

# Some Foundations for Probabilistic Abstract Argumentation

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**Abstract.** Recently, there has been a proposal by Dung and Thang and by Li *et al* to extend abstract argumentation to take uncertainty of arguments into account by assigning a probability value to each argument, and then use this assignment to determine the probability that a set of arguments is an extension. In this paper, we explore some of the assumptions behind the definitions, and some of the resulting properties, of the proposal for probabilistic argument graphs.

**Keywords.** Abstract argumentation; Uncertain arguments; Probabilistic arguments; Computational models of argument; Formal argument systems.

## 1. Introduction

Recent developments of abstract argumentation, that take into account the uncertainty of arguments, have been presented including probabilistic argument graphs [DT10,LON11]. Whilst the approach of probabilistic argument graphs is a useful development of abstract argumentation, allowing for better modelling of some real-world situations, its nature has been largely unexplored. In order to address this, we investigate in this paper the foundations of probabilistic argument graphs by providing a clarification of some of the assumptions behind the original definitions and by providing a number of properties that hold for it. We also extend the approach by qualifying the certainty of inferences (i.e. arguments) from probabilistic argument graphs.

## 2. Probabilistic argument graphs

In this section, we review probabilistic argument graphs [DT10,LON11].

**Definition 1.** A **probabilistic argument graph** is a tuple  $(\mathcal{A}, \mathcal{R}, p)$  where  $(\mathcal{A}, \mathcal{R})$  is an argument graph and  $p : \mathcal{A} \rightarrow [0, 1]$  is a probability function over arguments.

In general, there are no further constraints on the probability assignment beyond Definition 1. So, for example, it is possible for every argument in a graph to be assigned a probability of 1, in which case we will return to Dung's original proposal. Similarly, it is possible for any or every argument in a graph to be assigned a probability of 0.

Let  $G = (\mathcal{A}, \mathcal{R}, p)$  be an probabilistic argument graph, and let  $\mathcal{A}' \subseteq \mathcal{A}$ . The marginalization of  $\mathcal{R}$  to  $\mathcal{A}'$ , denoted  $\mathcal{R} \otimes \mathcal{A}'$ , is the subset of  $\mathcal{R}$  involving just the arguments in  $\mathcal{A}'$  (i.e.  $\mathcal{R} \otimes \mathcal{A}' = \{(A, B) \in \mathcal{R} \mid A, B \in \mathcal{A}'\}$ ). If  $G' = (\mathcal{A}', \mathcal{R}')$  is an argument graph, then  $G'$  is a **spanning subgraph** of  $G$ , denoted  $G' \sqsubseteq G$ , iff  $\mathcal{A}' \subseteq \mathcal{A}$  and  $\mathcal{R}'$  is  $\mathcal{R} \otimes \mathcal{A}'$ . For simplicity, we will refer to a spanning subgraph as a subgraph. Using these definitions, the probability distribution over subgraphs is obtained as follows.

**Definition 2.** Let  $G = (\mathcal{A}, \mathcal{R}, p)$  be probabilistic argument graph and  $G' = (\mathcal{A}', \mathcal{R}')$  be a subgraph such that  $G' \sqsubseteq G$ . The **probability of subgraph**  $G'$ , denoted  $p(G')$ , is  $(\prod_{\alpha \in \mathcal{A}'} p(\alpha)) \times (\prod_{\alpha \in \mathcal{A} \setminus \mathcal{A}'} (1 - p(\alpha)))$ .

For a probabilistic argument graph  $G = (\mathcal{A}, \mathcal{R}, p)$ , a set of arguments  $\Gamma \subseteq \mathcal{A}$ , and  $G' = (\mathcal{A}', \mathcal{R}')$  where  $G' \sqsubseteq G$ ,  $G' \Vdash \Gamma$  denotes that  $\Gamma$  is an admissible set in  $G'$  and  $G' \Vdash_X \Gamma$  denotes that  $\Gamma$  is an  $X$  extension of  $G'$  where  $X = \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ , and co denotes complete semantics, pr denotes preferred semantics, st denotes stable semantics, and gr denotes grounded semantics. When  $G' \Vdash \Gamma$  holds, we say that  $G'$  **entails**  $\Gamma$ . The set of subgraphs that entail a set of arguments  $\Gamma$ , denoted  $Q_X(\Gamma)$ , is  $\{G' \sqsubseteq G \mid G' \Vdash_X \Gamma\}$ .

The probability that a set of arguments is admissible, denoted  $p(\Gamma)$ , is the sum of the probability of each subgraph for which  $\Gamma$  is admissible.

**Definition 3.** Let  $G = (\mathcal{A}, \mathcal{R}, p)$  be a probabilistic argument graph and let  $\Gamma \subseteq \mathcal{A}$ . The probability that  $\Gamma$  is admissible is  $\sum_{G' \in Q(\Gamma)} p(G')$ .

**Example 1.** Consider the argument graph in Figure 1a where  $p(a) = 1$ ,  $p(b) = 1$ ,  $p(c) = 0.5$ , and  $p(d) = 0.5$ . So there are four subgraphs with non-zero probability. These are  $G^1$ ,  $G^2$ ,  $G^3$ , and  $G^4$ , as shown in 1. Each has probability 1/4. The admissible sets for each subgraph is given below.

Subgraph	Admissible sets
$G^1$	$\{a, b, d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{\}$
$G^2$	$\{a, b\}, \{a\}, \{b\}, \{\}$
$G^3$	$\{a, b, d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{d\}, \{\}$
$G^4$	$\{a, b\}, \{a\}, \{b\}, \{\}$

As a result, there are eight admissible sets with non-zero probability to consider:  $p(\emptyset) = 1$ ,  $p(\{a\}) = 1$ ,  $p(\{b\}) = 1$ ,  $p(\{d\}) = 1/4$ ,  $p(\{a, b\}) = 1$ ,  $p(\{a, d\}) = 1/2$ ,  $p(\{b, d\}) = 1/2$ , and  $p(\{a, b, d\}) = 1/2$ .

The probability that a set of arguments is an  $X$  extension, denoted  $p(\Gamma^X)$ , is the sum of the probability of each subgraph for which  $\Gamma$  is an  $X$  extension.

**Definition 4.** Let  $G = (\mathcal{A}, \mathcal{R}, p)$  be a probabilistic argument graph and let  $\Gamma \subseteq \mathcal{A}$ . The probability that  $\Gamma$  is an  $X$  extension is  $\sum_{G' \in Q_X(\Gamma)} p(G')$ .

**Example 2.** We consider a scenario involving a clinician considering two diagnoses for a patient given by arguments  $a$  and  $b$  respectively. These arguments rebut each other (assuming that normally only one diagnosis should be associated with a disorder). The clinician also has a reason, captured by counterargument  $c$ , to doubt the diagnosis in

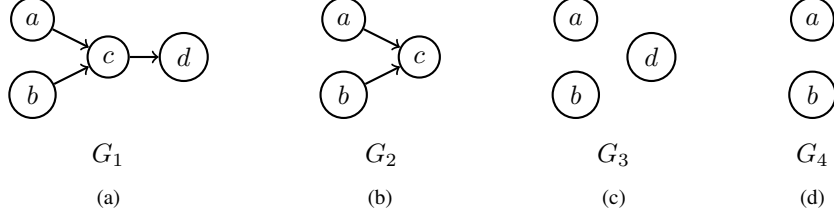


Figure 1. Spanning subgraphs for Examples 1.

argument  $a$ , and she has a reason, captured by counterargument  $d$ , to doubt the diagnosis in argument  $b$ . This is formalized in the following argument graph together with the probability function  $p$  where  $p(a) = 0.7$ ,  $p(b) = 0.2$ ,  $p(c) = 0.3$ , and  $p(d) = 0.8$ . The subgraphs are presented in Table 1. As a result, there are eight grounded extensions with non-zero probability to consider:  $p(\emptyset^{\text{gr}}) = 0.0053$ ,  $p(\{a\}^{\text{gr}}) = 0.0784$ ,  $p(\{b\}^{\text{gr}}) = 0.0084$ ,  $p(\{c\}^{\text{gr}}) = 0.048$ ,  $p(\{d\}^{\text{gr}}) = 0.168$ ,  $p(\{a, d\}^{\text{gr}}) = 0.392$ ,  $p(\{c, b\}^{\text{gr}}) = 0.0036$ , and  $p(\{c, d\}^{\text{gr}}) = 0.24$ .



We see that the most likely grounded extension is  $\{a, d\}$  with a probability of 0.392, and the next most likely grounded extension is  $\{c, d\}$  with a probability of 0.24. For the original argument graph, the grounded extension is  $\{c, d\}$ . So with the use of the probability function, the belief in  $a$  and the lack of belief in  $c$  means that  $\{a, d\}$  is the most probable grounded extension, and this does include one of the diagnoses. In contrast, using Dung's original definition, we get  $\{c, d\}$  which just captures the doubts involved.

So from [DT10,LON11], each argument in a graph can be assigned a value in the unit interval, which gives a probability distribution over the subgraphs of the argument graph, and this can then be used to give a probability assignment for a set of arguments being an admissible set or extension of the argument graph.

### 3. Probabilistic independence

There is an issue with respect to the assumption of independence of arguments when calculating the probability distribution over the spanning subgraphs (i.e. Definition 2). The proposals for probabilistic argument graphs [DT10,LON11] do not address this issue and so we attempt to address this now.

For this, we introduce the **justification perspective** on the probability of an argument: For an argument  $\alpha$  in a graph  $G$ , with a probability assignment  $p$ ,  $p(\alpha)$  is treated as the probability that  $\alpha$  is a justified point (i.e. each is a self-contained, and internally valid, contribution) and therefore should appear in the graph, and  $1 - p(\alpha)$  is the probability that  $\alpha$  is not a justified point and so should not appear in the graph. This means the probabilities of the arguments being justified are independent (i.e. knowing that one argument is a justified point does not affect the probability that another is a justified point).

	Subgraph	Probability of subgraph	Grounded extension	Preferred extensions
$G^1$	$c \rightarrow a \leftrightarrow b \leftarrow d$	0.0336	$\{c, d\}$	$\{c, d\}$
$G^2$	$c \rightarrow a \leftrightarrow b$	0.0084	$\{b, c\}$	$\{b, c\}$
$G^3$	$a \leftrightarrow b \leftarrow d$	0.0784	$\{a, d\}$	$\{a, d\}$
$G^4$	$a \leftrightarrow b$	0.0196	$\{\}$	$\{a\}, \{b\}$
$G^5$	$c \rightarrow a \quad d$	0.1344	$\{c, d\}$	$\{c, d\}$
$G^6$	$c \rightarrow a$	0.0336	$\{c\}$	$\{c\}$
$G^7$	$a \quad d$	0.3136	$\{a, d\}$	$\{a, d\}$
$G^8$	$a$	0.0784	$\{a\}$	$\{a\}$
$G^9$	$c \quad b \leftarrow d$	0.0144	$\{c, d\}$	$\{c, d\}$
$G^{10}$	$c \quad b$	0.0036	$\{c, b\}$	$\{c, b\}$
$G^{11}$	$b \leftarrow d$	0.0336	$\{d\}$	$\{d\}$
$G^{12}$	$b$	0.0084	$\{b\}$	$\{b\}$
$G^{13}$	$c \quad d$	0.0576	$\{c, d\}$	$\{c, d\}$
$G^{14}$	$c$	0.0144	$\{c\}$	$\{c\}$
$G^{15}$	$d$	0.1344	$\{d\}$	$\{d\}$
$G^{16}$		0.0336	$\{\}$	$\{\}$

**Table 1.** Subgraphs with probability of subgraphs and extensions for Example 2.

To illustrate the justification perspective, we consider logical arguments. If we have a knowledgebase containing just two formulae  $\{q, \neg q\}$ , we can construct arguments  $a_1 = \langle \{q\}, q \rangle$ , and  $a_2 = \langle \{\neg q\}, \neg q \rangle$ . The rebuttal relation holds so that the arguments attack each other. In terms of classical logic, it is not possible for both arguments to be true, but each of them is a justified point (i.e. each is a self-contained, and internally valid, contribution given the knowledgebase). So even though logically  $a_1$  and  $a_2$  are not independent (in the sense that if one is known to be true, then the other is known to be false), they are independent as justified points. This means we can construct an argument graph with both arguments appearing. Furthermore, we can use the probability assignment to each argument as reflecting the confidence that the argument makes a justified point. If each of  $a_1$  and  $a_2$  is assigned 1, then both are treated as fully justified points (and so we handle the argument graph using Dung's original definition). But if the assignment is less than 1 for either argument, then there is some explicit doubt that the argument is a justified point, and therefore there is some doubt that it should appear in the argument graph (in which case we can handle the uncertainty as a probabilistic argument graph).

#### 4. Sample spaces

The proposals for probabilistic argument graphs [DT10,LON11] do provide the following result, but they do not then investigate the nature of the probability distribution over subgraphs as a probability space. We attempt to address this shortcoming in this section.

**Proposition 1.** For any  $G = (\mathcal{A}, \mathcal{R}, p)$ ,  $\sum_{G' \sqsubseteq G} p(G') = 1$

So we can view  $\mathcal{G} = \{G' \sqsubseteq G\}$  as a sample space. This means we can assign a probability value to each subgraph  $G' \in \mathcal{G}$  so that the probabilities sum to 1.

Furthermore, for a graph  $G$  with  $n$  nodes, each element of the sample space (i.e. each subgraph  $G' \sqsubseteq G$ ) can be viewed as a conjunction of the form  $x_1 \wedge \dots \wedge x_n$ , where for each conjunct  $x_i$ , if the node  $\alpha_i$  is in  $G'$ , then  $x_i$  is  $\alpha_i$ , otherwise  $x_i$  is  $\bar{\alpha}_i$ . We call  $x_1 \wedge \dots \wedge x_n$  the conjunctive form of  $G'$ . Therefore, the probability distribution over  $\mathcal{G}$  is equivalent to the joint distribution over the conjunctive form of each subgraph in  $\mathcal{G}$ . Using this correspondence, we can obtain the probability of any argument as a marginal distribution.

**Example 3.** Let  $\mathcal{A} = \{a, b\}$ . The joint distribution is an assignment to each of  $p(a \wedge b)$ ,  $p(a \wedge \bar{b})$ ,  $p(\bar{a} \wedge b)$ , and  $p(\bar{a} \wedge \bar{b})$  such that these sum to 1. The marginal for  $p(a)$  is  $p(a \wedge b) + p(a \wedge \bar{b})$  and for  $p(b)$  is  $p(a \wedge b) + p(\bar{a} \wedge b)$ .

So calculating the marginal distribution using the joint distribution is the same as using the probability distribution over  $\mathcal{G}$  (i.e.  $p(\alpha) = \sum_{G' \sqsubseteq G \text{ s.t. } \alpha \in \text{Nodes}(G')} p(G')$  where  $\alpha$  is an argument).

**Example 4.** Consider the argument graph below. Let  $G^1$  be this graph,  $G^2$  be the graph containing  $a$ ,  $G^3$  be the graph containing  $b$ , and  $G^4$  be the empty graph. Suppose we have a probability distribution over subgraphs such that  $p(G^1) = 0.09$ ,  $p(G^2) = 0.81$ ,  $p(G^3) = 0.01$ , and  $p(G^4) = 0.09$ . So  $p(a) = p(G^1) + p(G^2)$  and  $p(b) = p(G^1) + p(G^3)$ .



However, as shown next, it is not the case that any probability distribution over subgraphs would give us an appropriate probability function over arguments.

**Example 5.** To illustrate how a probability distribution over subgraphs is not necessarily a probability function over arguments. Consider the argument graph  $G^1$  and its subgraphs as given in Example 4. Suppose we have a probability distribution over subgraphs such that  $p(G^1) = 0$ ,  $p(G^2) = 0.5$ ,  $p(G^3) = 0.5$ , and  $p(G^4) = 0$ . So  $p(G^1) + p(G^2) + p(G^3) + p(G^4) = 1$ . Then we obtain  $p(a) = 0.5$ , and  $p(b) = 0.5$ , as the marginal distributions. However, if we use these values with Definition 2, we get  $p(G^1) = 0.25$ ,  $p(G^2) = 0.25$ ,  $p(G^3) = 0.25$ , and  $p(G^4) = 0.25$ , which contradicts the original probability distribution over subgraphs.

So if we start with the sample space  $\mathcal{G} = \{G' \sqsubseteq G\}$ , and we want to find a probability distribution over  $\mathcal{G}$  that is consistent with the framework given in [DT10,LON11], and summarized in Section 2, then we need to restrict the choice of probability distribution over  $\mathcal{G}$  to being a regular distribution as defined next.

**Definition 5.** Let  $\mathcal{G} = \{G' \sqsubseteq G\}$ , where  $G$  has  $n$  nodes, and let  $P : \wp(\mathcal{G}) \rightarrow [0, 1]$ .  $p$  is a **regular distribution** over  $\mathcal{G}$  iff for each  $G' \in \mathcal{G}$ ,  $p(x_1 \wedge \dots \wedge x_n) = p(x_1) \times \dots \times p(x_n)$ , where  $x_1 \wedge \dots \wedge x_n$  is the conjunctive form of  $G'$ .

The probability distribution over  $\mathcal{G}$  given in Example 4 is a regular distribution, whereas that given in Example 5 is not a regular distribution.

**Proposition 2.** Let  $G = (\mathcal{A}, \mathcal{R}, p)$  be a probabilistic argument graph and let  $\mathcal{G} = \{G' \sqsubseteq G\}$ .  $p$  is a probability function over  $\mathcal{A}$  iff  $p$  is a regular distribution over  $\mathcal{G}$ .

The above result means that we can start with a sample space, viz.  $\mathcal{G}$ , and choose a regular distribution over  $\mathcal{G}$ , knowing that this would be a probability function over the arguments in the argument graph.

## 5. Probability functions over admissible sets

Whilst the proposals for probabilistic argument graphs [DT10,LON11] do not explicitly consider the probability of admissible sets, it is useful to consider the properties of them. For this, we require some further subsidiary definitions. For a probabilistic argument graph  $G = (\mathcal{A}, \mathcal{R}, p)$ ,  $p$  is **maximal** iff for all  $\alpha \in \mathcal{A}$ ,  $p(\alpha) = 1$ ,  $p$  is **minimal** iff for all  $\alpha \in \mathcal{A}$ ,  $p(\alpha) = 0$ , and  $p$  is **uniform** iff there is a  $k \in [0, 1]$  such that for all  $\alpha \in \mathcal{A}$ ,  $p(\alpha) = k$ .

**Proposition 3.** *If  $G = (\mathcal{A}, \mathcal{R}, p)$  is a probabilistic argument graph such that  $p$  is maximal, then  $p(G) = 1$ .*

So we see that we recover Dung's original definitions and results by assuming all arguments have probability 1.

**Proposition 4.** *If  $G = (\mathcal{A}, \mathcal{R}, p)$  is a probabilistic argument graph such that  $p$  is maximal, and  $\Gamma \subseteq \mathcal{A}$ , then  $\Gamma$  is admissible iff  $p(\Gamma) = 1$ .*

At the other extreme, if all the arguments in a probabilistic argument graph  $G$  have probability of 0, then the empty subgraph has probability 1, and so the only admissible set is the empty set (with probability 1).

We may regard the following as postulates that should hold for the probability  $p(\Gamma)$  that  $\Gamma$  is admissible.

- **(A0)**  $p(\emptyset) = 1$ .
- **(A1)** if  $\Gamma$  is not conflictfree, then  $p(\Gamma) = 0$ .
- **(A2)**  $p$  is maximal iff for all  $\Gamma \subseteq \mathcal{A}$ ,  $p(\Gamma) = 1$  or  $p(\Gamma) = 0$ .
- **(A3)**  $p(\Gamma) = 1$  iff  $\forall G' \sqsubseteq G$  ( $p(G') = 0$  or  $G' \Vdash \Gamma$ ).
- **(A4)**  $p(\Gamma_1) \leq p(\Gamma_2)$  if  $Q(\Gamma_1) \subseteq Q(\Gamma_2)$ .
- **(A5)** If  $Q(\Gamma_1) \subseteq Q(\Gamma_2)$ , then  $p(\Gamma_2) = p(\Gamma_1) + p(G_1) + \dots + p(G_i)$ , where  $Q(\Gamma_2) \setminus Q(\Gamma_1) = \{G_1, \dots, G_i\}$ .
- **(A6)** If  $\mathcal{A}$  is conflictfree, and  $\Gamma \subseteq \mathcal{A}$ , then  $p(\Gamma) = \prod_{\alpha \in \Gamma} p(\alpha)$

We can explain these postulates as follows: (A0) Since the empty set is always an admissible set, the probability of it is 1; (A1) Since a set that is not conflictfree can never be an admissible set, the probability of it is 0; (A2) If the probability of each argument is 1, then there is just one subgraph to consider, which is the original graph, and so each subset of argument is either an admissible set (with probability 1) or not an admissible set (with probability 0); (A3) The probability of a set being admissible is 1 iff all subgraphs are such that the subgraph entails that set or the subgraph has zero probability; (A4) The probability of a set being admissible increases monotonically as the membership of the subgraphs entailing it increases; (A5) If the set of subgraphs entailing  $\Gamma_1$  is a subset of those entailing  $\Gamma_2$ , then the extra probability assigned to  $\Gamma_2$  is just the sum of the probability of the extra subgraphs that entail  $\Gamma_2$ ; and (A6) If the set of all arguments is

conflictfree, then the probability of a set being admissible is the product of the probability of the arguments in the set. As shown next, the postulates are met by Definition 3.

**Proposition 5.** *Given a probabilistic argument graph  $G = (\mathcal{A}, \mathcal{R}, p)$ , a set of arguments  $\Gamma \subseteq \mathcal{A}$ , the definition for  $p(\Gamma)$  given in Definition 3 satisfies the postulates A0 to A6.*

*Proof.* Assume the probability function  $p$  satisfies Definition 3. (A0) For each  $G' \sqsubseteq G$ , the definition of admissible set implies that  $G' \Vdash \emptyset$  holds. Therefore, from Proposition 1, we have  $p(\emptyset) = 1$ . (A1) Assume  $\Gamma$  is not conflictfree. Therefore, from the definition of admissible set, there is no graph  $G'$  such that  $G' \Vdash \Gamma$ . Therefore,  $\sum_{G' \in Q(\Gamma)} p(G') = 0$ , and hence we have  $p(\Gamma) = 0$ . (A2) Assume the probability of each argument in  $\mathcal{A}$  is 1. Therefore by Proposition 3,  $p(G) = 1$ , and for all  $G' \sqsubset G$ ,  $p(G') = 0$ . Therefore,  $p(\Gamma) = 1$  iff  $G \Vdash \Gamma$ , and  $p(\Gamma) = 0$  iff  $G \not\Vdash \Gamma$ . Therefore, for all  $\Gamma \subseteq \mathcal{A}$ ,  $p(\Gamma) = 1$  or  $p(\Gamma) = 0$ . (A3) By Definition 3,  $p(\Gamma) = 1$  iff  $\sum_{G' \in Q(\Gamma)} p(G') = 1$ . Also as a corollary of Proposition 1,  $\sum_{G' \in Q(\Gamma)} p(G') = 1$  iff  $\forall G' \sqsubseteq G$  ( $p(G') = 0$  or  $G' \Vdash \Gamma$ ). Therefore,  $p(\Gamma) = 1$  iff  $\forall G' \sqsubseteq G$  ( $p(G') = 0$  or  $G' \Vdash \Gamma$ ). (A4) Assume  $Q(\Gamma_1) \subseteq Q(\Gamma_2)$ . Therefore, by Definition 3,  $p(\Gamma_2) \leq p(\Gamma_1)$ . (A5) Assume  $Q(\Gamma_1) \subseteq Q(\Gamma_2)$ . Let  $Q(\Gamma_2) \setminus Q(\Gamma_1) = \{G_1, \dots, G_i\}$  and  $Q(\Gamma_1) = \{G_{i+1}, \dots, G_{i+j}\}$ . Therefore, by Definition 3,  $p(\Gamma_1) = p(G_{i+1}) + \dots + p(G_{i+j})$ , and  $p(\Gamma_2) = p(G_1) + \dots + p(G_i) + p(G_{i+1}) + \dots + p(G_{i+j})$ . Therefore,  $p(\Gamma_2) = p(\Gamma_1) + p(G_1) + \dots + p(G_i)$ . (A6) Assume  $\mathcal{A}$  is conflictfree, and  $\Gamma \subseteq \mathcal{A}$ . Also, for any  $G' \sqsubseteq G$ , where  $G' = (\mathcal{A}', \mathcal{R}', p)$ , let  $\text{Nodes}(G')$  denote  $\mathcal{A}'$ . Hence, for any  $G' \sqsubseteq G$ , we have  $G' \Vdash \Gamma$  iff  $\Gamma \subseteq \text{Nodes}(G')$ . So from Definition 3, we have  $p(\Gamma) = \sum_{G' \sqsubseteq G \text{ s.t. } \Gamma \subseteq \text{Nodes}(G')} p(G')$ . Then using Definition 2 to substitute for  $p(G')$ ,

$$p(\Gamma) = \sum_{G' \sqsubseteq G \text{ s.t. } \Gamma \subseteq \text{Nodes}(G')} \left( \prod_{\alpha \in \text{Nodes}(G')} p(\alpha) \times \prod_{\alpha \in \mathcal{A} \setminus \text{Nodes}(G')} (1 - p(\alpha)) \right)$$

Let  $\Phi = \mathcal{A} \setminus \Gamma$ , and so we can rewrite the above to the following.

$$p(\Gamma) = \sum_{\Psi \subseteq \Phi} \left( \prod_{\alpha \in \Gamma \cup \Psi} p(\alpha) \times \prod_{\alpha \in \mathcal{A} \setminus (\Gamma \cup \Psi)} (1 - p(\alpha)) \right)$$

Therefore, we can rewrite the above to obtain  $p(\Gamma) = \prod_{\alpha \in \Gamma} p(\alpha)$ .  $\square$

The following is a representation result for the probability of admissibility.

**Proposition 6.** *For a probabilistic argument graph  $G = (\mathcal{A}, \mathcal{R}, p)$ ,  $p$  satisfies Definition 3 iff  $p$  satisfies A1, A5 and A6.*

*Proof.* ( $\Rightarrow$ ) Shown in Proposition 5. ( $\Leftarrow$ ) Assume A1, A5 and A6. Consider a probabilistic argument graph  $G = (\mathcal{A}, \mathcal{R}, p)$ , and a set of arguments  $\Gamma \subseteq \mathcal{A}$ . We use three cases to show the result. (Case 1) In this case,  $\Gamma$  is conflictfree and  $\mathcal{A}$  is conflictfree. So from A6,  $p(\Gamma) = \left( \prod_{\alpha \in \Gamma} p(\alpha) \right) \times \left( \prod_{\alpha \in \mathcal{A} \setminus \Gamma} (1 - p(\alpha)) \right)$ . Then we can use the proof in Proposition 5 (A6), since each step is equal to the previous step. Therefore, we can show that  $p(\Gamma) = \sum_{G' \in Q(\Gamma)} p(G')$  in this case. (Case 2) In this case,  $\Gamma$  is conflict-free and  $\mathcal{A}$  is not conflictfree. From A1,  $p(\mathcal{A}) = 0$ . Therefore,  $Q(\mathcal{A}) = \emptyset$ . Since  $Q(\Gamma) \neq \emptyset$ ,  $Q(\mathcal{A}) \subset Q(\Gamma)$ . Therefore,  $Q(\Gamma) \setminus Q(\mathcal{A}) = Q(\Gamma)$ . Therefore, by A5, if

$Q(\Gamma) = \{G_1, \dots, G_i\}$ , then  $p(\Gamma) = p(G_1) + \dots + p(G_i)$ . Therefore,  $\sum_{G' \in Q(\Gamma)} p(G')$  in this case. (Case 3) In this case,  $\Gamma$  is not conflictfree. So from A1,  $p(\Gamma) = 0$ . Also, by the definition of admissible set,  $Q(\Gamma) = \emptyset$ . So,  $p(\Gamma) = \sum_{G' \in Q(\Gamma)} p(G')$  in this case.  $\square$

In this section, we have shown some desirable properties hold for the probability function for admissible sets.

## 6. Probability functions over extensions

The proposals for probabilistic argument graphs [DT10,LON11] do introduce the idea of a probability function over extensions (i.e Definition 4), but they do not consider the properties of the definition. To address this, we start by proposing the following as constraints that should hold for any way of calculating  $p(\Gamma^X)$ .

- **(B1)** If  $p$  is maximal, then for all  $\Gamma \subseteq \mathcal{A}$ ,  $p(\Gamma^X) = 1$  or  $p(\Gamma^X) = 0$ .
- **(B2)** If  $p$  is uniform and not maximal, then for all  $\Gamma \subseteq \mathcal{A}$ ,  $p(\Gamma^X) < 1$ .
- **(B3)** For all  $\Gamma \subseteq \mathcal{A}$ ,  $p(\Gamma^{\text{gr}}) \leq p(\Gamma^{\text{pr}})$ , and  $p(\Gamma^{\text{pr}}) \leq p(\Gamma^{\text{co}})$ .
- **(B4)** For all  $\Gamma \subseteq \mathcal{A}$ ,  $p(\Gamma^X) \leq p(\Gamma)$ .

We explain these postulates as follows: (B1) If the probability of each argument is 1, then there is just one subgraph to consider, which is the original graph, and so each subset of argument is either an admissible set (with probability 1) or not an admissible set (with probability 0); (B2) If  $p$  is uniform and not maximal, then each subgraph is possible, and so no extension has a probability of 1; (B3) The probability that  $\Gamma$  is a complete extension is greater than it being a preferred extension, which in turn is greater than it being a grounded extension; and (B4) The probability that  $\Gamma$  is an admissible set is greater than the probability that  $\Gamma$  is an extension. We will show that the above constraints are met by the definition.

**Proposition 7.** *Given a probabilistic argument graph  $G = (\mathcal{A}, \mathcal{R}, p)$ , a set of arguments  $\Gamma \subseteq \mathcal{A}$ , and  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ , the definition for  $p(\Gamma^X)$  given in Definition 4 satisfies B1 to B4.*

*Proof.* Assume  $p(\Gamma^X)$  satisfies Definition 4. (B1) Assume  $p$  is maximal. Therefore, for all  $\alpha \in \mathcal{A}$ ,  $p(\alpha) = 1$ . Therefore,  $p(G) = 1$  and for all  $G' \sqsubset G$ ,  $p(G') = 0$ . Therefore, for all  $\Gamma \subseteq \mathcal{A}$ , if  $G \Vdash_X \Gamma$ , then  $p(\Gamma^X) = 1$ , else  $p(\Gamma^X) = 0$ . (B2) Assume  $p$  is uniform and not maximal. Therefore, for all  $\alpha \in \mathcal{A}$ ,  $p(\alpha) < 1$ . Therefore, for all  $G' \sqsubseteq G$ ,  $0 < p(G') < 1$ . Also, for each  $\Gamma \subseteq \mathcal{A}$ , there is a  $G' \sqsubseteq G$  such that not all the arguments in  $\Gamma$  appear as nodes in  $G'$ . Therefore,  $Q_X(\Gamma) \subset \{G' \mid G' \sqsubseteq G\}$ . Therefore,  $\sum_{G' \in Q_X(\Gamma)} p(G') < \sum_{G' \sqsubseteq G} p(G')$ . Hence, for all  $\Gamma \subseteq \mathcal{A}$ ,  $p(\Gamma^X) < 1$ . (B3) For all  $G' \sqsubseteq G$ ,  $G' \Vdash_{\text{gr}} \Gamma$  implies  $G' \Vdash_{\text{pr}} \Gamma$ , and  $G' \Vdash_{\text{pr}} \Gamma$  implies  $G' \Vdash_{\text{co}} \Gamma$ . Therefore,  $\sum_{G' \in Q_{\text{gr}}(\Gamma)} p(G') \leq \sum_{G' \in Q_{\text{pr}}(\Gamma)} p(G')$ , and  $\sum_{G' \in Q_{\text{pr}}(\Gamma)} p(G') \leq \sum_{G' \in Q_{\text{co}}(\Gamma)} p(G')$ . Therefore,  $p(\Gamma^{\text{gr}}) \leq p(\Gamma^{\text{pr}})$ , and  $p(\Gamma^{\text{pr}}) \leq p(\Gamma^{\text{co}})$ . (B4) For all  $G' \sqsubseteq G$ , and for all  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ ,  $G' \Vdash \Gamma$  implies  $G' \Vdash_X \Gamma$ . So,  $\sum_{G' \in Q_X(\Gamma)} p(G') \leq \sum_{G' \in Q(\Gamma)} p(G')$ , and hence,  $p(\Gamma^X) \leq p(\Gamma)$ .  $\square$

The following results show that by constraining the probability function, we can obtain extensions with unit probability. The first of these results shows that whatever argu-



ment graph we start with, we can choose a probability function that lets any conflictfree set be an extension with probability of 1.

**Proposition 8.** *For all argument graphs  $G = (\mathcal{A}, \mathcal{R})$ , for all  $\Gamma \subseteq \mathcal{A}$ , and for all  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ , if  $\Gamma$  is conflictfree, then there is a probability function  $p$ , such that  $(\mathcal{A}, \mathcal{R}, p)$  is a probabilistic argument graph and  $p(\Gamma^X) = 1$ .*

By disbelieving at least one argument in each cycle, we can ensure that there is a non-empty extension with non-zero probability. So by making some commitment to some arguments, cycles can be broken.

**Proposition 9.** *For all probabilistic graphs  $(\mathcal{A}, \mathcal{R}, p)$ , where  $\mathcal{A} \neq \emptyset$  and  $p$  is not minimal, if for all cycles  $(\alpha_1, \alpha_2), \dots, (\alpha_k, \alpha_1) \in \mathcal{R}$ , there is a  $\alpha_i \in \{\alpha_1, \dots, \alpha_n\}$  such that  $p(\alpha_i) = 0$ , then there is a  $\Gamma \subseteq \mathcal{A}$  such that  $p(\Gamma^X) > 0$  and  $\Gamma \neq \emptyset$ .*

*Proof.* Assume that for all cycles  $(\alpha_1, \alpha_2), \dots, (\alpha_k, \alpha_1) \in \mathcal{R}$ , there is a  $\alpha_i \in \{\alpha_1, \dots, \alpha_n\}$  such that  $p(\alpha_i) = 0$ . Therefore, for all  $G' \sqsubseteq G$ , if  $p(G') > 0$ , then  $G'$  contains no cycles. Therefore, for all  $G' \sqsubseteq G$ , and for all  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ , if  $p(G') > 0$ , there is a  $\Gamma \subseteq \mathcal{A}$  such that  $G' \Vdash_X \Gamma$ . Therefore, for all  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ , there is a  $\Gamma \subseteq \mathcal{A}$  such that  $p(\Gamma^X) > 0$  and  $\Gamma \neq \emptyset$ .  $\square$

In this section, we have introduced postulates for the probability of an extension, and we have considered some of the circumstances under which the probability function or extension can be chosen to meet certain conditions such as a non-zero probability.

## 7. Probability function over inferences

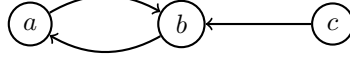
The proposals for probabilistic argument graphs [DT10,LON11] do not consider the probability of “inferences”. Yet given a probabilistic argument graph  $G = (\mathcal{A}, \mathcal{R}, p)$ , and an argument  $\alpha \in \mathcal{A}$ , we can calculate the probability that  $\alpha$  is an  $X$  inference (i.e. the probability that argument  $\alpha$  is in an  $X$  extension), which we denote by  $p(\alpha^X)$ , where  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ . To address this, we define the probability of a formula being in an  $X$  extension as the sum of the probabilities for the subgraphs that entail an  $X$  extension containing the formula, and this requires the following subsidiary definitions: For an argument  $\alpha \in \mathcal{A}$ , the set of subgraphs that imply an argument is an  $X$  extension, denoted  $I_X(\alpha)$ , is  $I(\alpha) = \{G' \sqsubseteq G \mid G' \Vdash \Gamma \text{ and } \alpha \in \Gamma\}$ .

**Definition 6.** *Let  $G = (\mathcal{A}, \mathcal{R}, p)$  be a probabilistic argument graph. For an argument  $\alpha$ , and  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ , the probability that it is in an  $X$  extension, denoted  $p(\alpha^X)$ , is  $\sum_{G' \in I_X(\alpha)} p(G')$ .*

As we have seen in the previous examples, the same argument can appear in multiple extensions with non-zero probability. So the above definition for probability of inferences allows this information to be drawn out.

**Example 6.** *Consider the following argument graph where  $p(a) = 1$ ,  $p(b) = 0.5$ , and  $p(c) = 0.5$ . There are four subgraphs,  $G^1$  to  $G^4$ , with non-zero probability. Each has probability 0.25.  $G^1$  is the graph,  $G^2$  is the subgraph composed of  $a$  and  $c$ ,  $G^3$  is the subgraph composed of  $a$  and  $b$ , and  $G^4$  is the subgraph composed of  $a$ . The grounded*

extension of  $G^1$  and  $G^2$  is  $\{a, c\}$ , the grounded extension of  $G^3$  is  $\{\}$ , and the grounded extension of  $G^4$  is  $\{a\}$ . Therefore  $p(a^{\text{gr}}) = 0.75$ ,  $p(b^{\text{gr}}) = 0$ , and  $p(c^{\text{gr}}) = 0.5$ .



We now propose the following as constraints that should hold for calculating  $p(\alpha^X)$ . For this, we require the the following subsidiary definitions: For an argument  $\alpha$ , we say that:  $\alpha$  is **self-attacking** when  $\alpha$  is an attacker of  $\alpha$ ;  $\alpha$  is **unattacked** when there is no attacker of  $\alpha$ ; and  $\alpha$  is **undefended** when there is an attacker  $\beta$  of  $\alpha$  but there is no attacker  $\gamma$  of  $\beta$ .

- (C1) If  $\alpha$  is self-attacking, then  $p(\alpha^X) = 0$ .
- (C2) If  $\alpha$  is unattacked, then  $p(\alpha^X) = p(\alpha)$ .
- (C3) If  $\alpha$  is undefended, then  $p(\alpha^X) \leq p(\alpha)$ .
- (C4) For all  $\alpha \in \mathcal{A}$ ,  $p(\alpha^{\text{gr}}) \leq p(\alpha^{\text{pr}})$ , and  $p(\alpha^{\text{pr}}) \leq p(\alpha^{\text{co}})$ .
- (C5) For all  $\alpha \in \mathcal{A}$ ,  $p(\alpha^X) \leq p(\{\alpha\})$ .

We explain these postulates as follows: (C1) If  $\alpha$  is self-attacking, there is no extension containing  $\alpha$ , and so it has zero probability; (C2) If  $\alpha$  is unattacked, then the probability that it is in an extension is equal to its original probability (dialectical consistency); (C3) If  $\alpha$  is undefended, then the probability that it is in an extension is less than its original probability (dialectical diminution); (C4) The probability that  $\alpha$  is in a complete extension is greater than being in a preferred extension, which in turn is greater than being in a grounded extension; and (C5) The probability that  $\{\alpha\}$  is an admissible set is greater than  $\alpha$  is in an extension.

**Proposition 10.** *Given a probabilistic argument graph  $G = (\mathcal{A}, \mathcal{R}, p)$ , an argument  $\alpha \in \mathcal{A}$ , and  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ , the definition for  $p(\alpha^X)$  given in Definition 6 satisfies C1 to C5.*

*Proof.* Assume  $p(\alpha^X)$  satisfies Definition 6. (C1) Assume  $\alpha$  is self-attacking. Therefore, for all  $\Gamma \in \mathcal{A}$ , if  $\alpha \in \Gamma$ , then  $\Gamma$  is not conflictfree. Therefore, for all  $G' \sqsubseteq G$ , for all  $\Gamma \subseteq \mathcal{A}$ , if  $\alpha \in \Gamma$ , then  $G' \not\models \Gamma$ . Therefore, for all  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ ,  $I_X(\alpha) = \emptyset$ , and hence  $p(\alpha^X) = 0$ . (C2) Assume  $\alpha$  is unattacked. Therefore, for all  $G' \sqsubseteq G$ , if  $\alpha$  is a node in  $G'$ , then  $\alpha$  is unattacked in  $G'$ . Therefore, for all  $G' \sqsubseteq G$ , if  $\alpha$  is a node in  $G'$ , then there is a  $\Gamma \subset \mathcal{A}$ , such that  $G' \Vdash_X \Gamma$  and  $\alpha \in \Gamma$ . Let  $J(\alpha) = \{G' \sqsubseteq G \mid \alpha \text{ is a node in } G'\}$ . Therefore,  $I_X(\alpha) = J(\alpha)$ . Also, from Definition 2, using a derivation analogous to that used in the proof of Proposition 1, it is the case that  $p(\alpha) = \sum_{G' \in J(\alpha)} p(G')$ . Since,  $p(\alpha^X) = \sum_{G' \in I_X(\alpha)} p(G')$ , we have that  $p(\alpha^X) = \sum_{G' \in J(\alpha)} p(G')$ . Hence,  $p(\alpha^X) = p(\alpha)$ . (C3) Assume  $\alpha$  is undefended. Therefore, there is a  $G' \sqsubseteq G$ , such that  $\alpha$  is a node in  $G'$ , then  $\alpha$  is undefended in  $G'$ . Therefore, there is a  $G' \sqsubseteq G$ , such that  $\alpha$  is a node in  $G'$ , and there is a  $\Gamma \subset \mathcal{A}$ , such that  $G' \Vdash_X \Gamma$  and  $\alpha \notin \Gamma$ . Let  $J(\alpha) = \{G' \sqsubseteq G \mid \alpha \text{ is a node in } G'\}$ . Therefore,  $I_X(\alpha) \subseteq J(\alpha)$ . Also, from Definition 2, using a derivation analogous to that used in the proof of Proposition 1, it is the case that  $p(\alpha) = \sum_{G' \in J(\alpha)} p(G')$ . Since,  $p(\alpha^X) = \sum_{G' \in I_X(\alpha)} p(G')$ , we have that  $p(\alpha^X) \leq p(\alpha)$ . (C4) For all  $G' \sqsubseteq G$ ,  $G' \Vdash_{\text{gr}} \Gamma$  implies  $G' \Vdash_{\text{pr}} \Gamma$ , and  $G' \Vdash_{\text{pr}} \Gamma$  implies  $G' \Vdash_{\text{co}} \Gamma$ . Therefore,  $I_{\text{gr}}(\alpha) \subseteq I_{\text{pr}}(\alpha)$  and  $I_{\text{pr}}(\alpha) \subseteq I_{\text{co}}(\alpha)$ . Therefore,  $\sum_{G' \in I_{\text{gr}}(\alpha)} p(G') \leq$

$\sum_{G' \in I_{pr}(\alpha)} p(G')$ , and  $\sum_{G' \in I_{pr}(\alpha)} p(G') \leq \sum_{G' \in I_{co}(\alpha)} p(G')$ . Therefore,  $p(\alpha^{gr}) \leq p(\alpha^{pr})$ , and  $p(\alpha^{pr}) \leq p(\alpha^{co})$ . (C5) For all  $G' \sqsubseteq G$ , and for all  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ ,  $G' \Vdash \Gamma$  implies  $G' \Vdash_X \Gamma$ . Therefore, for all  $X \in \{\text{co}, \text{pr}, \text{gr}, \text{st}\}$ ,  $I_X(\alpha) \subseteq I(\alpha)$ . Hence,  $\sum_{G' \in I_X(\alpha)} p(G') \leq \sum_{G' \in I(\alpha)} p(G')$ , and so,  $p(\alpha^X) \leq p(\{\alpha\})$ .  $\square$

The next result shows that we can equivalently define the probability of a formula being in a grounded extension in terms of the probability function over extensions.

**Proposition 11.** *Given a probabilistic argument graph  $G = (\mathcal{A}, \mathcal{R}, p)$ , an argument  $\alpha \in \mathcal{A}$ ,  $p(\alpha^{gr}) = \sum_{\Gamma \subseteq \mathcal{A} \text{ s.t. } \alpha \in \Gamma} p(\Gamma^{gr})$*

*Proof.*  $p(\alpha^{gr}) = \sum_{G' \in I_{gr}(\alpha)} p(G')$   
 $= \sum_{G' \in \{G' \sqsubseteq G \mid G' \Vdash_{gr} \Gamma \text{ and } \alpha \in \Gamma\}} p(G')$   
 $= \sum_{\Gamma \subseteq \mathcal{A} \text{ s.t. } \alpha \in \Gamma} \sum_{G' \in \{G' \sqsubseteq G \mid G' \Vdash_{gr} \Gamma\}} p(G')$   
 $= \sum_{\Gamma \subseteq \mathcal{A} \text{ s.t. } \alpha \in \Gamma} \sum_{G' \in \{G' \in Q_{gr}(\Gamma)\}} p(G')$   
 $= \sum_{\Gamma \subseteq \mathcal{A} \text{ s.t. } \alpha \in \Gamma} p(\Gamma^{gr})$   $\square$

In this section, we have defined the probability of a formula being in an extension as the sum of the probabilities of the subgraphs that entail an extension containing the formula. This satisfies a number of simple postulates that we have introduced, and we have shown in the case of grounded semantics this is equivalent to the sum of the probabilities of the extensions containing the formula.

## 8. Discussion

In this paper, we have reviewed the proposals for probabilistic argument graphs [DT10, LON11]. Probabilistic argument graphs are a valuable contribution to better understanding argumentation arising in the real-world. However, they also raise questions about what the probabilities over arguments mean, and can the definitions be justified. To address this need, this paper has made the following contributions: (1) A clarification for why independence can be assumed when generating the probability distribution over the spanning subgraphs; (2) An analysis of how the set of spanning subgraphs offers a probability space; (3) A proposal for sets of postulates for the probability function over admissible sets and extensions, plus a number of results concerning these and related properties; and (4) A proposal for a probability function for inferences from probabilistic argument graphs, together with a set of properties that hold for the probability function.

As well as first introducing the idea of probabilistic argument graphs, Dung and Thang [DT10] have used them in a version of assumption-based argumentation in which a subset of the rules are probability rules. In another rule-based system for argumentation by Riveret *et al* [RRS<sup>+</sup>07], the belief in the premises of an argument is used to calculate the belief in the argument. However, the proposal does not investigate further the nature of this assignment, in particular there is no investigation of how it relates to abstract argumentation, but rather its use in dialogue is explored. For logical arguments, a probability function on model has been used by Haenni *et al* [Hae98, HKL00, Hae01] for a notion of probabilistic argumentation for diagnosis. Arguments are constructed for and against particular diagnoses (i.e. only arguments and counterarguments that rebut each other are considered). However, they do not consider their proposal with respect to

abstract argumentation. In the LA system, another logic-based framework for argumentation, probabilities are also introduced into the rules, and these probabilities are propagated by the inference rules so that arguments are qualified by probabilities (such as via labels such as "likely", "very likely", etc). Again, there is no consideration of how this relates to abstract argumentation [EGKF93,FD00].

Whilst using weights on arguments (such as discussed in [BGW05]), allow for a notion of uncertainty to be represented, our understanding is incomplete for using such weights in a way that conforms with established theories of quantitative uncertainty. Preferences over arguments have been harnessed in argumentation theory (see for example [AC98,AC02]) in order to decide on a pairwise basis whether one argument defeats another argument. In some situations, this is a powerful and intuitive solution. Sometimes, the preferences seem to be based on the relative strength of belief in the arguments, though more research is required to better understand the relationship between the use of preferences over arguments and the use of a probability function over arguments. Of course probability theory is only one way of capturing uncertainty about arguments, and indeed, some interesting proposals have been made for using possibility theory in argumentation (see for example [AP04,ACGS08]).

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