Base Logics in Argumentation

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Abstract. There are a number of frameworks for modelling argumentation in logic. They incorporate a formal representation of individual arguments and techniques for comparing conflicting arguments. A common assumption for logic-based argumentation is that an argument is a pair $\langle \Phi, \alpha \rangle$ where Φ is a minimal subset of the knowledgebase such that Φ is consistent and Φ entails the claim α. We call the logic used for consistency and entailment, the base logic. Different base logics provide different definitions for consistency and entailment and hence give us different options for argumentation. This paper discusses some of the commonly used base logics in logic-based argumentation, and considers various criteria that can be used to identify commonalities and differences between them.

Keywords. logic-based argumentation, logical argument systems, consequence relations, defeasible logic, classical logic

1. Introduction

Proposals for logic-based argumentation rely on an underlying logic, which we call a *base logic*, for generating logical arguments and for defining the counterargument relationships (using inference of conflict or existence of inconsistency). For logic-based argumentation, we assume that an argument is a pair $\langle \Phi, \alpha \rangle$ where Φ entails the claim α. Let $\vdash_x$ be the consequence relation of the base logic, and so Φ entails the claim α means $\Phi \vdash_x \alpha$. Many proposals for logic-based argumentation also stipulate that for $\langle \Phi, \alpha \rangle$ to be an argument, Φ is minimal (i.e. there is no $\Phi' \subset \Phi$ such that $\Phi' \vdash_x \alpha$), and/or Φ is consistent (which in most proposals for argumentation systems means that it is not the case that $\Phi \vdash_x \alpha$ and $\Phi \vdash_x \neg\alpha$ for any atom α).

The choice of base logic is an important design decision for a logic-based argumentation system. This then raises the questions of what are the minimal requirements for a base logic and what are the factors that need to be considered for a base logic? In this paper, we consider these questions in terms of general properties and in terms of the base logics that arise in key approaches to logic-based argumentation. The net result is that we can see some useful properties holding for all the key approaches (including the important properties of cut, monotonicity, and a restricted form of reflexivity), and some useful properties that differentiate approaches. We also suggest that given the wide range of logics being developed in the knowledge representation field, there are further interesting opportunities for using different base logics in argumentation.

2. Examples of base logics in argument systems

To help us explore the nature of base logics, we consider some key proposals for logic-based argumentation, and draw out the base logics used. We start with simple proposals that use classical logic $\vdash_c$ as base logic [10,3,5], and for which $\langle \Phi, \alpha \rangle$ is an argument iff $\Phi \vdash_c \alpha$ and there is no $\Phi' \subset \Phi$ such that $\Phi' \vdash_c \alpha$ and $\Phi \not\vdash_c \bot$.

Example 1. *Let* $\Delta = \{\neg\neg a, \neg b \rightarrow \neg a, \neg b \vee (c \wedge d), b \wedge c \wedge \neg b, \neg f \rightarrow g \vee h\}$. *So according to the above,* $\langle \{\neg\neg a, \neg b \rightarrow \neg a, \neg b \vee (c \wedge d)\}, e \rightarrow d \rangle$ *is an argument.*

The most common kind of base logic is a form of defeasible logic such as used in defeasible logic programming [16], defeasible argumentation with specificity-based preferences [27], the ASPIC system [9], and argument-based extended logic programming [24]. For a general coverage of defeasible logics in argumentation see [11,25,26].

The language for defeasible logic is based on rules of the following form where $\beta_1, \ldots, \beta_j, \beta_{j+1}$ are literals and $\rightarrow_k$ is an implication symbol.

$$\beta_1 \wedge \ldots \wedge \beta_j \rightarrow_k \beta_{j+1}$$

For the defeasible logic approaches to argumentation, such as [16], there can be more than one type of implication symbol $\rightarrow_k$, and the proof theory for the base logic is given by a derivation using modus ponens as defined next. Note, the consequence relation ignores any differences between the different types of implication symbol that may appear in the knowledgebase[1].

Definition 1. *Let* Δ *be the union of a set of rules and a set of literals. The defeasible logic consequence relation* $\vdash_d$ *is defined as follows.*

$\Delta \vdash_d \psi$ *iff there is a sequence of literals* $\alpha_1, \ldots, \alpha_n$
such that ψ *is* α_n *and for each* $\alpha_i \in \{\alpha_1, \ldots \alpha_n\}$
either α_i *is a literal in* Δ
or there is a $\beta_1 \wedge \ldots \wedge \beta_j \rightarrow_k \alpha_i \in \Delta$
and $\{\beta_1, \ldots, \beta_j\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\}$

In the following example, $\rightarrow_1$ denotes a strict rule and $\rightarrow_2$ denotes a defeasible rule, though, as defined above, this denotation is ignored by the $\vdash_d$ consequence relation.

Example 2. *Let* $\Delta = \{p, \neg q, p \rightarrow_1 \neg r, \neg q \wedge \neg r \rightarrow_2 s, s \rightarrow_1 t, p \wedge t \rightarrow_2 u\}$. *Therefore* $\Delta \vdash_d u$ *where the sequence of literals in the derivation is* $p, \neg r, \neg q, s, t, u$.

For defeasible logic programming [16], $\langle \Phi, \alpha \rangle$ is an argument iff $\Phi \vdash_d \alpha$ and there is no $\Phi' \subset \Phi$ such that $\Phi' \vdash_d \alpha$ and it is not the case that there is a β such that $\Phi \vdash_d \beta$ and $\Phi \vdash_d \neg\beta$ (i.e. Φ is a minimal consistent set entailing α), whereas for assumption-based argumentation [12], $\langle \Phi, \alpha \rangle$ is an argument iff $\Phi \vdash_d \alpha$. Note, in [16] only the defeasible rules are explicitly represented in the support of the argument, and in [12] only the literals are explicitly represented in the support of the argument, but in both cases it is a trivial change (as we do here) to explicitly represent both the rules and literals used in the derivation in the support of the argument.

[1]The type of implication appearing in each formula in the support of an argument is used for determining the relative preference of the argument when compared with other arguments.

Example 3. *Continuing Example 2, the following is an argument in defeasible logic programming [16].*

$$\langle\{p, \neg q, p \rightarrow_1 \neg r, \neg q \wedge \neg r \rightarrow_2 s, s \rightarrow_1 t, p \wedge t \rightarrow_2 u\}, u\rangle$$

Example 4. *For* $\Delta = \{p, \neg q, s, p \rightarrow \neg r, \neg q \wedge \neg r \wedge s \rightarrow t, t \wedge p \rightarrow u, v\}$*, the following is an argument in assumption-based argumentation [12].*

$$\langle\{p, \neg q, s, p \rightarrow \neg r, \neg q \wedge \neg r \wedge s \rightarrow t\}, t\rangle$$

Also note that in [12], there is the notion of a backward argument which is an argument that can be constructed by backward chaining reasoning: This means that the derivation is constructed starting from the goal, which generates subgoals, and which are addressed by recursion. If $\langle\Phi, \alpha\rangle$ is a backward argument, then $\langle\Phi, \alpha\rangle$ is an argument, and so $\Phi \vdash_d \alpha$. Furthermore, this backward reasoning will avoid some unnecessary formulae appearing in the support of the argument, but it is not guaranteed that the support is minimal (i.e. it is possible that there is a $\Phi' \subset \Phi$ such that $\Phi' \vdash_d \alpha$).

Clearly, classical logic has a richer language and proof theory than defeasible logic. Even if we restrict classical logic to the same language as defeasible logic, then we see many simple situations where classical logic gives an intuitive inference and defeasible logic fails to give the inference, such as in the following example.

Example 5. *Let* $\Delta = \{a \rightarrow b, \neg a \rightarrow b\}$*. Hence,* $\Delta \vdash_c b$*, but* $\Delta \not\vdash_d b$*.*

However, we should not regard classical logic as better than defeasible logic, or vice versa. Rather, there is a range of logics available as base logics, and that we should choose the base logic according to the needs of the application. Moreover, we should not restrict consideration to those base logics already considered in the literature on argumentation. There are many other candidates in the literature on artificial intelligence that could be harnessed as base logics.

In some approaches to defeasible logic, such as argument-based extended logic programming [24], a more complex defeasible rule is used that is based on two types of negation (strong $\neg$ and weak $\sim$). For an atom γ, both γ and $\neg\gamma$ are strong literals, and for a strong literal δ, $\sim\delta$ is a weak literal. An enhanced defeasible rule is a formula of the following form where $\alpha_0, \ldots, \alpha_m$ are strong literals, $\beta_0, \ldots, \beta_n$ are weak literals, and δ is a strong literal and $\rightarrow_k$ is an implication symbol.

$$\alpha_0 \wedge \ldots \wedge \alpha_m \wedge \beta_0 \wedge \ldots \wedge \beta_n \rightarrow_k \delta$$

Using this language, we can obtain a refined form of a defeasible consequence relation, which we call the enhanced consequence relation $\vdash_e$ as follows. In this, the antecedent of a defeasible rule is satisfied when the strong literals can be obtained earlier in the derivation. The meaning of the weak literals is that they are assumed to not hold, and if there is evidence to the contrary in Δ, this will be manifested in the existence of a counterargument.

Definition 2. *Let* Δ *be the union of a set of enhanced defeasible rules and a set of strong literals. The enhanced consequence relation* $\vdash_e$ *is defined as follows.*

$$\begin{array}{l} \Delta \vdash_e \psi \textit{ iff there is a sequence of literals } \alpha_1, \ldots, \alpha_n \\ \quad\quad \textit{such that } \psi \textit{ is } \alpha_n \textit{ and for each } \alpha_i \in \{\alpha_1, \ldots \alpha_n\} \\ \quad\quad\quad \textit{either } \alpha_i \textit{ is a strong literal in } \Delta \\ \quad\quad\quad \textit{or there is a } \gamma_0 \wedge \ldots \wedge \gamma_m \wedge \beta_0 \wedge \ldots \wedge \beta_n \rightarrow_k \delta \in \Delta \\ \quad\quad\quad\quad \textit{and } \{\gamma_0, \ldots, \gamma_m\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\} \end{array}$$

In the following example, $\rightarrow_1$ denotes a strict rule and $\rightarrow_2$ denotes an enhanced defeasible rule, though as before, this differentiation does not affect the consequence relation.

Example 6. *Let* $\Delta = \{p, p \rightarrow_1 \neg r, \neg r \wedge \sim q \rightarrow_2 s, s \rightarrow_1 t, p \wedge t \rightarrow_1 u\}$. *Therefore* $\Delta \vdash_d u$ *where the sequence of literals in the derivation is* $p, \neg r, s, t, u$.

For argument-based extended logic programming [24], we can define $\langle \Phi, \alpha \rangle$ as an argument iff $\Phi \vdash_e \alpha$ and there is no $\Phi' \subset \Phi$ such that $\Phi' \vdash_e \alpha$.

Example 7. *Continuing Example 6, the following is an argument in argument-based extended logic programming [24].*

$$\langle \{p, p \rightarrow_1 \neg r, \neg r \wedge \sim q \rightarrow_2 s, s \rightarrow_1 t, p \wedge t \rightarrow_1 u\}, u \rangle$$

In [8,9], various proposals for argumentation based on defeasible logic were criticized for violating some postulates that they proposed for acceptable argumentation. They suggested introducing contraposition into the reasoning of the base logic offered a way to address this problem. We introduce contraposition by defining a consequence relation as follows where $\mathsf{Contrapositives}(\Delta)$ is the set of contrapositives formed from the rules in Δ.

Definition 3. *Let* Δ *be a set of rules and literals. The defeasible logic consequence relation* $\vdash_f$ *is defined as follows.*

$$\begin{array}{l} \Delta \vdash_f \psi \textit{ iff there is a sequence of literals } \alpha_1, \ldots, \alpha_n \\ \quad\quad \textit{such that } \psi \textit{ is } \alpha_n \textit{ and for each } \alpha_i \in \{\alpha_1, \ldots \alpha_n\} \\ \quad\quad\quad \textit{either } \alpha_i \textit{ is a literal in } \Delta \\ \quad\quad\quad \textit{or there is a } \beta_1 \wedge \ldots \wedge \beta_j \rightarrow_k \alpha_i \in \Delta \cup \mathsf{Contrapositives}(\Delta) \\ \quad\quad\quad\quad \textit{and } \{\beta_1, \ldots, \beta_j\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\} \end{array}$$

Example 8. *Let* $\Delta = \{q, \neg r, p \wedge q \rightarrow r, \neg p \rightarrow u\}$. *So* $\mathsf{Contrapositives}(\Delta) = \{\neg r \wedge q \rightarrow \neg p, p \wedge \neg r \rightarrow \neg q, \neg u \rightarrow p\}$. *Therefore,* $\Delta \vdash_f u$*, where the sequence of literals in the derivation is* $q, \neg r, \neg p, u$.

Further base logics considered for logic-based argumentation include (i) variants of defeasible logic with annotations for lattice-theoretic truth values (such as for Belnap's four-valued logic) [28] and for possibility theory [1], (ii) temporal reasoning calculi used with defeasible logic [4] and with classical logic [21], (iii) minimal logic (which is intuitionistic logic without the $\perp \rightarrow \phi$ axiom) [18], and (iv) a form of modal logic [13].

A more general approach to logic-based argumentation is to leave the logic for deduction as a parameter. This was proposed in abstract argumentation systems [29], and developed in assumption-based argumentation (ABA) [12]. However, since a substantial part of the development of the theory and implementation of ABA is focused on defeasible logic (e.g. [15]), we have considered the base logic of ABA as being given by the $\vdash_d$ consequence relation.

$\Delta \cup \{\alpha\} \vdash_x \alpha$	(Reflexivity)
$\Delta \cup \{\alpha\} \vdash_x \alpha$ if α is a literal	(Literal reflexivity)
$\Delta \cup \{\beta\} \vdash_x \gamma$ if $\Delta \cup \{\alpha\} \vdash_x \gamma$ and $\vdash \alpha \leftrightarrow \beta$	(Left logical equivalent)
$\Delta \vdash_x \alpha$ if $\Delta \vdash_x \beta$ and $\vdash \beta \rightarrow \alpha$	(Right weakening)
$\Delta \vdash_x \alpha \wedge \beta$ if $\Delta \vdash_x \alpha$ and $\Delta \vdash_x \beta$	(And)
$\Delta \cup \{\alpha\} \vdash_x \beta$ if $\Delta \vdash_x \beta$	(Monotonicity)
$\Delta \vdash_x \beta$ if $\Delta \vdash_x \alpha$ and $\Delta \cup \{\alpha\} \vdash_x \beta$	(Cut)
$\Delta \vdash_x \alpha \rightarrow \beta$ if $\Delta \cup \{\alpha\} \vdash_x \beta$	(Conditionalization)
$\Delta \cup \{\alpha\} \vdash_x \beta$ if $\Delta \vdash_x \alpha \rightarrow \beta$	(Deduction)
$\Delta \cup \{\alpha\} \vdash_x \beta$ if $\Delta \cup \{\neg\beta\} \vdash_x \neg\alpha$	(Contraposition)
$\Delta \cup \{\alpha \vee \beta\} \vdash_x \gamma$ if $\Delta \cup \{\alpha\} \vdash_x \gamma$ and $\Delta \cup \{\beta\} \vdash_x \gamma$	(Or)

Figure 1. Some properties of a consequence relation $\vdash_x$ adapted from [20].

3. Properties of base logics

We have defined a base logic as the logic for defining entailment in constructing arguments. Given that many logics have been proposed in philosophy, linguistics, and artificial intelligence, a natural question to ask is what are the required properties of a consequence relation. The list of properties of a consequence relation given in Figure 1 provides a good starting point for considering this question. These properties have been proposed as desirable conditions of a consequence relation. Furthermore, according to Gabbay [14] and Makinson [20], the minimal properties of a consequence relation are reflexivity, monotonicity (or a variant of it) and cut, and the need for each of them can be justified as follows:

- Reflexivity captures the idea of "transparency"; If a formula α is assumed (i.e. $\alpha \in \Delta$), then α can be declared (i.e $\Delta \vdash_x \alpha$).
- Monotonicity captures the idea of "irreversibility"; Once a formula α is declared (i.e $\Delta \vdash_x \alpha$), then there is no assumption that can cause α to be withdrawn (i.e. there is no β such that $\Delta \cup \{\beta\} \not\vdash_x \alpha$).
- Cut captures the idea of "equitability" of assumptions and inferences. Once a formula α is declared (i.e $\Delta \vdash_x \alpha$), it can be used for further reasoning.

These three properties can be seen equivalently in terms of the following three properties based on the consequence closure C_x of a logic x [20], where $C_x(\Delta) = \{\alpha \mid \Delta \vdash \alpha\}$: (inclusion) $\Delta \subseteq C_x(\Delta)$; (idempotence) $C_x(\Delta) = C_x(C_x(\Delta))$; and (monotony) $C_x(\Delta') \subseteq C_x(\Delta)$ whenever $\Delta' \subseteq \Delta$.

Classes of base logic can be identified using properties of the consequence relation, and then argument systems can be developed in terms of them. For instance, to instantiate abstract argumentation, in [2], the class of Tarskian logics has been used. This is the class defined by inclusion, idempotence, finiteness (i.e. $C_x(\Delta)$ is the union of $C_x(\Gamma)$ for all finite subsets Γ of Δ), absurdity (i.e. $C_x(\{\phi\}) = \mathcal{L}$ for some ϕ in the language $\mathcal{L}$), and coherence (i.e. $C_x(\emptyset) \neq \mathcal{L}$). Classical logic is an example of a Tarskian logic.

We now consider the base logics $\vdash_c$, $\vdash_d$, $\vdash_e$, and $\vdash_f$, reviewed in the previous section, in terms of the properties of the consequence relation given in Figure 1.

Proposition 1. *Each property holding for each of $\vdash_c$, $\vdash_d$, $\vdash_e$ and $\vdash_f$ is denoted by $\times$ in the following table.*

	$\vdash_c$	$\vdash_d$	$\vdash_e$	$\vdash_f$
Reflexivity	$\times$			
Literal reflexivity	$\times$	$\times$	$\times$	$\times$
Left logical equivalence	$\times$			
Right weakening	$\times$			
And	$\times$			
Monotonicity	$\times$	$\times$	$\times$	$\times$
Cut	$\times$	$\times$	$\times$	$\times$
Conditionalization	$\times$			
Deduction	$\times$			
Contraposition	$\times$			$\times$
Or	$\times$			

The good news from the above proposition is that the base logics $\vdash_c$, $\vdash_d$, $\vdash_e$, and $\vdash_f$, that appear in the main proposals for logic-based argumentation, satisfy the properties of monotonicity and cut. Furthermore, by a trivial adaptation of the $\vdash_d$, $\vdash_e$, and $\vdash_f$ consequence relations, they could also all satisfy reflexivity. For instance, for $\vdash_d$ if we add the meta-rule that "if ψ is a rule in Δ, then $\Delta \vdash_d \psi$", then reflexivity also holds. This means that we can say that the main base logics used in argumentation meet the minimal requirements for being consequence relations. Furthermore, if we add the above meta-rule, then we will also get the deduction property holding.

We can also compare base logics according to inferential strength of their consequence relations as follows.

Definition 4. *For $\vdash_x$ and $\vdash_y$, $\vdash_x$ is at* **least as strong** *as $\vdash_y$ iff for all knowledgebases Δ, and all formulae α, if $\Delta \vdash_y \alpha$, then $\Delta \vdash_x \alpha$. Furthermore, $\vdash_x$ is* **stronger** *than $\vdash_y$ iff $\vdash_x$ is at least as strong as $\vdash_y$ and it is not the case that $\vdash_y$ is at least as strong as $\vdash_x$. Finally, $\vdash_x$ and $\vdash_y$ are* **equally strong** *iff $\vdash_x$ is at least as strong as $\vdash_y$ and $\vdash_y$ is at least as strong as $\vdash_x$.*

Proposition 2. *Let Δ be the union of a set of rules (excluding enhanced defeasible rules) and a set of literals (excluding weak literals): (1) $\vdash_c$ is stronger than $\vdash_d$, $\vdash_e$, and $\vdash_f$; (2) $\vdash_f$ is stronger than $\vdash_d$ and $\vdash_e$; and (3) $\vdash_d$ and $\vdash_e$ are equally strong.*

Another way that we can compare the consequence relations is with how they deal with inconsistent assumptions. For this, we consider the trivializable and the purity properties. The former characterizes the situations where any formula of the language follows from an inconsistent set of premises, and the later characterizes a notion of relevancy between premises and consequences.

Definition 5. *The consequence relation $\vdash_x$ is trivializable iff for all Δ, there is an atom α such that if $\Delta \vdash \alpha$ and $\Delta \vdash \neg\alpha$ then $\Delta \vdash_x \beta$ for all atoms β.*

Definition 6. *Let $\mathsf{Atoms}(\Gamma)$ give the atoms appearing in a set of formulae Γ. A formula α is pure with respect to Δ iff $\mathsf{Atoms}(\Delta) \cap \mathsf{Atoms}(\{\alpha\}) \neq \emptyset$. A consequence relation $\vdash_x$ is pure iff for all α and Δ, if $\Delta \vdash_x \alpha$, then α is pure with respect to Δ.*

Proposition 3. *If a consequence relation $\vdash_x$ is pure, then $\vdash_x$ is not trivializable. However, the converse does not necessarily hold.*

Proposition 4. *The $\vdash_c$ consequence relation is trivializable and not pure, whereas the $\vdash_d$, $\vdash_e$, and $\vdash_f$ consequence relations are not trivializable and they are pure.*

The trivialization and lack of purity of classical logic does not appear to be a shortcoming in argumentation since for $\langle \Phi, \alpha \rangle$ to be an argument, most proposals have that Φ is consistent.

Another dimension for comparing base logics is with respect to computational complexity. For $\vdash_c$, it is well-known that the decision problem for entailment is co-NP complete and for satisfiability it is NP complete [17]. This results in the problem of deciding whether a tuple $\langle \Phi, \alpha \rangle$ is an argument (i.e. the support entails the claim, it is minimal for this, and it is consistent) being Σ_2^p complete [23]. We can also regard $\vdash_f$ as being co-NP complete for entailment and NP complete for satisfiability since generating the contrapostives for each defeasible rule can equivalently be captured by treating each rule as a clause (i.e. a disjuntion of literals) and with the proof rule being disjunctive syllogism instead of modus ponens. Hence, we can formalize the $\vdash_f$ decision problems as decision problems of Boolean satisfiability and its complement. In contrast, the $\vdash_d$ and $\vdash_e$ consequence relations are much more efficient for entailment and consistency checking (where $\Delta \not\vdash_x \perp$ is an abbreviation for $\Delta \not\vdash_x \alpha$ and $\Delta \not\vdash_x \neg\alpha$). For these, we can define a polynomial algorithm (adapting an algorithm by Mahler [19]) to decide whether $\Delta \vdash_x \alpha$ holds and whether $\Delta \not\vdash_x \perp$ holds.

In this section, we have considered how we can compare base logics used in argumentation. We see that the key properties of cut and monotonicity that have been proposed as being essential properties for a logic, together with the restricted form of reflexivity (called literal reflexivity), hold for the base logics for a number of key proposals. We also see that a number of key proposals are essentially equivalent in terms of the base logic, and that differences between the proposals can be identified in terms of these properties. We do not suggest that any particular proposal is superior to others because of its properties. Rather, different applications call for different base logics. See for instance the discussion of when contraposition is desirable [8] and undesirable [7].

4. Framework for combined base logics

An approach to defining a base logic is to compose it from two other logics. Before we define this idea, we consider an example taken from a proposal for ontology-based argumentation with the base logic $\vdash_\Gamma$ as follows [30]. The essential idea is that a set of defeasible rules can be used with an ontology so that the ontology contains the information that is certain and the defeasible rules contain the information that is uncertain. In the prototype system presented in [30], a specialized description logic software was harnessed for the ontology. See also [22] for a similar proposal.

Definition 7. *Let Δ be a set of defeasible rules of the form $\beta_1 \wedge \ldots \beta_j \rightarrow \beta_{j+1}$ and let Γ be an ontology in classical logic or description logic.*

$\Delta \vdash_\Gamma \psi$ *iff* *there is a sequence of literals* $\alpha_1, \ldots, \alpha_n$
such that ψ *is* α_n *and for each* $\alpha_i \in \{\alpha_1, \ldots \alpha_n\}$
either $\Gamma \cup \{\alpha_1, \ldots, \alpha_{i-1}\} \vdash_c \alpha_i$
or there is a $\beta_1 \wedge \ldots \wedge \beta_j \rightarrow \alpha_i \in \Delta$
such that $\{\beta_1, \ldots, \beta_j\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\}$

Example 9. *$\{a \to b, c \to \neg d\} \vdash_{\{a,b \to c\}} \neg d$ because of the sequence $a, b, c, \neg d$.*

An alternative to this definition would be to only allow for the ontology to be called for literal inferences, and no inferences from the defeasible reasoning could be passed back to the ontology for further inferences (i.e. we have $\Gamma \vdash_c \alpha_i$ instead of $\Gamma \cup \{\alpha_1, \ldots, \alpha_{i-1}\} \vdash_c \alpha_i$). This is a more cautious form of reasoning.

Definition 8. *Let Δ be a set of defeasible rules of the form $\beta_1 \wedge \ldots \beta_j \to \beta_{j+1}$ and let Γ is an ontology in classical/description logic.*

$$\begin{array}{ll} \Delta \vdash'_\Gamma \psi \textit{ iff} & \textit{there is a sequence of literals } \alpha_1, \ldots, \alpha_n \\ & \textit{such that } \psi \textit{ is } \alpha_n \textit{ and for each } \alpha_i \in \{\alpha_1, \ldots \alpha_n\} \\ & \qquad \textit{either } \Gamma \vdash_c \alpha_i \\ & \qquad \textit{or there is a } \beta_1 \wedge \ldots \wedge \beta_j \to \alpha_i \in \Delta \\ & \qquad\qquad \textit{such that } \{\beta_1, \ldots, \beta_j\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\} \end{array}$$

Example 10. *Continuing Example 9, for $\Delta = \{a \to b, c \to \neg d\}$, and $\Gamma = \{a, b \to c\}$, we get $\Delta \vdash_\Gamma b$, but $\Delta \not\vdash_\Gamma c$, and $\Delta \not\vdash_\Gamma \neg d$,*

Now we generalize Definition 7 into a form of combined base logic, in what we call a bilogic, as follows.

Definition 9. *Let $\vdash_x$ be the consequence relation for a logic x, and let $\vdash_y$ be the consequence relation for a logic y. Also let $\Delta \subseteq \mathcal{L}_x$ be a knowledgebase in the language x, and let $\Gamma \subseteq \mathcal{L}_y$ be a knowledgebase in the language y. The consequence relation for the* **bidirectional bilogic** *$\vdash_{x \oplus y}$, is defined as follows.*

$$\begin{array}{ll} (\Delta, \Gamma) \vdash_{x \oplus y} \alpha \textit{ iff} & \textit{either } \Delta \cup \{\beta_1, \ldots, \beta_n\} \vdash_x \alpha \textit{ or } \Gamma \cup \{\beta_1, \ldots, \beta_n\} \vdash_y \alpha \\ & \textit{where } (\Delta, \Gamma) \vdash_{x \oplus y} \beta_1 \textit{ and } \ldots \textit{ and } (\Delta, \Gamma) \vdash_{x \oplus y} \beta_n \end{array}$$

Using the notion of the bidirectional bilogic, we see in the following proposition that we can define the consequence relation $\vdash_\Gamma$ (i.e. Definition 7) equivalently just using the $\vdash_d$ and $\vdash_c$ consequence relations.

Proposition 5. *For a set of defeasible rules and literals Δ and a set of formulae Γ, $\Delta \vdash_\Gamma \psi$ iff $(\Delta, \Gamma) \vdash_{c \oplus d} \psi$.*

We can also see that other base logics can be captured as bidirectional bilogics, as illustrated next, and this may help us better understand existing definitions.

Proposition 6. *If Δ is a set of defeasible rules and literals, and Γ is a set of strict rules and literals, then $(\Delta, \Gamma) \vdash_{d \oplus d} \psi$ iff $\Delta \cup \Gamma \vdash_d \psi$.*

Now, we consider an alternative notion of bilogic, generalizing Definition 8, that lets one of the constituent logics be used as a service for providing formulae without conditional reasoning.

Definition 10. *Let $\vdash_x$ be the consequence relation for a logic x, and let $\vdash_y$ be the consequence relation for a logic y. Also let $\Delta \subseteq \mathcal{L}_x$ be a knowledgebase in the language x, and let $\Gamma \subseteq \mathcal{L}_y$ be a knowledgebase in the language y. The consequence relation for the* **unidirectional bilogic** *$\vdash_{x \ominus y}$, is defined as follows.*

$$(\Delta, \Gamma) \vdash_{x \ominus y} \alpha \textit{ iff } \textit{ either } \Delta \vdash_x \alpha \textit{ or } \Gamma \cup \{\beta_1, \ldots, \beta_n\} \vdash_y \alpha$$
$$\textit{where } (\Delta, \Gamma) \vdash_{x \ominus y} \beta_1 \textit{ and } \ldots \textit{ and } (\Delta, \Gamma) \vdash_{x \ominus y} \beta_n$$

Using unidirectional bilogic, we can define the consequence relation $\vdash'_\Gamma$ equivalently just using the $\vdash_d$ and $\vdash_c$ consequence relations.

Proposition 7. *For a set of defeasible rules and literals Δ and a set of formulae Γ, $\Delta \vdash'_\Gamma \psi$ iff $(\Delta, \Gamma) \vdash_{c \ominus d} \psi$.*

We can also consider new proposals for combining existing base logics. For instance, if Δ is a set of strict rules and literals and Γ is a set of defeasible rules and literals, then $(\Delta, \Gamma) \vdash_{d \ominus d}$ is a cautious defeasible logic (as opposed to the $\vdash_{d \oplus d}$ considered in Proposition 6) that is cautious with its use of strict rules (i.e. those in Δ).

Example 11. *Let $\Delta = \{a, b, c \rightarrow e, d \rightarrow \neg e\}$ be a set of strict rules, and let $\Gamma = \{a \rightarrow c, b \rightarrow d\}$ be a set of defeasible rules. Using $\vdash_d$, with $\Delta \cup \Gamma$, we get e and $\neg e$ as inferences (i.e. $\Delta \cup \Gamma \vdash_d e$ and $\Delta \cup \Gamma \vdash_d \neg e$), which may be regarded as unacceptable, since a contradiction follows from the strict rules). As an alternative, we can use $\vdash_{d \ominus d}$, and we do not get a contradiction since $(\Delta, \Gamma) \not\vdash_{d \ominus d} e$ and $(\Delta, \Gamma) \not\vdash_{d \ominus d} \neg e$.*

In general, a unidirectional bilogic is more cautious than its bidirectional bilogic counterpart, and hence, it gives fewer inferences. Therefore, the bidirectional bilogic is stronger for a given choice of base logics x and y.

Proposition 8. *For any base logics x and y, if $(\Delta, \Gamma) \vdash_{x \ominus y} \alpha$, then $(\Delta, \Gamma) \vdash_{x \oplus y} \alpha$.*

Considering existing base logics as bilogics allows us to decompose existing, perhaps complex, definitions and consider them in terms of the simpler constituent logics. Furthermore, combining base logics in the form of bilogics gives us the possibility for designing and implementing new base logics more quickly. It also raises opportunities for using existing technology (e.g. description logic reasoners, defeasible logic reasoners, logic programming systems, database systems, etc) for implementing base logics, and then combining them as bilogics to give systems appropriate for applications.

5. Impact of choice of base logic

The choice of base logic has a significant impact on the arguments generated by an argument system. For instance, if we use $\vdash_c$ as our base logic, then $\langle\{a \rightarrow b, \neg a \rightarrow b\}, b\rangle$ is an argument, whereas if we use $\vdash_d$ as our base logic, then $\langle\{a \rightarrow b, \neg a \rightarrow b\}, b\rangle$ is not an argument. There are many examples where it is debatable whether an inference is intuitive or not, and it seems that whether to choose a logic that permits or prohibits certain inferences depends on the application.

So far we have focused our discussion on propositional logics as base logics. But, there are first-order logics that we can use as base logics [6]. By choosing a first-order logic, we get further choices for defining arguments. Consider the knowledgebase $\Delta = \{p(a), \forall x.(p(x) \rightarrow q(x)\}$. We can let $\langle\{p(a), \forall x.(p(x) \rightarrow q(x)\}, q(a)\rangle$ be an argument since the support is a minimal consistent set of formulae that entails the claim. However, we may also want to let $\langle\{p(a), p(a) \rightarrow q(a)\}, q(a)\rangle$ be an argument since we may regard forming a ground version of the premises as being an acceptable step in forming the argument [21]. In other words, if Δ is a knowledgebase, and $\mathsf{Ground}(\Delta)$ is formed from Δ by universal specialization (i.e. grounding of universally quantified formulae), then we may allow $\langle \Phi, \alpha \rangle$ as an argument when Φ is a minimal consistent subset of $\mathsf{Ground}(\Delta)$. This definition seems intuitive. Furthermore, it allows for arguments to be formed for a claim when it is not possible to do so from the original knowledgebase as illustrated in the next example.

Example 12. *Consider* $\Phi = \{\forall x.p(x) \rightarrow q(x), p(a) \land p(b) \land p(c) \land \neg q(b)\}$*. Since* $\Phi \vdash_c \bot$*,* $\langle \Phi, q(a) \rangle$ *is not an argument. However, there is a* $\Psi \subseteq \mathsf{Ground}(\Phi)$ *such that* $\langle \Psi, q(a) \rangle$ *is an argument, namely* $\Psi = \{p(a) \rightarrow q(a), p(a) \land p(b) \land p(c) \land \neg q(b)\}$*.*

Richer logics also lead to more possibilities for counterarguments. For example, using defeasible logic as a base logic, a counterargument $\langle \Phi, \alpha \rangle$ for an argument $\langle \Psi, \beta \rangle$ is often defined as being such that α is the negation of a literal occurring in the derivation of β from Ψ. In other words, there is a γ such that $\Psi \vdash_d \gamma$ and γ is the complement of α (for instance, $\langle \{p, p \rightarrow q, q \rightarrow r\}, r \rangle$ is an argument and $\langle \{s, s \rightarrow \neg q, \}, \neg q \rangle$ is a counterargument to it). Now, if we consider a richer logic as a base logic, such as classical logic, then we see we have more counterarguments (as illustrated in the following example).

Example 13. *Consider* $\Delta = \{a, b, a \rightarrow c, b \rightarrow d, \neg a \lor \neg b\}$*. used to generate the following arguments using* $\vdash_c$ *as the base logic.*

$$A_1 = \langle \{a, b, a \rightarrow c, b \rightarrow d\}, c \land d \rangle \qquad A_2 = \langle \{a, \neg a \lor \neg b\}, \neg b \rangle$$
$$A_3 = \langle \{a, \neg a \lor \neg b\}, \neg a \rangle \qquad A_4 = \langle \{\neg a \lor \neg b\}, \neg a \lor \neg b \rangle$$

Here, we see that A_2 *and* A_3 *are counterarguments to* A_1 *as discussed above. However, we see that the claim of* A_4 *also contradicts some of the support of* A_1*. It does not contradict an individual literal, but rather contradicts a conjunction of literals. It is a weaker counterargument than* A_2 *and* A_3 *in the sense it has a subset of the support and the claim is implied by the claim of each of* A_2 *and* A_3*.*

In [5], the nature of counterarguments in a rich logic such as classical logic was explored, and the proposal made that only the maximally conservative counterarguments (the arguments with the weakest claim necessary for contradicting the argument) need to be considered since they subsume the other counterarguments. The value of maximally conservative counterarguments can even be seen with a language of defeasible rules as illustrated next.

Example 14. *Consider* $\Delta = \{a, b, a \land b \rightarrow c, a \rightarrow \neg b\}$*. used to generate the following arguments using* $\vdash_d$ *as the base logic. Here, we see that* A_2 *is a counterargument to* A_1 *as discussed above. However, we may have preferred to have* $\langle \{a \rightarrow \neg b\}, a \rightarrow \neg b \rangle$ *as the counterargument since it is based on fewer premises.*

$$A_1 = \langle \{a, b, a \land b \rightarrow c\}, c \rangle \qquad A_2 = \langle \{a, a \rightarrow \neg b\}, \neg b \rangle$$

When dealing with richer logics, the need to avoid unnecessary counterarguments is important. Richer logics can create many more inferences, and therefore they can create many more counterarguments. Often, it seems there is much redundancy, and so selecting a subset of counterarguments can render the use of argumentation more manageable by eliminating potentially many redundant counterarguments.

One issue that we have conflated so far in this paper is the dichotomy identified between assumption-based and derivation-based approaches to the definition of arguments. In the former, the support of an argument is a set of premises that proves the claim (as we have considered in this paper), and in the later, the support of an argument is a proof resulting in the claim. For defining individual arguments, the assumption-based approach seems sufficient since the proof can be generated from the assumptions: Given an argument $\langle \Phi, \alpha \rangle$ and a base logic x, there is a function $\mathsf{Proofs}_x(\langle \Phi, \alpha \rangle)$ which returns the set of proofs of α from Φ. The reason that proofs become important is that some approaches to comparing arguments take into account the sequence in which formulae are brought into the proof and the relative "strength" of those premises. For instance, consider the following arguments.

$$A_1 = \langle \{a, a \rightarrow_1 b, b \rightarrow_2 c\}, c \rangle \qquad A_2 = \langle \{a, a \rightarrow_2 \neg b, \neg b \rightarrow_1 \neg c\}, \neg c \rangle$$

So A_1 and A_2 rebut each other, and furthermore each has a subargument that undercuts the other: $A_3 = \langle \{a, a \rightarrow_1 b\}, b \rangle$ undercuts A_2 and $A_4 = \langle \{a, a \rightarrow_2 \neg b\}, \neg b \rangle$ undercuts A_1. Now, suppose $\rightarrow_1$ denotes strict implication, and $\rightarrow_2$ denotes defeasible implication, then we may regard A_3 as sufficient to defeat A_2, in which case A_1 has no counterargument. For more discussion of these issues, see [25].

6. Discussion

There are a number of proposals for logic-based formalizations of argumentation. Often these proposals are quite complex in that they are based on number of defined notions (e.g. definition of an argument, counterargument, preference criteria, acceptability or warrant criteria, etc), and as a result they become difficult to compare. Therefore, attempts to draw out features of logic-based argument systems, in order to find commonalities and differences, is potentially valuable.

In this paper, we have seen how base logics are an important part of a logic-based argument system. By considering the base logic, we can identify properties to compare and contrast the base logics. There are some properties of the consequence relation in common for all the key approaches (including the important properties of cut, monotonicity, and a restricted form of reflexivity), and there are properties of the consequence relation to differentiate base logics (e.g. and, or, left logical equivalence, etc). But, obviously, increasing the strength of the consequence relation can affect the computational complexity of decision problems (such as validity and consistency) for the logic. Also, increasing the strength of the consequence relation, and the language over which it operates, can also lead to an increasing range of options for how to define notions such as argument, counterargument, attack, and defeat.

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