end{split}$$

Figure 24: Semantics of iteration

## 4.14 Requirements

Requirements are the constitutive blocks of a cat specification. Their evaluation goes as given in Figure 25. Requirements can be statements, or with bindings.

#### 4.14.1 Statements

and their semantics have been presented in the sections above. At the level of requirements, we evaluate lists of statements, gather their evaluation  $\langle j, f, \rho, \omega \rangle$ , and the final verdict forgets the environment  $\rho$  to built the result (see Figure 2).

## 4.14.2 Candidate extension via with binding

happens through the construct with id from expr. This construct extends the current environment by one binding (see Figure 25). The grammar only allows with bindings to occur at the top-level. The expression expr is evaluated to a set S. Then the remainder of the specification is evaluated for each choice of element e in S in an environment extended by a binding of the name id to e.

The final verdict at top level in Figure 2 gets rid of the environment and returns the communication relations obtained by finding the value of the communication relation identifiers idin the environment. Requirements requirements requirements  $\in$ requirements ::= statement  $statement\ requirements$ with *id* from *expr* requirements (ii) [requirements]]  $\in$  Candidate  $\rightarrow \mathcal{R} \rightarrow \wp(\mathcal{R})$  $( \bullet \bullet )$  [*requirements*]] X error  $\triangleq$  error  $\underbrace{\text{````}} \llbracket requirements \rrbracket X \langle j, f, \rho, \omega \rangle \triangleq \text{ if } j = \texttt{forbidden then } \langle j, f, \rho, \omega \rangle$ else match *requirements* with  $| statement \rightarrow \{ \textcircled{oo} [statement] ] X \langle \texttt{allowed}, f, \rho, \omega \rangle \}$  $| \textit{ statement requirements'} \twoheadrightarrow$ let  $r = \bigotimes [statement] X$  (allowed,  $f, \rho, \omega$ ) in if (r = error) then error else let  $\langle j, \, f', \, \rho', \, \omega' \rangle = r \; \text{ in }$ if (j = allowed) then else  $\{r\}$ | with *id* from *expr* requirements'  $\rightarrow$ let type:  $S = \bigotimes [expr] X \rho$  in if (type  $\neq$  set) then error  $\mathsf{else} \ \bigcup \ \widetilde{\mathsf{Go}}[\![requirements']\!] \ X \ \langle \texttt{allowed}, \ f, \ \rho[id := e], \ \omega \cup \{id\} \rangle$  $e \in S$ 

Figure 25: Semantics of requirements

## 5 cat library functions

For reference, we give the code of three library functions that operate over relations and sets (fold, map and cross).

## 5.1 Definition of fold

Given a function f, a set  $S = \{e_1, e_2, \ldots, e_n\}$  and an element y, the call fold f (S, y) returns the value  $f(e_{i_1}, f(e_{i_2}, \ldots, f(e_{i_n}, y)))$ , where  $i_1, i_2, \ldots, i_n$  is a permutation of  $1, 2, \ldots, n$ :

```
let fold f =
  let rec fold_rec (es,y) = match es with
  || {} -> y
  || e ++ es -> fold_rec (es, f(e,y))
  end in
  fold_rec
```

## 5.2 Definition of map

Given a function f and a set  $S = \{e_1, \ldots, e_n\}$ , the call map f S returns the set  $\{f(e_1), \ldots, f(e_n)\}$ . This function can be implemented directly or more concisely by calling the fold function:

let map f = fun es -> fold (fun (e,y) -> f e ++ y) (es,{})

## 5.3 Definition of cross

The function **cross** takes a set of sets  $S = \{S_1, S_2, \ldots, S_n\}$  as argument and returns all possible unions built by picking elements from each of the  $S_i$ :

$$\{ e_1 \cup e_2 \cup \cdots \cup e_n \mid e_1 \in S_1, e_2 \in S_2, \dots, e_n \in S_n \}$$

Note that if S is empty, then **cross** should return one relation exactly: the empty relation  $\emptyset$ , *i.e.*, the neutral element of the union operator. This choice for **cross** ( $\emptyset$ ) =  $\emptyset$  is natural when we define **cross** inductively:

$$\operatorname{cross}(S_1 + S) = \bigcup_{e_1 \in S_1, t \in \operatorname{cross}(S)} \{e_1 \cup t\}$$

In this specification, we simply build **cross**  $(S_1 + + S)$  by building the set of all unions of one relation  $e_1$  picked in  $S_1$  and of one relation t picked in **cross**(S). From this inductive specification for **cross**, one writes the following concise code:

```
let rec cross S = match S with
    || {} -> { 0 }
    || S1 ++ S ->
    let yss = cross S in
    fold
        (fun (e1,r) -> map (fun t -> e1 | t) yss | r)
        (S1,{})
    end
```

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