A Relevance-Theoretic Framework for Constructing and Deconstructing Enthymemes

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Abstract

In most proposals for logic-based models of argumentation dialogues between agents, the arguments exchanged are logical arguments of the form $\langle \Phi, \alpha \rangle$ where Φ is a set of formulae (called the support) and α is a formula (called the claim) such that Φ is consistent and Φ entails α. However, arguments presented by real-world agents do not normally fit the mould of being logical arguments. They are normally enthymemes, and so they only explicitly represent some of the premises for entailing their claim and/or they do not explicitly state their claim. For example, for a claim that "you need an umbrella today", a husband may give his wife the premise "the weather report predicts rain". Clearly, the premise does not entail the claim, but it is easy for the wife to identify the assumed knowledge used by the husband in order to reconstruct the intended argument correctly (i.e. "if the weather report predicts rain, then you need an umbrella"). Whilst humans are constantly handling examples like this, proposals for logic-based formalisations of the process remain underdeveloped. In this paper, we present a logic-based framework for handling enthymemes, some design features of which are influenced by aspects of relevance theory (proposed by Sperber and Wilson). In particular, we use the ideas of maximising cognitive effect and minimising cognitive effort in order to enable a proponent of an intended logical argument to construct an enthymeme appropriate for the intended recipient, and for the intended recipient to deconstruct the intended logical argument from the enthymeme. We relate our framework back to Sperber and Wilson's relevance theory via some formal properties.

1 Introduction

Argumentation is a vital aspect of intelligent behaviour by humans. Consider diverse professionals such as politicians, journalists, clinicians, scientists, and administrators, who all need to collate and analyse information looking for pros and cons for consequences of importance when attempting to understand problems and make decisions.

There are a number of proposals for logic-based formalisations of argumentation (for reviews see [CML00, PV02, BH08a]). These proposals allow for the representation of arguments for and against some claim, and for attack relationships between arguments. In a number of key examples of argumentation systems, an argument is a pair where the first item in the pair is a minimal consistent set of formulae that proves the second item, which is a formula. Furthermore, in these approaches, a key form of counterargument is an undercut: One argument undercuts another argument when the claim of the first argument negates the premises of the second argument.

Unfortunately, real arguments do not normally fit this mould. Real arguments (i.e. those presented by people in general) are normally enthymemes [Wal89]. We consider two types which we will refer to as *implicit support enthymemes* and *implicit claim enthymemes*. An implicit support enthymeme only explicitly represents some of the premises for entailing its claim. An implicit claim enthymeme not only misses some of the premises for entailing its claim, but also does not explicitly represent its claim. So if Γ is the set of premises explicitly given for an implicit support enthymeme, and α is the claim, then Γ does not entail α, but there are some implicitly assumable premises Γ' such that $\Gamma \cup \Gamma'$ is a minimal consistent set of formulae that entails α. For example, for a claim that *you need an umbrella today*, a husband may give his wife the premise *the weather report predicts rain*. Clearly, the premise does not entail the claim, but it is easy for the wife to identify the common knowledge used by the husband (i.e. *if the weather report predicts rain, then you need an umbrella today*) in order to reconstruct the intended argument correctly.

Whilst humans are constantly handling examples like this, the logical formalisation that characterises the process remains underdeveloped. Therefore, we need to investigate enthymemes because of their ubiquity in the real world, and because of the difficulties they raise for formalising and automating argumentation. If we want to build agents that can understand real arguments coming from humans, they need to identify the missing premises and missing claims with some reliability. And if we want to build agents that can generate real arguments for humans, they need to identify the premises and claims that can be missed without causing undue confusion.

Clearly, deciding how to construct or deconstruct enthymemes is difficult, and proposals for logic-based formalisations of the process remain underdeveloped. In this paper, we refine a logic-based framework for enthymemes [Hun07, BH08b], by harnessing aspects of relevance theory (proposed by Sperber and Wilson) in order to support a proponent of an intended logical argument to construct an enthymeme appropriate for the intended recipient, and for the intended recipient to deconstruct the intended logical argument from the enthymeme.

In [Hun07, BH08b], we introduced a way for each agent in a dialogue to have information about what it can use as shared knowledge, and then a proponent can use this information to remove redundant premises from an intended argument (creating an implicit support enthymeme), and a recipient can use this information to identify the necessary premises in order to recover the intended argument. In this paper, we extend and refine the proposal, by allowing each agent to also have a representation of information requirements. These are formulae that the agent would like to receive arguments about. So for example if an agent asks a question, it is making an explicit declaration of an information requirement. By introducing the notion of information requirements, we can formalise a key idea from relevance theory that the relevance of an utterance depends on maximising cognitive effect and minimising cognitive effort. This allows proponents to construct both implicit support and implicit claim enthymemes that are relevant for the intended recipient, and the recipient can deconstruct such enthymemes by using relevance criteria to overcome some of the ambiguities that normally arise when trying to understand enthymemes.

Although we do not claim that our framework formalises relevance theory in its entirety, we relate our framework back to relevance theory and it is in this sense that it is relevance-theoretic. We define properties that characterise aspects of relevance theory (in particular, the idea that an argument is relevant if it maximises cognitive effect whilst minimising cognitive effort) and show that these properties hold for our system for enthymemes.

In the following sections, we present our new framework that draws on relevance theory to facilitate construction and deconstruction of enthymemes. We start, in Section 2, by explaining in general terms how we can harness the relevance-theoretic ideas of maximising cognitive effect and minimising cognitive effort, and then present our framework in detail. For this, in Section 3, we review aspects of an existing framework for argumentation based on classical logic, that incorporates the notion of approximate arguments. We will represent each enthymeme as an approximate argument. Then by using relevance theory we show, in Section 4, how enthymemes can be constructed by a proponent for consignment to a recipient, and how they can be deconstructed by a recipient. For this, a proponent of an enthymeme can miss premises and perhaps also the claim from the intended argument that it perceives to be shared knowledge and expectations, and the recipient of an enthymeme can aim to identify the missing premises and, if necessary, the claim for the intended argument from what it perceives to be shared knowledge and expectations. In Section 4.5, we define what it means to be the most relevant enthymeme for a particular argument, proponent and recipient (i.e. the enthymeme that maximises cognitive effect whilst minimising cognitive effort), and show that, when constructing an enthymeme from an intended argument, a proponent within our framework always selects the most relevant of all, given the recipient's information requirements and what the proponent perceives to be shared knowledge between itself and the recipient. The recipient of the enthymeme is able to draw on its perception of the shared knowledge in order to deconstruct the enthymeme, taking into account the fact that the proponent will have maximised its relevance.

2 Harnessing Relevance Theory

In this section, we provide our high-level proposal. It is based on our interpretation of certain aspects of relevance theory. The proposal by Sperber and Wilson [SW86, WS02] is extensive and multifaceted, and we will focus on only part of their domain. We will only consider simple verbal or textual communications that arise in simple dialogues such as information-seeking dialogues (where participants aim to gain or share personal knowledge) or inquiry dialogues (where participants aim to jointly find a 'proof' for something) from the influential Walton and Krabbe dialogue typology [WK95]. Although many dialogue systems have been proposed for the different Walton and Krabbe type dialogues (e.g. information-seeking [Hul00, PWA03]; inquiry [MP01, PWA03, BH07, BH09]; persuasion [AMP00a, DDV00, ABM05]; negotiation [AMP00b, Hul00, STT01, MEPA02]; deliberation [HMP01]), there are no existing dialogue frameworks that consider the use of implicit claim enthymemes.

In our proposal, we assume that there are two agents x_1 and x_2, and two roles, a proponent and a recipient. They take it in turns to say something, and so each agent takes it in turns to be a proponent, and to be a recipient.

The kinds of situation we want to deal with include those that are illustrated by Examples 1 to 4 below, taken from [SW86, WS02]. In each case, background knowledge about the shared beliefs of the agents and the information requirements of the recipient is required to disambiguate and deconstruct the enthymeme, and to draw the useful relevant intended inferences.

Example 1. *Here x_2 is the recipient who has to determine the useful relevant intended inferences that should follow from the statement by x_1. For example, should either "John has bought the company that publishes The Times" be an inference or "John has bought a copy of The Times" be an inference?*

x_1 *John has bought The Times.*

In other words, we will treat "John has bought The Times" as some of the premises for an intended argument with either "John has bought the company that publishes The Times" or "John has bought a copy of The Times" being the claim of the intended argument (i.e. "John has bought The Times" is an implicit claim enthymeme).

Example 2. *Here, x_1 has asked a question, and x_1 is the recipient of the reply. In response to the the question, x_2 gives an implicit support enthymeme (and so does not give all the premises) for the argument* *with the claim "not(John has paid back the money he owed)".*

x_1	*Did John pay back the money he owed to you?*
x_2	*No, he forgot to go to the bank.*

In this example, we see that by asking the question, x_1 has made public an information requirement (i.e. x_1 wants to know if "John has paid back the money he owed" or "not(John has paid back the money he owed)" is true).

Example 3. *Here, x_1 has asked a question, and x_1 is the recipient of the reply. In response to the question, x_2 appears to have constructed an argument, without giving all the support for the argument and without giving the claim for that argument (and so the argument given by x_2 is an implicit claim enthymeme). That claim could be "I want a coffee" if x_2 wants to stay awake or it could be "not(I want a coffee)" if x_2 wants to go to sleep. So this example involves an information requirement by x_1 (i.e. x_1 has the information requirement to know whether "x_2 would like a coffee" or "not(x_2 would like a coffee)" is true) and this information requirement is met by the statement by x_2.*

x_1	*Would you like a coffee?*
x_2	*Coffee will keep me awake.*

Example 4. *Here, x_1 has asked a question, and x_1 is the recipient of the reply. In response to the question, x_2 gives an implicit support enthymeme for the argument with the claim "not(I would like to buy a flag for the RNLI)". So this example involves an information requirement by x_1 (i.e. x_1 has the information requirement to know whether "x_2 would like to buy a flag for the RNLI" or "not(x_2 would like to buy a flag for the RNLI)" is true) and this information requirement is met by the statement by x_2.*

x_1	*Would you like to buy a flag for the RNLI?*
x_2	*No, I always spend my holidays in Birmingham.*

Note, the RNLI provides lifeboat services around the UK coast, and Birmingham is a city that is not near the sea, but it does have a lot of canals.

The following is a paraphrasing of some of the key points of relevance theory pertinent to our concerns [SW86, WS02].

- In general, cognition by an agent aims to maximise the relevance of any intellectual process to the needs or interests of that agent.
- In processing input, cognition by an agent aims to maximise the useful relevant inferences that can be drawn from that input.
- In processing communicated input from another agent, cognition by an agent aims to maximise the useful relevant intended inferences that can be drawn from that input.

These observations lead to the following "relevance-theoretic comprehension procedure": When processing communicated input from another agent, "take the path of least resistance" when searching for useful relevant intended inferences of the input, and stop processing when these inferences meet the expectations of the agent for that input.

Since our aim is to provide a logic-based characterisation of the relevance-theoretic comprehension principle in an argumentation-theoretic dialogue framework, and in particular for using enthymemes in dialogues, we need to conceptualise this principle in terms of enthymemes. For this, we summarise in our own words the relevance principle that will be used by the proponent and recipient when constructing and deconstructing (respectively) enthymemes. We call this the **enthymeme relevance principle**, and we define it informally as follows.

- The intended argument which the proponent wishes to make manifest to the recipient is relevant enough (for the recipient's information needs) to make it worthwhile for the recipient to process the enthymeme that is sent by the proponent.
- The enthymeme sent by the proponent is the most relevant one (in the sense of being understandable but not redundant) that the proponent could have used to communicate the intended argument.

In other words, the enthymeme used for an intended argument should be beneficial (in providing useful information to the recipient) and it should be efficient (in providing sufficient but not excessive information to the recipient). We leave a formalisation of this principle until Section 4.5, at which point we will be able to show how our proposed framework meets this principle.

3 Logical Argumentation

In this section, we review part of an existing proposal for logic-based argumentation [BH01]. We consider a classical propositional language $\mathcal{L}$ with classical deduction denoted by the symbol $\vdash$. We use $\alpha, \beta, \gamma, \ldots$ to denote formulae and $\Delta, \Phi, \Psi, \ldots$ to denote sets of formulae. Also, $\top$ denotes tautology, and $\bot$ denotes falsity, as usual. For the following definitions, we first assume a knowledgebase Δ (a finite set of formulae) and use this Δ throughout.

The paradigm for the approach is a large repository of information, represented by Δ, from which arguments can be constructed for and against arbitrary claims. Apart from information being understood as declarative statements, there is no *a priori* restriction on the contents, and the pieces of information in the repository can be arbitrarily complex. Therefore, Δ is not expected to be consistent. It need not even be the case that every single formula in Δ is consistent.

The framework adopts a very common intuitive notion of a logical argument. Essentially, a logical argument is a set of formulae that can be used to classically prove some claim, together with that claim. Each claim is represented by a formula.

Definition 1. *A* **logical argument** *is a pair $\langle \Phi, \alpha \rangle$ such that: (1) $\Phi \subseteq \Delta$; (2) $\Phi \nvdash \bot$; (3) $\Phi \vdash \alpha$; and (4) there is no $\Phi' \subset \Phi$ such that $\Phi' \vdash \alpha$. We say that $\langle \Phi, \alpha \rangle$ is a logical argument for α. We call α the* **claim** *of the argument and Φ the* **support** *of the argument (we also say that Φ is a support for α).*

Example 5. *Let $\Delta = \{\alpha, \alpha \to \beta, \gamma \to \neg\beta, \gamma, \delta, \delta \to \beta, \neg\alpha, \neg\gamma\}$. Some arguments are:*

$$\langle \{\alpha, \alpha \to \beta\}, \beta \rangle$$
$$\langle \{\neg\alpha\}, \neg\alpha \rangle$$
$$\langle \{\alpha \to \beta\}, \neg\alpha \vee \beta \rangle$$
$$\langle \{\neg\gamma\}, \delta \to \neg\gamma \rangle$$

We impose the constraint that the support of an argument must be minimal (condition (4) of the above definition). Although there is a computational cost involved in assuming a minimal set of premises for an argument [EG95, PWA03], this constraint is often imposed when specifying concrete argumentation systems (e.g. [PS97, AC02, BH01, GS04, Hun07]), which assume that efficient algorithms for argumentation will continue to be developed that ameliorate the associated cost (see [BH08a] for a discussion). The minimality constraint is a form of relevancy for arguments: By minimising the set of premises, unnecessary premises are avoided and we are able to identify precisely the reasons for inferring the claim. The minimality constraint also avoids any untoward illucutionary effects that may be caused by unnecessary premises.

Since argumentation is often used when there is conflicting information or situations, it is useful to conceptualise the notion of counterargument. We will consider two commonly considered types, namely rebuttals and undercuts. A rebuttal of an argument directly negates the claim of the argument, whereas an undercut for an argument directly opposes the support of the argument. More formally, a logical argument $\langle \Psi, \beta \rangle$ is a **rebuttal** for a logical argument $\langle \Phi, \alpha \rangle$ if and only if $\beta \leftrightarrow \neg\alpha$ is a tautology, and an **undercut** for a logical argument $\langle \Phi, \alpha \rangle$ is a logical argument $\langle \Psi, \neg(\phi_1 \wedge \ldots \wedge \phi_n) \rangle$ such that $\{\phi_1, \ldots, \phi_n\} \subseteq \Phi$.

Example 6. *Let $\Delta = \{\alpha, \gamma, \alpha \rightarrow \neg\beta, \gamma \rightarrow \neg\alpha, \neg\alpha \rightarrow \beta\}$. So for the following logical argument (below left), we have counterarguments (below right).*

$$\langle \{\alpha, \alpha \rightarrow \neg\beta\}, \neg\beta \rangle \quad \left\{ \begin{array}{l} \textit{An undercut is } \langle \{\gamma, \gamma \rightarrow \neg\alpha\}, \neg\alpha \rangle \\ \textit{A rebuttal is } \langle \{\gamma, \gamma \rightarrow \neg\alpha, \neg\alpha \rightarrow \beta\}, \beta \rangle \end{array} \right.$$

In the development of logic-based argumentation systems, identifying and evaluating constellations of logical arguments and counterarguments has been the focus of interest (for example see [Pol92, SL92, Dun95, PS97, Vre97, AC02, Ben03, GS04, DKT06, ABC07, Mod07] and for reviews see [CML00, PV02, BH08a]). However, for our purposes, in this paper, we will not explicitly consider counterarguments further, although they do play a significant role in the use of enthymemes. Indeed, it is easy to extend our running examples to include counterarguments.

We now turn to a review of the notion of approximate arguments [Hun07], and we will use this to conceptualise enthymemes. An **approximate argument** is a pair $\langle \Phi, \alpha \rangle$ where $\Phi \subseteq \mathcal{L}$ and $\alpha \in \mathcal{L}$. This is a very general definition. It does not assume that Φ is consistent, or even that it entails α. Following our terminology for logical arguments, we will refer to Φ as the support and α as the claim.

In this paper, we restrict consideration to particular kinds of approximate arguments that relax the definition of a logical argument: If $\Phi \vdash \alpha$, then $\langle \Phi, \alpha \rangle$ is **valid**; If $\Phi \not\vdash \bot$, then $\langle \Phi, \alpha \rangle$ is **consistent**; If $\Phi \vdash \alpha$, and there is no $\Phi' \subset \Phi$ such that $\Phi' \vdash \alpha$, then $\langle \Phi, \alpha \rangle$ is **minimal**; And if $\Phi \vdash \alpha$, and $\Phi \not\vdash \bot$, then $\langle \Phi, \alpha \rangle$ is **expansive** (i.e. it is valid and consistent, but it may have unnecessary premises).

In addition, we require a further kind of approximate argument that has the potential to be transformed into a logical argument: If $\Phi \not\vdash \alpha$, and $\Phi \not\vdash \neg\alpha$, then $\langle \Phi, \alpha \rangle$ is a **precursor** (i.e. it is a precursor for a logical argument). Therefore, if $\langle \Phi, \alpha \rangle$ is a precursor, then there exists some $\Psi \subset \mathcal{L}$ such that $\Phi \cup \Psi \vdash \alpha$ and $\Phi \cup \Psi \not\vdash \bot$, and hence $\langle \Phi \cup \Psi, \alpha \rangle$ is expansive.

Example 7. *Let $\Delta = \{\alpha, \neg\alpha \vee \beta, \gamma, \neg\beta, \beta, \neg\gamma, \neg\beta \vee \gamma\}$. Some approximate arguments from Δ that are valid include $\{A_1, A_2, A_3, A_4, A_5\}$ of which $\{A_1, A_3, A_5\}$ are expansive, $\{A_2, A_5\}$ are minimal, and A_5 is a logical argument. Also, some approximate arguments that are not valid include $\{A_6, A_7\}$ of which A_6 is a precursor.*

$$\begin{array}{l} A_1 = \langle \{\alpha, \neg\alpha \vee \beta, \gamma, \beta\}, \beta \rangle \\ A_2 = \langle \{\gamma, \neg\gamma\}, \beta \rangle \\ A_3 = \langle \{\alpha, \neg\alpha \vee \beta, \gamma\}, \beta \rangle \\ A_4 = \langle \{\alpha, \neg\alpha \vee \beta, \gamma, \neg\gamma\}, \beta \rangle \\ A_5 = \langle \{\alpha, \neg\alpha \vee \beta\}, \beta \rangle \\ A_6 = \langle \{\neg\alpha \vee \beta\}, \beta \rangle \\ A_7 = \langle \{\neg\alpha \vee \beta, \neg\beta \vee \gamma, \neg\gamma\}, \beta \rangle \end{array}$$

Some observations that we can make concerning approximate arguments include: (1) If $\langle \Gamma, \alpha \rangle$ is expansive, then there is a $\Phi \subseteq \Gamma$ such that $\langle \Phi, \alpha \rangle$ is a logical argument; (2) If $\langle \Phi, \alpha \rangle$ is minimal, and $\langle \Phi, \alpha \rangle$ is expansive, then $\langle \Phi, \alpha \rangle$ is a logical argument; (3) If $\langle \Phi, \alpha \rangle$ is a logical argument, and $\Psi \subset \Phi$, then $\langle \Psi, \alpha \rangle$ is a precursor; and (4) If $\langle \Gamma, \alpha \rangle$ is a precursor, then $\langle \Gamma, \alpha \rangle$ is consistent.

Note also, the definitions presented in this section can be used directly with first-order classical logic, so Δ and α are from the first-order classical language [BH08a].

4 A Relevance-Theoretic Framework for Enthymemes

In this section, we present our relevance-theoretic framework for enthymemes. It constitutes our detailed proposal, and later we will explain how it relates to our informal notion of the Enthymeme Relevance Principle.

4.1 Representing enthymemes

Given a logical argument, an enthymeme is simply an approximate argument that can be generated from it. It may be a precursor for the intended argument (i.e. an implicit support enthymeme), or it may be precursor of the intended argument that has a weakened claim (i.e. an implicit claim enthymeme).

Definition 2. *Let $\langle \Phi, \beta \rangle$ be an approximate argument and $\langle \Psi, \alpha \rangle$ be a logical argument.*

$\langle \Phi, \beta \rangle$ *is an* **enthymeme** *for* $\langle \Psi, \alpha \rangle$ *iff* $\Phi \subset \Psi$ *and* $\alpha \vdash \beta$

So if a proponent has a logical argument that it wishes a recipient to be aware of (we refer to this argument as the **intended argument**), then the proponent may send an enthymeme instead of the intended argument to the recipient. We refer to whatever the proponent sends to the recipient (whether the intended argument or an enthymeme for that intended argument) as the **real argument**.

Example 8. *Let α be "you need an umbrella today", and β be "the weather report predicts rain". So for an intended argument $\langle \{\beta, \beta \rightarrow \alpha\}, \alpha \rangle$, the real argument sent by the proponent to the recipient may be $\langle \{\beta\}, \alpha \rangle$. This is an example of an implicit support enthymeme.*

Example 9. *Returning to Example 3, let α = "I would like a coffee", β = "Coffee keeps me awake", and γ = "I want to stay awake". The intended argument that x_2 wishes to communicate to x_1 is $\langle \{\beta, \gamma, \beta \wedge \gamma \rightarrow \alpha\}, \alpha \rangle$, and the enthymeme is $\langle \{\beta\}, \top \rangle$. This is an example of an implicit claim enthymeme.*

We can see the use of enthymemes both in monological argumentation, for example by a politician giving a lecture (as illustrated next) or a journalist writing an article, and in dialogical argumentation, for example lawyers arguing in court, or academics debating in a seminar.

Example 10. *Consider a politician who says "The government will support the expansion of JFK airport with new legislation because it will be good for the local and national economy. And we will address the disturbance to local people with tighter regulations on night time flights and on older more polluting aircraft". This short speech can be analysed as follows: Let α be "The government will support the expansion of JFK airport with new legislation", let β be "the expansion of JFK airport will be good for everyone", let γ be "expansion will improve the local and national economy", let δ be "the local environment will suffer pollution", let ϕ be "there will be tighter regulations on night time flights", and let ψ be "there will be tighter regulations on older more polluting aircraft". So in the first sentence of the speech, the politician effectively gives the enthymeme $\langle \{\gamma\}, \alpha \rangle$, and then in the second sentence, the politician gives the enthymemes $\langle \{\delta \rightarrow \neg\beta\}, \neg\beta \rangle$, and $\langle \{\phi, \psi\}, \neg\delta \rangle$. The intended arguments for each of these enthymemes are as follows. As an aside, A_2 is an undercut to A_1 and A_3 is an undercut to A_2.*

$$
\begin{aligned}
A_1 &= \langle \{\gamma, \gamma \rightarrow \beta, \beta \rightarrow \alpha\}, \alpha \rangle \\
A_2 &= \langle \{\delta, \delta \rightarrow \neg\beta\}, \neg\beta \rangle \\
A_3 &= \langle \{\phi, \psi, \phi \wedge \psi \rightarrow \neg\delta\}, \neg\delta \rangle
\end{aligned}
$$

In the next section, we consider the information required for enthymemes, and then in the subsequent sections, we consider how to construct and deconstruct enthymemes.

4.2 Kinds of information in framework

In order for a proponent to construct an intended argument, it needs a knowledgebase from which to obtain premises. Then if it wants to construct the relevant enthymeme for a particular recipient, it needs further knowledge about the recipient. Similarly, for the recipient to deconstruct an enthymeme from a proponent, it also needs knowledge about the proponent to determine the most relevant deconstructions. In this, we are assuming the agents are cooperative in that the proponent wants the recipient to get the intended argument.

In general, since there can be more than one enthymeme that can be generated from an intended argument, a proponent i needs to choose which to send to a recipient j. To facilitate this selection, the proponent consults what it believes is shared knowledge for i and j.

We assume that each agent i has a finite knowledgebase Δ_i of beliefs (i.e. formulae from $\mathcal{L}$), called a **perbase**, that is its personal knowledgebase, and so if i is a proponent, the support of the intended argument comes from Δ_i.

In addition, agent i has a function $\sigma_{i,j} : \mathcal{L} \mapsto [0, 1]$, called a **cobase function**, that represents the degree to which an agent i believes each formula in the language can be used as shared knowledge between i and j. For $\alpha \in \mathcal{L}$, agent i believes α is knowledge that can be used as shared knowledge between i and j if and only if $\sigma_{i,j}(\alpha) > 0$. The higher the value of $\sigma_{i,j}(\alpha)$, the more i regards it is possible to use α as shared knowledge between i and j. So if $\sigma_{i,j}(\alpha) = 0$, then i believes that it cannot use α as shared knowledge between i and j, whereas if $\sigma_{i,j}(\alpha) = 1$, then i believes that there is no knowledge that it is more able to use as shared knowledge between i and j than α.

Example 11. *In Example 8, with $\beta, \beta \rightarrow \alpha \in \Delta_i$, proponent i could have the cobase function $\sigma_{i,j}$ where $\sigma_{i,j}(\beta \rightarrow \alpha) = 1$, representing that the premise $\beta \rightarrow \alpha$ is superfluous in any real argument consigned by proponent i to recipient j.*

We assume that for each cobase function, only a finite part of the language has a non-zero value, and so using $\sigma_{i,j}$ and its **cobase threshold** τ_i ($0 < \tau_i \leq 1$), agent i can construct a finite set of knowledge $\Sigma_{i,j}$, called a **cobase**, that represents what an agent i believes it can use as shared knowledge between i and j where $\Sigma_{i,j} = \{\alpha \in \mathcal{L} \mid \sigma_{i,j}(\alpha) \geq \tau_i\}$.

As an illustration of how we may set up the cobase, we may have $\sigma_{i,j}(\phi) = 1$ for any ϕ that either agent has previously expressed in the dialogue between i and j. Furthermore, for other formulae ψ, we may have $\sigma_{i,j}(\psi)$ denotes the proportion of other agents that have expressed ψ in a previous dialogue with i (and thereby offering a probabilistic interpretation for $\sigma_{i,j}(\psi)$).

Given that $\sigma_{i,j}$ reflects the perception i has of the shared knowledge between i and j, and $\sigma_{j,i}$ reflects the perception j has of the shared knowledge between i and j, it is not necessarily the case that $\sigma_{i,j} = \sigma_{j,i}$. Furthermore, it is not necessarily the case that i regards the shared knowledge between i and j as being consistent, and so it is possible, for some α, that $\sigma_{i,j}(\alpha) > 0$ and $\sigma_{i,j}(\neg\alpha) > 0$.

The third type of information is an agenda which stores the information requirements of an agent. An information requirement for an agent is a formula α that the agent would like to know if there are reasons to believe it holds (i.e. whether there is an argument with the claim α is for example an answer to a question). For an agent i, an **agenda function** is a function $\pi_i : \mathcal{L} \mapsto [0, 1]$ such that for a formula ϕ, if $\pi_i(\phi) > 0$, then ϕ is an **information requirement** for the agent i. The magnitude of $\pi_i(\phi)$ denotes the degree to which ϕ is an information requirement for i, and so the higher the value, the more pressing the information requirement is in the dialogue. In addition, we denote the **agenda** as $\Pi_i = \{\phi \mid \pi_i(\phi) > 0\}$. We assume that $\phi \in \Pi_i$ if and only if $\neg\phi \in \Pi_i$.

Importantly, we assume that the agenda is public knowledge, and so all agents can see the contents of every agent's agenda. So given π_i, other agents can determine what may to some degree be relevant claims for agent i. After asking a question, an agent normally has an information requirement (indeed that is why they asked the question in the first place). In most of the examples in this paper, we will represent an information requirement ϕ generated by a simple yes/no type of question by $\pi_i(\phi) = 1$ and $\pi_i(\neg\phi) = 1$.

Example 12. *Returning to Example 3, for the question by x_1 "Would x_2 like a coffee?", x_1 has the information requirements "x_2 would like a coffee" and "not(x_2 wants a coffee)", such that π_{x_1}(x_2 wants a coffee) $= \pi_{x_1}$(not(x_2 wants a coffee)) $= 1$.*

Information requirements can arise for other reasons. For example, an agent normally has an interest in arguments that concern its own safety, welfare, or economic interests. This means that there will be information requirements that reflect this.

As summary of the information used in our framework, there are three types we have available for construction and deconstruction of enthymemes.

- Perbase Δ_i for representing personal knowledge of an agent i.
- Cobase function $\sigma_{i,j}$ for representing what i believes to be shared knowledge for i and j.
- Agenda function π_i for representing the publicly available information requirements for agent i.

Given these three kinds of knowledge, we can explain in the following section how a proponent can construct enthymemes for a recipient, and how a recipient can deconstruct enthymemes from a proponent.

4.3 Constructing enthymemes

In general, we see that the construction process involves the proponent first generating the intended argument $\langle\Phi, \alpha\rangle$ and then forming an enthymeme $\langle\Psi, \alpha\rangle$ from that argument by letting Ψ be a subset of Φ by removing shared knowledge (we call this the **encodation step**). The proponent may then perhaps choose to weaken the claim to a tautology, to give the approximate argument $\langle\Psi, \top\rangle$, and so $\alpha \vdash \top$. We illustrate some of the issues in the construction process by the following example.

Example 13. *Let $\alpha =$ "John has bought The Times", $\beta =$ "John has bought a copy of The Times" and $\gamma =$ "John has bought the company that publishes The Times". Agent x_1 has the beliefs $\{ \alpha, \alpha \to \beta, \beta, \alpha \to \gamma \}$ in its perbase. Agent x_1 intends to communicate to agent x_2 the argument $\langle\{\alpha, \alpha \to \beta\}, \beta\rangle$.*
Agent x_1 believes that the beliefs $\alpha \to \beta$ and $\alpha \to \gamma$ are part of the shared knowledge between itself and agent x_2 (i.e. $\alpha \to \beta, \alpha \to \gamma \in \Sigma_{x_1,x_2}$), and so agent x_1 is able to remove the belief $\alpha \to \beta$ to get the enthymeme $\langle\{\alpha\}, \beta\rangle$.
Agent x_1 decides that it does not need to send the implicit support enthymeme $\langle\{\alpha\}, \beta\rangle$ but is able to send the weaker implicit claim enthymeme $\langle\{\alpha\}, \top\rangle$. This must mean that x_1 is confident that x_2 will be able to correctly deconstruct the approximate argument $\langle\{\alpha\}, \top\rangle$ into the intended argument $\langle\{\alpha, \alpha \to \beta\}, \beta\rangle$, and so must believe $\langle\{\alpha, \alpha \to \beta\}, \beta\rangle$ is more relevant to x_2 than $\langle\{\alpha, \alpha \to \gamma\}, \gamma\rangle$.
A reason for this might be that β is a more pressing information requirement for x_2 than γ (and so creates greater cognitive effect). Another reason for this (assuming that β and γ are equally pressing information requirements for x_2) might be that x_1 believes that there are only two formulae that are part of the shared knowledge with x_2 and that use the atom α, namely $\alpha \to \beta$ and $\alpha \to \gamma$, and that it is more possible to use $\alpha \to \beta$ as shared knowledge with x_2 than $\alpha \to \gamma$ (i.e. $\sigma_{x_1,x_2}(\alpha \to \beta) > \sigma_{x_1,x_2}(\alpha \to \gamma)$, and so using $\alpha \to \beta$ requires less cognitive effort). Therefore x_1 is confident that x_2 will correctly deconstruct the approximate argument.

Clearly, this sketch leaves many important and interesting questions unanswered. The construction process needs to be formalised in such a way that the relevance-theoretic principles of maximising cognitive effect and minimising cognitive effort are preserved, and the deconstruction process needs to be formalised so as to draw on the fact that the proponent has aimed for the preservation of these principles.

Now we formalise the construction process starting with the encodation step. Consider an agent i who has an intended argument $\langle \Phi, \alpha \rangle$ that it wants agent j to be aware of. So Φ is a subset of Δ_i, and i is the proponent of the argument and j is the recipient of the argument. By reference to its representation of the shared knowledge $\sigma_{i,j}$, agent i will remove premises ϕ from Φ for which $\sigma_{i,j}(\phi)$ is greater than or equal to its cobase threshold τ_i (i.e. agent i will remove premises ϕ from Φ which are in its cobase for j, $\Sigma_{i,j}$). The result of this encodation process is either the intended argument or an enthymeme for that argument.

Definition 3. *For an argument $\langle \Phi, \alpha \rangle$, the* **encodation** *of $\langle \Phi, \alpha \rangle$ from a proponent i for a recipient j, denoted $C(\langle \Phi, \alpha \rangle, \Sigma_{i,j})$, is the approximate argument $\langle \Psi, \alpha \rangle$, where $\Psi = \Phi \setminus \Sigma_{i,j}$.*

Example 14. *In Example 11, when $\sigma_{i,j}(\beta \rightarrow \alpha) = 1$, and for all τ_i (as $0 < \tau_i \leq 1$), $C(\langle \{\beta, \beta \rightarrow \alpha\}, \alpha \rangle, \Sigma_{i,j})$ is $\langle \{\beta\}, \alpha \rangle$.*

So given a cobase function $\sigma_{i,j}$ and a cobase threshold τ_i, it is simple for a proponent i to obtain an encodation for a recipient j. Note, for an intended argument A, it is possible that $C(A, \Sigma_{i,j}) = B$ where the support of B is the emptyset. This raises the question of whether a proponent would want to send a real argument with empty support to another agent, since it is in effect "stating the obvious". Nevertheless, there may be a rhetorical or pragmatic motivation for such a real argument. For example, when a husband issues a reminder like *don't forget your umbrella* to his wife when the shared knowledge includes the facts that the month is April, the city is London, and London has many showers in April. Hence, *don't forget your umbrella* is the claim, and the support for this real argument is empty.

In order for a proponent i to judge the ability of the intended recipient j to be able to deconstruct an enthymeme, the proponent can use the information from the cobase function to construct an ordering over subsets of the cobase in order to ascertain the possibility for confusion. For this, we use the following preference relation over $\wp(\mathcal{L})$ adapted from [CRS93].

Definition 4. *Let Φ and Ψ be two finite non-empty subsets of $\mathcal{L}$. Φ is* **preferred** *to Ψ, denoted $\Phi >_{i,j} \Psi$ iff for all $\phi \in \Phi \setminus \Psi$, there is a $\psi \in \Psi \setminus \Phi$ such that $\sigma_{i,j}(\phi) > \sigma_{i,j}(\psi)$. For all non-empty subsets Φ of $\mathcal{L}$, $\emptyset >_{i,j} \Phi$.*

As we see in the following proposition, this preference relation means that if Φ is a proper subset of Ψ, then Φ is preferred to Ψ.

Proposition 1. *Let Φ and Ψ be two finite subsets of $\mathcal{L}$. If $\Phi \subset \Psi$, then $\Phi >_{i,j} \Psi$.*

Proof: As $\Phi \subset \Psi$, we have $\Phi \setminus \Psi = \emptyset$, hence it is clear that for all $\phi \in \Phi \setminus \Psi$ there is a $\psi \in \Psi \setminus \Phi$ such that $\sigma_{i,j}(\phi) > \sigma_{i,j}(\psi)$, hence $\Phi >_{i,j} \Psi$. $\square$

We use this preference relation, together with the definition of encodation, in the following definition for construction. The construction process is simple with regard to removing unnecessary premises to create the encodation: If a premise is in the cobase above the threshold, then remove it. The construction process is slightly more complex with regard to the claim. In order to simplify the claim to $\top$: (1) It should not be the case that there is an information requirement δ that is more pressing that α, such that there is some subset of the cobase that can be used with the premises that are not removed in the encodation to produce an expansive argument for the claim δ; and (2) It should not be the case that there is an information requirement δ that is as equally pressing as α, such that there is a subset of the cobase that is not less preferred to that removed in the encodation step (using the $>_{i,j}$ preference relation) and that could be used with the premises that are not removed in the encodation to generate an expansive argument for δ.

Definition 5. *The* **construction** *of the enthymeme for the intended argument $\langle \Phi, \beta \rangle$ from a proponent i for a recipient j results in an approximate argument $\langle \Psi, \beta \rangle$ where $C(\langle \Phi, \alpha \rangle, \Sigma_{i,j})$ is $\langle \Psi, \alpha \rangle$, $\equiv$ denotes logical* *equivalence, and β is assigned as follows:*

$$\beta = \begin{cases} \top & \textit{iff} \quad \textit{(1) there does not exist } \delta \textit{ such that } \pi_i(\alpha) < \pi_i(\delta) \\ & \qquad \textit{and there exists } \Gamma \subseteq \Sigma_{i,j} \textit{ such that } \langle \Psi \cup \Gamma, \delta \rangle \textit{ is an expansive argument} \\ & \quad\ \textit{(2) there does not exist } \delta \textit{ such that } \pi_i(\alpha) = \pi_i(\delta) \textit{ and } \alpha \not\equiv \delta \\ & \qquad \textit{and there exists } \Gamma \subseteq \Sigma_{i,j} \textit{ such that } \langle \Psi \cup \Gamma, \delta \rangle \textit{ is an expansive argument} \\ & \qquad \textit{and either } \Gamma >_{i,j} (\Phi \setminus \Psi) \textit{ or } (\Gamma \not>_{i,j} (\Phi \setminus \Psi) \textit{ and } (\Phi \setminus \Psi) \not>_{i,j} \Gamma) \\ \alpha & \textit{otherwise.} \end{cases}$$

Overall, the construction process is intended to capture consideration of efficiency in both the premises and in the claim of the enthymeme, but not at the expense of making it problematical for the recipient to recover the intended argument. We now illustrate the intuition behind the construction process with the use of some examples, and then return to the running examples from earlier in the paper.

Example 15. *Let x_1 be the proponent and x_2 be the recipient. The intended argument is $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$. We have $\sigma_{x_1,x_2}(\alpha \rightarrow \beta) = \sigma_{x_1,x_2}(\alpha \rightarrow \gamma) = 1$. Also, $\pi_{x_2}(\gamma) = \pi_{x_2}(\beta) > 0$.*
x_1 first applies the encodation step to get the enthymeme $\langle\{\alpha\}, \beta\rangle$.
Using the definition of construction, the proponent is unable to reduce the claim to $\top$ and so the real argument constructed is $\langle\{\alpha\}, \beta\rangle$.
This matches our intuition. We would not want x_1 to send the real argument $\langle\{\alpha\}, \top\rangle$ as (assuming x_2's view of the shared knowledge is the same as that of x_1) the arguments $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$ and $\langle\{\alpha, \alpha \rightarrow \gamma\}, \gamma\rangle$ are of equal relevance to x_2 in the sense that they each create equal magnitude of cognitive effect whilst requiring the same amount of cognitive effort to deconstruct (as β and γ are equally pressing information requirements for x_2 and each argument is as easy as the other, in the sense of using equally preferred subsets of the cobase, for x_2 to reconstruct) and so x_2 would have no way of knowing which was the intended argument.

Example 16. *Let x_1 be the proponent and x_2 be the recipient. The intended argument is $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$. We have $\sigma_{x_1,x_2}(\alpha \rightarrow \beta) = 1$, $\sigma_{x_1,x_2}(\alpha \rightarrow \gamma) = 0.5$. Also, $\pi_{x_2}(\gamma) = \pi_{x_2}(\beta) > 0$.*
x_1 first applies the encodation step to get the enthymeme $\langle\{\alpha\}, \beta\rangle$.
Using the definition of construction, the proponent reduces the claim to $\top$ and so the real argument constructed is $\langle\{\alpha\}, \top\rangle$.
This matches our intuition. Assuming that x_2's view of the shared knowledge is the same as that of x_1, although β and γ are equally pressing information requirements for x_2, the argument $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$ is easier for x_2 to reconstruct than the argument $\langle\{\alpha, \alpha \rightarrow \gamma\}, \gamma\rangle$ (given the ranking over the cobase) and so is more relevant to x_2 (as it requires less cognitive effort).

Example 17. *Let x_1 be the proponent and x_2 be the recipient. The intended argument is $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$. We have $\sigma_{x_1,x_2}(\alpha \rightarrow \beta) = \sigma_{x_1,x_2}(\alpha \rightarrow \gamma) = 1$. Also, $\pi_{x_2}(\gamma) < \pi_{x_2}(\beta)$.*
x_1 first applies the encodation step to get the enthymeme $\langle\{\alpha\}, \beta\rangle$.
Using the definition of construction, the proponent reduces the claim to $\top$ and so the real argument constructed is $\langle\{\alpha\}, \top\rangle$.
This matches our intuition. Assuming that x_2's view of the shared knowledge is the same as that of x_1, although x_2 would find it as easy to reconstruct the argument $\langle\{\alpha, \alpha \rightarrow \gamma\}, \gamma\rangle$ as $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$, β is a more pressing information requirement for x_2 than γ, and so the argument $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$ is more relevant to x_2 than $\langle\{\alpha, \alpha \rightarrow \gamma\}, \gamma\rangle$ (as it produces greater cognitive effect).

Example 18. *Returning to Example 1, let α = "John has bought The Times", β = "John has bought a copy of The Times" and γ = "John has bought the company that publishes The Times". Agent x_1 has the beliefs $\{\alpha, \alpha \rightarrow \beta, \beta, \alpha \rightarrow \gamma\}$. Agent x_1 intends to communicate to agent x_2 the argument $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$. Now assume that $\sigma_{x_1,x_2}(\alpha \rightarrow \beta) = 0.9$ and $\sigma_{x_1,x_2}(\alpha \rightarrow \gamma) = 0.6$. Hence, the encodation is $\langle\{\alpha\}, \beta\rangle$. Also assume that $\pi_{x_1}(\beta) = \pi_{x_1}(\gamma) > 0$. However, because $\sigma_{x_1,x_2}(\alpha \rightarrow \beta) > \sigma_{x_1,x_2}(\alpha \rightarrow \gamma)$, we have $\{\alpha \rightarrow \beta\} >_{i,j} \{\alpha \rightarrow \gamma\}$, and therefore the real argument constructed is $\langle\{\alpha\}, \top\rangle$.*

Example 19. *Returning to Example 2, let* α = *"John paid back the money he owed",* β = *"John forgot to go to the bank",* γ = *"John needs to borrow some money", and* δ = *"John can't afford to come out tonight". Suppose the intended argument that* x_2 *wants to communicate to* x_1 *is* $\langle\{\beta, \beta \rightarrow \neg\alpha\}, \neg\alpha\rangle$*. Also suppose that* $\sigma_{x_1,x_2}(\beta \rightarrow \neg\alpha) = \sigma_{x_1,x_2}(\beta \rightarrow \gamma) = \sigma_{x_1,x_2}(\beta \rightarrow \delta) = 0.5$*, and* $\tau_{x_1} = 0.5$*. And suppose that* $\pi_{x_1}(\neg\alpha) = \pi_{x_1}(\gamma) = \pi_{x_1}(\delta) = 1$*. So even though* $\neg\alpha \in \Pi_i$*, the real argument constructed is* $\langle\{\beta\}, \neg\alpha\rangle$*, and not* $\langle\{\beta\}, \top\rangle$*. This is because there is an information requirement* γ *such that* $\pi_{x_1}(\neg\alpha) = \pi_{x_1}(\gamma)$ *and a* $\Gamma = \{\beta \rightarrow \gamma\}$ *such that* $\Gamma \not\succ_{i,j} \{\beta \rightarrow \neg\alpha\}$ *and* $\{\beta \rightarrow \neg\alpha\} \not\succ_{i,j} \Gamma$*, and* $\langle\{\beta\} \cup \Gamma, \gamma\rangle$ *is an expansive argument. (There is also an information requirement* δ *such that* $\pi_{x_1}(\neg\alpha) = \pi_{x_1}(\delta)$ *and a* $\Gamma' = \{\beta \rightarrow \delta\}$ *such that* $\Gamma' \not\succ_{i,j} \{\beta \rightarrow \neg\alpha\}$ *and* $\{\beta \rightarrow \neg\alpha\} \not\succ_{i,j} \Gamma'$*, and* $\langle\{\beta\} \cup \Gamma', \gamma\rangle$ *is an expansive argument.)*

Example 20. *Returning to Example 3, let* α = *"I would like a coffee",* β = *"Coffee keeps me awake", and* γ = *"I want to stay awake". The intended argument that* x_2 *wishes to communicate to* x_1 *is* $\langle\{\beta, \gamma, \beta \wedge \gamma \rightarrow \alpha\}, \alpha\rangle$*. Now assume that* $\sigma_{x_1,x_2}(\beta \wedge \gamma \rightarrow \alpha) = \sigma_{x_1,x_2}(\beta \wedge \neg\gamma \rightarrow \neg\alpha) = \sigma_{x_1,x_2}(\gamma) = 1$*, and* $\tau_{x_1} = 0.5$*. Also assume that* $\pi_{x_1}(\alpha) = \pi_{x_1}(\neg\alpha) = 1$*. So the encodation of the intended argument is* $\langle\{\beta\}, \alpha\rangle$*. In order to decide whether to reduce the claim to* $\top$*, the only other information requirement to consider is* $\neg\alpha$*, but there is no* $\Gamma \subseteq \Sigma_{i,j}$ *such that* $\langle\{\beta\} \cup \Gamma, \neg\alpha\rangle$ *is an expansive argument, and so the real argument constructed is* $\langle\{\beta\}, \top\rangle$*.*

Example 21. *Returning to Example 4, let* α = *"I would like to buy a flag for RNLI",* β = *"I always spend my holidays in Birmingham",* γ = *"I have no need of the services of the RNLI",* δ = *"There are a lot of canals in Birmingham", and* ϵ = *"It would be good if the RNLI expanded their services to Birmingham canals". Let* $\sigma_{x_1,x_2}(\beta \rightarrow \gamma) = \sigma_{x_1,x_2}(\gamma \rightarrow \neg\alpha) = \sigma_{x_1,x_2}(\beta \wedge \delta \rightarrow \epsilon) = \sigma_{x_1,x_2}(\epsilon \rightarrow \alpha) = \sigma_{x_1,x_2}(\delta) = 0.5$*, and* $\tau_{x_1} = 0.5$*. Also let* $\pi_{x_1}(\alpha) = \pi_{x_1}(\neg\alpha) = 1$*. The intended argument by* x_2 *is* $\langle\{\beta, \beta \rightarrow \gamma, \gamma \rightarrow \neg\alpha\}, \neg\alpha\rangle$*, and the encodation of this is* $\langle\{\beta\}, \neg\alpha\rangle$*. Since there is a* $\Gamma \subseteq \Sigma_{x_1,x_2}$ *(*$\Gamma = \{\delta, \beta \wedge \delta \rightarrow \epsilon, \epsilon \rightarrow \alpha\}$*) such that* $\langle\{\beta\} \cup \Gamma, \neg\alpha\rangle$ *is a logical argument and* $\Gamma \not\succ_{i,j} \{\beta \rightarrow \gamma, \gamma \rightarrow \neg\alpha\}$ *and* $\{\beta \rightarrow \gamma, \gamma \rightarrow \neg\alpha\} \not\succ_{i,j} \Gamma$*, it is not possible to weaken the claim of the enthymeme and the real argument constructed is* $\langle\{\beta\}, \neg\alpha\rangle$*.*

Now we consider another example from [WS02] in order to see how the interplay of benefit and efficiency is taken into account in the construction process.

Example 22. *Consider an agent* x_1 *who is wanting to know what is for lunch. Furthermore, imagine that they have a preference for vegetarian food, but in any case they definitely do not want chicken (and perhaps will seek lunch elsewhere if this is served). This could perhaps be represented by the information requirements* υ = *"It is a vegetarian dish for lunch" and* χ = *"It is a chicken dish for lunch" where*

$$\pi_{x_1}(\chi \wedge \neg\upsilon) > \pi_{x_1}(\chi) > \pi_{x_1}(\neg\chi \wedge \upsilon) > \pi_{x_1}(\upsilon) > \pi_{x_1}(\neg\chi \wedge \neg\upsilon) > \pi_{x_1}(\neg\chi) > \pi_{x_1}(\neg\upsilon)$$

Now consider following statements, each of which might be given in response.

1. *It is a meat dish for lunch*
2. *It is a chicken dish for lunch*
3. *Either it is a chicken dish for lunch or* $(7^2 - 3)$ *is not* 46

We use the following extra propositions to encode the possible response: μ = *"It is a meat dish for lunch" and* θ = *"*$(7^2 - 3)$ *is not* 46*". Also assume the following shared knowledge* $\sigma_{i,j}(\mu \rightarrow \neg\upsilon) = \sigma_{i,j}(\chi \rightarrow \neg\upsilon) > \sigma_{i,j}(\neg\theta)$*. Now, we can represent each response in the form of the intended argument and its enthymeme.*

Response	*Intended argument*	*Enthymeme*
1	$\langle\{\mu, \mu \rightarrow \neg\upsilon\}, \neg\upsilon\rangle$	$\langle\{\mu\}, \top\rangle$
2	$\langle\{\chi, \chi \rightarrow \neg\upsilon\}, \chi \wedge \neg\upsilon\rangle$	$\langle\{\chi\}, \top\rangle$
3	$\langle\{\chi \vee \theta, \neg\theta, \chi \rightarrow \neg\upsilon\}, \chi \wedge \neg\upsilon\rangle$	$\langle\{\chi \vee \theta\}, \top\rangle$

So each of the arguments meets some information requirements, but responses 2 and 3 meet a more pressing information requirement than response 1 (and implicitly meet more than one information requirement), and responses 2 and 3 are equal to each other in terms of the information requirements met. However, in terms of the shared knowledge required, we see that since the following holds, it is the case that response 3 is more costly.

$$\{\chi \rightarrow \neg\upsilon\} >_{x_1,x_2} \{\neg\theta, \chi \rightarrow \neg\upsilon\}$$

So the construction method allows for a proponent to send an enthymeme that is understandable but not redundant and that (based on the proponent's understanding of the shared knowledge and the recipient's agenda) not only is worthwhile for the recipient to deconstruct, but of all the possible worthwhile deconstructions it is the one that the proponent believes the recipient is most likely to access (given the proponent's cobase function for the recipient).

4.4 Deconstructing enthymemes

When an agent receives an enthymeme $\langle\Psi, \beta\rangle$, it deconstructs it by first seeing if the claim of the enthymeme is $\top$: If no, then it proceeds with decoding the enthymeme; If yes, then it takes the path of 'least resistance' (and so tries to use the highest ranked parts of its cobase first) to try to find a way of meeting the most pressing information requirement it can, given the premises that have been provided and the shared knowledge. In the following we start by explaining decoding, and then consider the details of deconstructing.

When $\langle\Psi, \alpha\rangle$ is an encodation of $\langle\Phi, \alpha\rangle$, it is either the intended argument or an enthymeme for the intended argument. If it is an enthymeme, then the recipient has to decode it using the shared knowledge $\Sigma_{j,i}$ (i.e. the knowledge that j believes is shared knowledge between i and j) by adding formulae Ψ' to the support of the enthymeme ($\Psi' \subseteq \Sigma_{j,i}$), creating $\langle\Psi \cup \Psi', \alpha\rangle$, which will be expansive but not necessarily minimal. It would be desirable for $\langle\Psi \cup \Psi', \alpha\rangle$ to be the intended argument, but this cannot be guaranteed. It may be that the wrong formulae from $\Sigma_{j,i}$ are used, or it could be that shared knowledge as viewed by agent i is not the same as that viewed by agent j (i.e. $\sigma_{i,j} \neq \sigma_{j,i}$). Nevertheless, using the ranking information in a cobase, we can aim for a reasonable decoding of an enthymeme.

Definition 6. *For an encodation $\langle\Psi, \alpha\rangle$ from a proponent i for a recipient j, a* **decodation** *is of the form $\langle\Psi \cup \Psi', \alpha\rangle$, where $\Psi' \subseteq \Sigma_{j,i}$, and $\langle\Psi \cup \Psi', \alpha\rangle$ is expansive, and there is no Ψ'' such that $\Psi'' >_{j,i} \Psi'$ and $\langle\Psi \cup \Psi'', \alpha\rangle$ is expansive. Let $D(\langle\Psi, \alpha\rangle, \Sigma_{j,i})$ denote the set of decodations of $\langle\Psi, \alpha\rangle$.*

Example 23. *If $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$ is an intended argument from proponent i to recipient j, where $\sigma_{i,j}(\alpha) = 0$, $\sigma_{i,j}(\alpha \rightarrow \beta) = 1$, and $\tau_i = 0.9$, then the encodation is $\langle\{\alpha\}, \beta\rangle$. Now suppose, $\tau_j = 0.9$, $\sigma_{j,i}(\alpha \rightarrow \beta) = 1$, $\sigma_{j,i}(\alpha \rightarrow \epsilon) = 1$, $\sigma_{j,i}(\epsilon \rightarrow \beta) = 1$, and for all other ϕ, $\sigma_{j,i}(\phi) = 0$. So for $\langle\{\alpha\}, \beta\rangle$, the decodations are $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$ and $\langle\{\alpha, \alpha \rightarrow \epsilon, \epsilon \rightarrow \beta\}, \beta\rangle$. If we change the cobase so that $\sigma_{j,i}(\alpha \rightarrow \beta) = 0.5$, then we get the second decodation as the unique decodation.*

Example 24. *If $\langle\{\beta, \gamma, \beta \wedge \gamma \rightarrow \alpha\}, \alpha\rangle$ is an intended argument from proponent i to recipient j, where $\sigma_{i,j}(\beta) = \sigma_{i,j}(\gamma) = 0$, $\sigma_{i,j}(\beta \wedge \gamma \rightarrow \alpha) = 1$, and $\tau_i = 0.9$, then the encodation is $\langle\{\beta, \gamma\}, \alpha\rangle$. Now suppose $\tau_j = 0.9$, $\sigma_{j,i}(\beta \wedge \gamma \rightarrow \alpha) = 0.5$, and $\sigma_{j,i}(\beta \rightarrow \alpha) = 0.9$, and for all other ϕ, $\sigma_{j,i}(\phi) = 0$. So for the enthymeme $\langle\{\beta, \gamma\}, \alpha\rangle$, the decodation is $\langle\{\beta, \gamma, \beta \rightarrow \alpha\}, \alpha\rangle$.*

Now we return to the question of how an agent deconstructs an enthymeme $\langle\Psi, \top\rangle$. Since the recipient is assuming that the enthymeme $\langle\Psi, \top\rangle$ maximises cognitive effect, there is the presumption that it should address one of the information requirements of the agent.

Definition 7. *For an enthymeme $\langle\Psi, \beta\rangle$, a* **deconstruction** *of $\langle\Psi, \beta\rangle$ from a proponent i for a recipient j, is $\langle\Phi, \alpha\rangle$ where $\langle\Phi, \alpha\rangle$ is a decodation of $\langle\Psi, \alpha\rangle$ and α is β if $\beta \neq \top$ otherwise $\alpha \in \Pi^j$. Let $F(\langle\Psi, \beta\rangle) = \{\langle\Phi, \alpha\rangle \mid \langle\Phi, \alpha\rangle$ is a deconstruction of $\langle\Psi, \beta\rangle\}$.*

Often there will be a number of deconstructions for an enthymeme, and so the ranking information available in the cobase and the agenda can be used to direct the search for the "preferred" deconstructions (those that produce the greatest cognitive effect with the least cognitive effort).

Definition 8. *The following algorithm, called the* **deconstruct algorithm***, returns a single element from the set of the most preferred deconstructions of an enthymeme $\langle \Psi, \beta \rangle$ sent by proponent i to recipient j.*

If $\beta \neq \top$,
then return $\langle \Phi, \beta \rangle$ *such that* $\langle \Phi, \beta \rangle \in D(\langle \Psi, \beta \rangle, \Sigma_{j,i})$
Otherwise let `REQ` *be* Π_j
while nothing has been returned
let `MAXREQ` *be the maximal subset of* `REQ`
such that $\forall \phi, \psi \in \texttt{MAXREQ}, \pi_j(\phi) = \pi_j(\psi)$
and $\forall \phi \in \texttt{MAXREQ}, \nexists \psi \in \texttt{REQ}$ *s.t.* $\pi_j(\psi) > \pi_j(\psi)$
remove `MAXREQ` *from* `REQ`
let $\texttt{DECONS} = \{\langle \Psi \cup \Gamma, \delta \rangle \mid \delta \in \texttt{MAXREQ}$ *and* $\langle \Psi \cup \Gamma, \delta \rangle$ *is a decoding of* $\langle \Psi, \delta \rangle\}$
if $\texttt{DECONS} \neq \emptyset$
then return $\langle \Psi \cup \Gamma, \delta \rangle$ *such that* $\langle \Psi \cup \Gamma, \delta \rangle \in \texttt{DECONS}$
and for all $\langle \Psi \cup \Gamma', \delta' \rangle \in \texttt{DECONS}$ *it is not the case that* $\Gamma' >_{j,i} \Gamma$

The above deconstruct algorithm returns an element of $F(\langle \Psi, \beta \rangle)$ that is at least as preferred as all other elements of $F(\langle \Psi, \beta \rangle)$, as dictated by the rankings in the agenda and the cobase. So (assuming the enthymeme received is an implicit claim enthymeme) the recipient starts by looking at its most pressing information requirements and generates each decodation that has one of these information requirements as its claim. If the set of such decodations is not empty, then it returns a member of this set $\langle \Psi \cup \Gamma, \delta \rangle$ such that there is no other decodation in this set $\langle \Psi \cup \Gamma', \delta' \rangle$ such that $\Gamma' >_{j,i} \Gamma$. If there is no decodation whose claim is one of the most pressing information requirements, then the agent repeats the process with the set of the next most pressing information requirements, and so on. In this way, arguments pertaining to higher degree information requirements will be returned above those pertaining to lower degree information requirements.

Example 25. *Returning to Example 1, let* α *= "John has bought The Times",* β *= "John has bought a copy of The Times" and* γ *= "John has bought the company that publishes The Times". Let* $\sigma_{x_2,x_1}(\alpha \to \beta) = 0.9$*,* $\sigma_{x_2,x_1}(\alpha \to \gamma) = 0.6$*, and* $\pi_{x_2}(\beta) = \pi_{x_2}(\gamma) = 1$*. Assume that* x_2 *has received the enthymeme* $\langle \{\alpha\}, \top \rangle$ *and that* $\tau_{x_2} = 0.5$*. Even though both* β *and* γ *are equally pressing information requirements for* x_2*,* $\{\alpha \to \beta\} >_{j,i} \{\alpha \to \gamma\}$ *and so* x_2 *prefers* $\{\alpha \to \beta\}$ *and correctly deconstructs the real argument it received to get* $\langle \{\alpha, \alpha \to \beta\}, \beta \rangle$*.*

Example 26. *Returning to Example 2, Let* α *= "John payed back the money he owed",* β *= "John forgot to go to the bank",* γ *= "John needs to borrow some money", and* δ *= "John can't afford to come out tonight". Suppose that* $\sigma_{x_2,x_1}(\beta \to \neg\alpha) = \sigma_{x_2,x_1}(\beta \to \gamma) = \sigma_{x_2,x_1}(\beta \to \delta) = 0.5$*, and* $\tau_{x_2} = 0.5$*. Given the enthymeme* $\langle \{\beta\}, \neg\alpha \rangle$*, there is only one deconstruction generated which is* $\langle \{\beta, \beta \to \neg\alpha\}, \neg\alpha \rangle$*.*

Example 27. *Returning to Example 3, Let* α *= "I would like a coffee",* β *= "Coffee keeps me awake", and* γ *= "I want to stay awake". Also let* $\sigma_{x_2,x_1}(\gamma) = \sigma_{x_2,x_1}(\beta \wedge \gamma \to \alpha) = \sigma_{x_2,x_1}(\beta \wedge \neg\gamma \to \neg\alpha) = 1$*, and* $\sigma_{x_2,x_1}(\neg\gamma) = 0.1$*. Assume the enthymeme from* x_2 *to* x_1 *is* $\langle \{\beta\}, \top \rangle$ *and* $\tau_{x_2} = 0.5$*. Also assume that* $\pi_{x_2}(\alpha) = \pi_{x_2}(\neg\alpha) = 1$ *(and so either* $\langle \{\beta, \gamma, \beta \wedge \gamma \to \alpha\}, \alpha \rangle$ *or* $\langle \{\beta, \neg\gamma, \beta \wedge \neg\gamma \to \neg\alpha\}, \neg\alpha \rangle$ *would meet one of these equally pressing information requirements). However, since* $\sigma_{x_2,x_1}(\gamma) = 1$ *and* $\sigma_{x_2,x_1}(\neg\gamma) = 0.1$*, the only decodation that is generated is* $\langle \{\beta, \gamma, \beta \wedge \gamma \to \alpha\}, \alpha \rangle$*.*

Example 28. *Returning to Example 4, consider the following propositions:* α *= "I would like to buy a flag for RNLI",* β *= "I always spend my holidays in Birmingham",* γ *= "I have no need of the services of the RNLI",* δ *= "There are a lot of canals in Birmingham", and* ϵ *= "It would be good if the RNLI expanded their services to Birmingham canals". Let* $\sigma_{x_2,x_1}(\beta \to \gamma) = \sigma_{x_2,x_1}(\gamma \to \neg\alpha) = \sigma_{x_2,x_1}(\beta \wedge \delta \to \epsilon) =$

$\sigma_{x_2,x_1}(\epsilon \to \alpha) = \sigma_{x_2,x_1}(\delta) = 0.5$. Also, let $\pi_{x_2}(\alpha) = \pi_{x_2}(\neg\alpha) = 1$. Given the enthymeme $\langle\{\beta\}, \neg\alpha\rangle$ and $\tau_{x_2} = 0.5$, there is one deconstruction which is $\langle\{\beta, \beta \to \gamma, \gamma \to \neg\alpha\}, \neg\alpha\rangle$.
If the enthymeme sent from x_1 to x_2 were instead $\langle\{\beta\}, \top\rangle$, x_2 would be as likely to generate the argument $\langle\{\beta, \delta, \beta \wedge \delta \to \epsilon, \epsilon \to \alpha\}, \alpha\rangle$ as it would be to generate the argument $\langle\{\beta, \beta \to \gamma, \gamma \to \neg\alpha\}, \neg\alpha\rangle$ (as each of these arguments meets an equally pressing information requirement and neither $\{\delta, \beta \wedge \delta \to \epsilon, \epsilon \to \alpha\} >_{j,i} \{\beta \to \gamma, \gamma \to \neg\alpha\}$ nor $\{\beta \to \gamma, \gamma \to \neg\alpha\} >_{j,i} \{\delta, \beta \wedge \delta \to \epsilon, \epsilon \to \alpha\}$, and so each argument is equally relevant to x_2).

In the above examples (i.e. Examples 25 to 28), we have focussed on when the deconstruction returns the intended argument. However, as we suggested with decoding, the overall deconstruction process does not necessarily return the intended argument. We return to this issue in the next section.

4.5 Properties of our framework

In this section we relate our framework back to relevance theory by providing a formal characterisation of relevance (that draws on the relevance-theoretic ideas of maximising cognitive effect whilst minimising cognitive effort) and showing that, when constructing an enthymeme from an intended argument, a proponent will always maximise what it perceives to be the relevance of the enthymeme, given what it believes to be true about the recipient (as represented by the proponent's cobase function for the recipient and the recipient's agenda). Given that this is the case, the recipient of an enthymeme is able to draw on its perception of the shared knowledge (i.e. its cobase function for the proponent) in order to deconstruct the enthymeme to give the most relevant expansive argument. If the cobase functions of the two agents are the same then we can be reasonably confident that the deconstruction that the recipient arrives at is the intended argument, and we can be certain that the claim of the deconstruction that the recipient arrives at is the same as the claim of the intended argument.

The relevance of an input to an individual can be characterised in terms of two competing dimensions that we quote from Wilson and Sperber.

- "Other things being equal, the greater the positive cognitive effects achieved by processing an input, the greater the relevance of the input to the individual at that time [WS02]."
- "Other things being equal, the greater the processing effort expended, the lower the relevance of the input to the individual at that time [WS02]."

So when a proponent of an intended argument wants to construct an enthymeme to send to a recipient, the proponent aims to make the enthymeme relevant to the recipient by ensuring that it is beneficial (in providing useful information to the recipient) and efficient (in providing sufficient but not excessive information to the recipient for deconstructing the enthymeme). With this in mind, we define the following relation over the expansive approximate arguments.

Definition 9. *Let $\langle\Phi', \beta'\rangle$ and $\langle\Phi, \beta\rangle$ be expansive arguments, let $\Psi \subseteq \mathcal{L}$, and let p be the proponent and r be the recipient. The* **relevance relation** *denoted $\succeq_\Psi$ is defined as follows.*

$$\langle\Phi', \beta'\rangle \succeq_\Psi \langle\Phi, \beta\rangle \text{ iff } \pi_r(\beta') > \pi_r(\beta) \text{ or } [\pi_r(\beta') = \pi_r(\beta) \text{ and } \Phi \setminus \Psi \not>_{p,r} \Phi' \setminus \Psi]$$

The relevance relation $\succeq_\Psi$ is a pairwise comparison for expansive approximate arguments. With the following lemma, which we use in the proof of Proposition 3, we see a clarification of the cases for which the relation holds.

Lemma 1. *Let $\langle\Phi', \beta'\rangle$ and $\langle\Phi, \beta\rangle$ be expansive arguments, let $\Psi \subseteq \mathcal{L}$, and let p be the proponent and r be the recipient. $\langle\Phi', \beta'\rangle \succeq_\Psi \langle\Phi, \beta\rangle$ if and only if one of the following conditions hold.*

1. $\pi_r(\beta') > \pi_r(\beta)$
2. $\pi_r(\beta') = \pi_r(\beta)$ *and* $\Phi' \setminus \Psi >_{p,r} \Phi \setminus \Psi$
3. $\pi_r(\beta') = \pi_r(\beta)$ *and* $\Phi' \setminus \Psi \not>_{p,r} \Phi \setminus \Psi$ *and* $\Phi \setminus \Psi \not>_{p,r} \Phi' \setminus \Psi$

So the relevance relation gives priority to those expansive approximate arguments that have a more highly valued information requirement as claim, and for those with equally preferred information requirements, it gives higher priority to those that have a more preferred set of formulae in the implicit support (i.e. the support of the expansive argument minus the explicit support Ψ) or those that have incomparable sets of formulae in the implicit support. Note in general it is not transitive nor antisymmetric, though it is reflexive.

The agenda, then, allows us to determine the magnitude of cognitive effect of an intended argument (i.e. the more pressing an information requirement is a claim of an intended argument, the greater the cognitive effect and so the higher the relevance), whilst the proponent's cobase function for the recipient (and the preference relation this defines) allows us to determine the magnitude of cognitive effort required to deconstruct a real argument (i.e. the less preferred the implicit support is of a real argument, the greater the processing effort required to deconstruct it and so the lower the relevance). So whilst removing premises from an intended argument to create a real argument is beneficial with respect to the cost of communicating information (if we assume fewer pieces of information sent from one agent to another costs less), this should not be at the expense of the recipient being unable to obtain the claim of the intended argument because the implicit support of the real argument is not sufficiently preferred. Hence, we regard the cognitive (or processing) effort as consisting of both the cost of communicating the real argument and the cost of deconstructing the real argument.

Now using the relevance relation, we can formalise the Enthymeme Relevance Principle given in Section 2 in the following definition for defining when an enthymeme is most relevant for a logical argument.

Definition 10. *An enthymeme* $\langle \Psi, \beta \rangle$ *is* **most relevant** *for a logical argument* $\langle \Phi, \alpha \rangle$ *and a proponent* p *and a recipient* r *as follows, assuming* $\Psi = \Phi \setminus \Sigma_{p,r}$ *and* $\beta \in \{\alpha, \top\}$.

For $\beta \neq \alpha$, $\langle \Psi, \beta \rangle$ *is most relevant for* $\langle \Phi, \alpha \rangle$ *w.r.t.* p *and* r
iff there is no $\Gamma \subseteq \Sigma_{p,r}$ *and* γ *s.t.* $\gamma \not\equiv \alpha$ *and* $\langle \Psi \cup \Gamma, \gamma \rangle \succeq_\Psi \langle \Phi, \alpha \rangle$

For $\beta = \alpha$, $\langle \Psi, \beta \rangle$ *is most relevant for* $\langle \Phi, \alpha \rangle$ *w.r.t.* p *and* r
iff there is a $\Gamma \subseteq \Sigma_{p,r}$ *and a* γ *s.t.* $\gamma \not\equiv \alpha$ *and* $\langle \Psi \cup \Gamma, \gamma \rangle \succeq_\Psi \langle \Phi, \alpha \rangle$

When constructing an enthymeme from its intended argument, a proponent selects the one it perceives to be most relevant given its cobase function for the recipient and the recipient's agenda (as we will shortly show). On receiving this enthymeme, the recipient must then deconstruct it. As a decision problem, deconstruction is a form of abduction, and so the computational complexity is in the second level of the polynomial hierarchy (and therefore intractable) [EG95]. Nonetheless, we believe that it is unavoidable to deconstruct enthymemes via abduction, and that this cost needs to be compared with the cost savings gained by using enthymemes—such as it is quicker to communicate fewer premises and it involves less effort in attention to listen to fewer premises being articulated. Whilst our framework does not currently take these different costs into account, this is one way in which we would like to develop our framework in the future. Furthermore, the ordering on the cobase can be thought of as ameliorating the cost involved in deconstructing enthymemes: Rather than considering any arbitrary formula in the language in the abduction, we consider those more highly ordered by the cobase function.

Given a logical argument, an enthymeme which is most relevant is uniquely determined as demonstrated by the following result.

Proposition 2. *If* $\langle \Psi, \beta \rangle$ *is most relevant for* $\langle \Phi, \alpha \rangle$ *w.r.t.* p *and* r *and* $\langle \Psi', \beta' \rangle$ *is most relevant for* $\langle \Phi, \alpha \rangle$ *w.r.t.* p *and* r *then* $\Psi = \Psi'$ *and* $\beta = \beta'$.

Proof: We get $\Psi = \Psi'$ because each is $\Phi \setminus \Sigma_{p,r}$. We get $\beta = \beta'$ because according to Definition 10, $\beta = \top$ iff the condition of the first rule holds, and $\beta = \alpha$ iff the condition of the second rule holds. Since, the conditions of each of these rules are complementary, the first holds iff the second does not hold. So either β and β' are α or β and β' are $\top$. Hence, in either case, $\beta = \beta'$. $\square$

Now, we can show an equivalence between the construction of an enthymeme for a logical argument, and the identification of the most relevant enthymeme for a logical argument. In this way, we claim that our construction process for enthymemes meets the Enthymeme Relevance Principle given in Section 2.

Proposition 3. *$\langle \Psi, \beta \rangle$ is the result of construction of the enthymeme for the intended argument $\langle \Phi, \alpha \rangle$ from the proponent p to recipient r iff $\langle \Psi, \beta \rangle$ is the most relevant for $\langle \Phi, \alpha \rangle$ w.r.t. p and r.*

Proof: ($\Rightarrow$) Assume $\langle \Psi, \beta \rangle$ is the result of construction of the enthymeme for the intended argument $\langle \Phi, \alpha \rangle$ from the proponent p to recipient r. Either $\beta \neq \alpha$ or $\beta = \alpha$. First assume $\beta \neq \alpha$. So $\beta = \top$. Hence, conditions (1) and (2) for construction definition hold. From condition (1), we infer (a) that there is no $\Gamma \subseteq \Sigma_{p,r}$, and γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) > \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive. From condition (2), we infer (b) that there is no $\Gamma \subseteq \Sigma_{p,r}$, and γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) = \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive and $\Gamma >_{p,r} \Phi \setminus \Psi$. Also from condition (2), we infer (c) that there is no $\Gamma \subseteq \Sigma_{p,r}$, and γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) = \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive and $\Gamma \not>_{p,r} \Phi \setminus \Psi$ and $\Phi \setminus \Psi \not>_{p,r} \Gamma$. From (a), (b), and (c), together with Lemma 1, there is no $\Gamma \subseteq \Sigma_{p,r}$, and γ s.t. $\alpha \not\equiv \gamma$ and $\langle \Psi \cup \Gamma, \gamma \rangle \succeq_\Psi \langle \Phi, \alpha \rangle$. Therefore, $\langle \Psi, \beta \rangle$ is the most relevant enthymeme for $\langle \Phi, \alpha \rangle$ w.r.t. p and r. Now assume $\beta = \alpha$. Therefore, conditions (1) or (2) for construction definition does not hold. If condition (1) fails, then we infer (d) that there is a $\Gamma \subseteq \Sigma_{p,r}$, and a γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) > \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive. If condition (2) fails, then we infer (e) that there is a $\Gamma \subseteq \Sigma_{p,r}$, and a γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) = \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive and $\Gamma >_{p,r} \Phi \setminus \Psi$ or $\Gamma \not>_{p,r} \Phi \setminus \Psi$ and $\Phi \setminus \Psi \not>_{p,r} \Gamma$. From (d) and (e), together with Lemma 1, there is a $\Gamma \subseteq \Sigma_{p,r}$, and γ s.t. $\alpha \not\equiv \gamma$ and $\langle \Psi \cup \Gamma, \gamma \rangle \succeq_\Psi \langle \Phi, \alpha \rangle$. Therefore, $\langle \Psi, \alpha \rangle$ is the most relevant enthymeme for $\langle \Phi, \alpha \rangle$ w.r.t. p and r.
($\Leftarrow$) Assume $\langle \Psi, \beta \rangle$ is the most relevant enthymeme for $\langle \Phi, \alpha \rangle$ w.r.t. p and r. Either $\beta \neq \alpha$ or $\beta = \alpha$. First assume $\beta \neq \alpha$. So $\beta = \top$. Therefore, we infer (f) that there is no $\Gamma \subseteq \Sigma_{p,r}$ and γ s.t. $\gamma \not\equiv \alpha$ and $\langle \Psi \cup \Gamma, \gamma \rangle \succeq_\Psi \langle \Phi, \alpha \rangle$. From (f), and Lemma 1, we infer (g) that there is no $\Gamma \subseteq \Sigma_{p,r}$, and γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) > \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive. Also from (f), and Lemma 1, we infer (h) that there is no $\Gamma \subseteq \Sigma_{p,r}$, and γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) = \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive and $\Gamma >_{p,r} \Phi \setminus \Psi$. Finally from (f), and Lemma 1, we infer (i) that there is no $\Gamma \subseteq \Sigma_{p,r}$, and γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) = \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive and $\Gamma \not>_{p,r} \Phi \setminus \Psi$ and $\Phi \setminus \Psi \not>_{p,r} \Gamma$. Taking (g), (h), and (i) together, we get that the conditions (1) and (2) for the construction definition hold. Therefore, $\langle \Psi, \beta \rangle$ is the result of construction of the enthymeme for the intended argument $\langle \Phi, \alpha \rangle$ from the proponent p to recipient r. Now assume $\beta = \alpha$. Therefore, we infer that there is a $\Gamma \subseteq \Sigma_{p,r}$ and a γ s.t. $\gamma \not\equiv \alpha$ and $\langle \Psi \cup \Gamma, \gamma \rangle \succeq_\Psi \langle \Phi, \alpha \rangle$. Therefore, together with Lemma 1, we infer either that there is a $\Gamma \subseteq \Sigma_{p,r}$, and a γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) > \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive, or there is a $\Gamma \subseteq \Sigma_{p,r}$, and a γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) = \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive and $\Gamma >_{p,r} \Phi \setminus \Psi$ or there is a $\Gamma \subseteq \Sigma_{p,r}$, and a γ s.t. $\alpha \not\equiv \gamma$ and $\pi_r(\gamma) = \pi_r(\alpha)$ and $\langle \Psi \cup \Gamma, \gamma \rangle$ is expansive and $\Gamma \not>_{p,r} \Phi \setminus \Psi$ and $\Phi \setminus \Psi \not>_{p,r} \Gamma$. Taking this disjunction, we get that the conditions (1) and (2) for the construction definition do not hold. Therefore, $\langle \Psi, \beta \rangle$ is the result of construction of the enthymeme for the intended argument $\langle \Phi, \alpha \rangle$ from the proponent p to recipient r. $\square$

If a proponent i sends an implicit claim enthymeme as the real argument to the recipient j, then it is the case that (given i's view of the shared knowledge between i and j) i believes that if there is an information requirement that is more pressing for j than the claim of the intended argument, then it is not possible for j to reconstruct from the real argument an argument that has this more pressing information requirement as its claim.

Proposition 4. *Let $\langle \Psi, \top \rangle$ be the enthymeme constructed for the intended argument $\langle \Phi, \alpha \rangle$ from i to j. If there is a $\delta \in \Pi_j \setminus \{\alpha\}$ such that $\pi_j(\delta) > \pi_j(\alpha)$, then there does not exist $\Gamma \subseteq \Sigma_{i,j}$ such that $\langle \Psi \cup \Gamma, \delta \rangle$ is expansive.*

Proof: This follows from condition (1) of the definition of construction (Definition 5). $\square$

If a proponent i sends an implicit claim enthymeme as the real argument to the recipient j, then it is the case that (given i's view of the shared knowledge between i and j) i believes that if there is an argument that is easier for j to reconstruct from the real argument than the intended argument, its claim is a less pressing information requirement for j than the claim of the intended argument.

Proposition 5. *Let $\langle\Psi, \top\rangle$ be the enthymeme constructed for the intended argument $\langle\Phi, \alpha\rangle$ from i to j. If there is a $\Gamma \subseteq \Sigma_{i,j}$ such that $\Gamma >_{i,j} (\Phi \setminus \Psi)$ and $\langle\Psi \cup \Gamma, \delta\rangle$ is a logical argument and $\delta \in \Pi^j$, then $\pi_j(\delta) < \pi_j(\alpha)$.*

Proof: This follows from conditions (1) and (2) of the definition of construction (Definition 5). □

Of course the proponent's intention to be as relevant as possible is not necessarily reflected in being perceived to be relevant by the recipient. The problem is that the cobases $\sigma_{i,j}$ and $\sigma_{j,i}$ used by the two agents i and j are not necessarily the same, and indeed normally there would be substantial differences between the two cobases. This disparity between knowledge of the proponent of an enthymeme and the recipient of an enthymeme leads to problems in the recipient determining the correct deconstruction.

Even if the proponent and recipient have identical shared knowledge and so the intended argument is one of the deconstructions, it is still not necessarily the case that the deconstruction returned by the deconstruct algorithm is the intended argument, as we show with the following example.

Example 29. *Proponent x_1 has the intended argument $\langle\{\alpha, \alpha \rightarrow \beta, \beta \rightarrow \gamma\}, \gamma\rangle$ that it wishes to make known to the recipient x_2. Let $\sigma_{x_1,x_2}(\alpha \rightarrow \beta) = \sigma_{x_2,x_1}(\alpha \rightarrow \beta) = 1$, $\sigma_{x_1,x_2}(\beta \rightarrow \gamma) = \sigma_{x_2,x_1}(\beta \rightarrow \gamma) = 1$, $\sigma_{x_1,x_2}(\alpha \rightarrow \delta) = \sigma_{x_2,x_1}(\alpha \rightarrow \delta) = 1$, and $\sigma_{x_1,x_2}(\delta \rightarrow \gamma) = \sigma_{x_2,x_1}(\delta \rightarrow \gamma) = 1$. Also let $\tau_i = \tau_j = 0.5$ and $\pi_j(\gamma) = \pi_j(\neg\gamma) = 1$.*
The proponent x_1 constructs the real argument $\langle\{\alpha\}, \gamma\rangle$.
On receiving the real argument $\langle\{\alpha\}, \gamma\rangle$, the recipient x_2 applies the deconstruct algorithm to return the deconstruction that it perceives to be most relevant. However, there are two equally relevant deconstructions, $\langle\{\alpha, \alpha \rightarrow \beta, \beta \rightarrow \gamma\}, \gamma\rangle$ and $\langle\{\alpha, \alpha \rightarrow \delta, \delta \rightarrow \gamma\}, \gamma\rangle$, and so the non-deterministic deconstruction algorithm may return either of these.

So when a recipient generates a deconstruction of an enthymeme $\langle\Psi, \alpha\rangle$, it does not know for certain what the intended argument is, and it is not guaranteed to find it even if the number of possible deconstructions that are logical arguments is exactly one. However, if the proponent and recipient have identical shared knowledge, then the intended argument is one of the deconstructions.

Proposition 6. *Let $\sigma_{i,j} = \sigma_{j,i}$ and $\tau_i = \tau_j$. For a logical argument $\langle\Phi, \alpha\rangle$, if $C(\langle\Phi, \alpha\rangle, \Sigma_{i,j}) = \langle\Psi, \beta\rangle$, then $\langle\Phi, \alpha\rangle \in F(\langle\Psi, \beta\rangle)$.*

Proof: From $\sigma_{i,j} = \sigma_{j,i}$ and $\tau_i = \tau_j$, we have that $\Sigma_{i,j} = \Sigma_{j,i}$. So the implicit support removed in coding by the proponent (i.e. $\Phi \setminus \Sigma_{i,j}$) will be available for recovery by the recipient in decoding. If $\beta = \alpha$, then $\langle\Phi, \alpha\rangle$ is a decodation, and hence $\langle\Phi, \alpha\rangle \in F(\langle\Psi, \beta\rangle)$. If $\beta = \top$, then the recipient by the construction process has ensured that either α is the most highly ranked information requirement for which $\langle\Phi, \alpha\rangle$ is expansive, or for any equally ranked information requirement, $\Phi \setminus \Psi$ is the more preferred implicit support according to the $>_{i,j}$ ranking. □

As a corollary of the above result, if the agents have the same cobase, then the deconstruct algorithm is guaranteed to return an argument whose claim matches that of the intended argument.

In the case that arises when the real argument is a logical argument, then the deconstruction is unique and correct.

Proposition 7. *For any $\langle\Phi, \alpha\rangle$, if $\langle\Phi, \alpha\rangle$ is a logical argument, then $F(\langle\Phi, \alpha\rangle) = \{\langle\Phi, \alpha\rangle\}$.*

Proof: For any cobase, $\emptyset$ is the most preferred subset of $\mathcal{L}$ in the $>_{i,j}$ ordering. Therefore $\langle\Phi \cup \emptyset, \alpha\rangle$ (which is equivalent to $\langle\Phi, \alpha\rangle$) is the unique decoding, and the unique deconstruction obtained. □

More generally, if there is a unique deconstruction that is a logical argument, and a high confidence that $\sigma_{i,j} = \sigma_{j,i}$, then the recipient may have high confidence that the deconstruction is the same as the intended argument.

5 Discussion

Argumentation is an important cognitive activity that needs to be better understood if we are to build intelligent systems better able to deal with conflicts arising in information and between agents. Enthymemes are a ubiquitous phenomenon in the real-world, and so if we are to build intelligent systems that generate arguments (e.g. to justify their actions, to persuade other agents, etc), and process arguments from other agents, then we need to build the capacity into these systems to generate and process enthymemes.

In [Hun07, BH08b], we introduced a way for each agent in a dialogue to have information about what it can use as shared knowledge, and then a proponent can use this information to remove redundant premises from an intended argument (creating an implicit support enthymeme), and a recipient can use this information to identify the necessary premises in order to recover the intended argument. In this paper, we have extended and refined the proposal by allowing each agent to also have a representation of information requirements. By introducing the notion of information requirements, we can use aspects of relevance theory in such a way as to evaluate the benefit and efficiency of enthymemes from the perspective of the proponent and of the recipient.

We have defined a relevance relation that allows us to compare two arguments. This relation gives priority to arguments that have a more highly valued information requirement as a claim, and for those with equally preferred information requirements, it gives higher priority to those that have a more preferred set of formulae in the implicit support. We have used our relevance relation to formalise the Enthymeme Relevance Principle that we gave in Section 2, and we have shown that the construction process we have defined meets the Enthymeme Relevance Principle. The recipient is then able to use its view of what is the most relevant deconstruction of an enthymeme to guide it in the deconstruction process, where it tries to recover the intended argument from the real argument it has received.

The proponent's view of the shared knowledge may be different to the recipient's view of the shared knowledge, and this can lead to difficulties when the recipient comes to deconstruct a real argument. However, we have shown that if the recipient's view of the shared knowledge is the same as that of the proponent, then the deconstruction process is guaranteed to return an argument whose claim is the same as that of the intended argument. One approach to improving the match between an agent i's cobase for an agent j and agent j's cobase for agent i is to update the agents' cobase functions as a result of the interactions that go on between the two agents when participating in dialogues (we explore this possibility in more detail in [BH08b]). For example: If j opens an information-seeking or inquiry dialogue with γ as its topic (and so is searching for a reason to believe γ), then i may wish to decrease its belief that it can use γ as shared knowledge with agent j (and so decrease the value of $\sigma_{i,j}(\gamma)$); If i presents an implicit support enthymeme to j and j subsequently questions this enthymeme as it is unable to deconstruct it, then i may have decreased belief that it can use the premises that it removed from its intended argument in order to construct the enthymeme as shared knowledge between itself and j.

In this paper we have only considered information requirements that an agent has made explicit. In future work we intend to also consider information requirements that the recipient has not explicitly declared but that the proponent has inferred somehow from its knowledge of the recipient. For example, if the recipient has an explicit information requirement to know if it is sunny then the proponent may use this, along with ontological knowledge that it has, to infer that the recipient also has an implicit information requirement to know that it is raining. We expect to use a ranking over what an agent i believes are implicit information requirements for an agent j, similar to the ranking over the cobase.

We also intend, as future work, to explore the relationship between enthymemes and counterarguments. For instance, an undercut can be represented by an implicit claim enthymeme, in which case the recipient needs to determine which formulae are being attacked. In such an adversarial situation, the deconstruction process may depend less on the recipient's information requirements and more on what has previously been said in the dialogue.

We believe this proposal could be adapted for a variety of other argumentation systems (e.g. [GS04, AC02]), and there are diverse ways that the notion of shared knowledge could be refined (e.g. [SW86]). Finally, decodation is a form of abduction, and so techniques and algorithms developed for abduction could be harnessed for improving the quality of decodation (e.g. [EGL97]).

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