

Presentation of arguments and counterarguments for tentative scientific knowledge

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Abstract. A key goal for a scientist is to find evidence to argue for or against universal statements (in effect first-order formulae) about the world. Building logic-based tools to support this activity could be potentially very useful for scientists to analyse new scientific findings using experimental results and established scientific knowledge. In effect, these logical tools would help scientists to present arguments and counterarguments for tentative scientific knowledge, and to share and discuss these with other scientists. To address this, in this paper, we explain how tentative and established scientific knowledge can be represented in logic, we show how first-order argumentation can be used for analysing scientific knowledge, and we extend our framework for evaluating the degree of conflict arising in scientific knowledge. We also discuss the applicability of recent developments in optimizing the impact and believability of arguments for the intended audience.

1 Introduction

Argumentation is a vital aspect of intelligent behaviour by humans. There are a number of proposals for logic-based formalisations of argumentation (for reviews see [14, 7]). These proposals allow for the representation of arguments for and against some claim, and of attack or undercut relationships between arguments. Whilst many proposals are essentially propositional, there are argumentation formalisms for reasoning with full first-order classical logic [3].

In many professional domains, such as science, it is apparent that there is a need to support first-order argumentation. For example, one of the key goals of scientists is to find evidence to argue for/against universal statements (in effect first-order formulae) about the world. Scientists have much knowledge about their area of expertise, and they have new findings which they want to consider with respect to the established knowledge. With this “knowledgebase”, a scientist will often identify arguments and counterarguments for new proposals for scientific knowledge (tentative scientific knowledge). This presentation of arguments and counterarguments will be for their own analytical purposes, and for other scientists to consider and to counter.

Arguments and counterarguments can be systematically, though not necessarily exhaustively, identified by hand in the free text of individual scientific papers using annotation methodologies [15]. Tools have also been developed to support scientists in analysing free text arguments obtained from a collection of papers, allowing the scientist to flag relationships between evidence from different papers such as “supports”, “contradicts”, etc., using a graphical notation (see for example ClaimMaker [6]).

However, logic-based argumentation has not been adequately harnessed for capturing arguments and counterarguments from scientific knowledge. Potential advantages would include a more precise representation of scientific knowledge that is tolerant of conflicts that inevitably arise, and automated reasoning for incorporation in tools for checking or generating arguments and counterarguments from scientific knowledge.

To address this need, we present a new framework for first-order argumentation with scientific knowledge. However, we are not intending to consider scientific theory formation here. Whilst argumentation theory is being considered for the process of generating new scientific theories [13], we assume that the scientist has generated a theory, and wants to analyse it with the respect to the rest of the relevant scientific knowledge.

In the following, we explain how tentative and established scientific knowledge can be represented in logic, we review our framework for first-order argumentation, we show how first-order argumentation can be used for analysing scientific knowledge, and we extend our framework for evaluating the degree of conflict arising in scientific knowledge.

2 Scientific Knowledge in Logic

Much established scientific knowledge can be represented by statements in first-order logic such as the following universal statements concerning cell biology.

$$\begin{aligned} \forall x.(\text{cell}(x) \rightarrow \text{contains}(x, \text{chromosomes})) \\ \forall x.(\text{chromosomes}(x) \rightarrow \text{contains}(x, \text{dna})) \end{aligned}$$

Here we assume much established scientific knowledge derived from experimental research is represented by a set of formulae each of which is a scientific statement as defined below.

Definition 1. *A scientific statement is a closed formula of first-order logic of the following format where (1) for $0 \leq i \leq m$, μ_i is either a \forall or a \exists quantifier and x_i is a variable; and (2) α and β are conjunctions of literals.*

$$\mu_0 x_0, \dots, \mu_m x_m. (\beta \rightarrow \alpha)$$

So the formulae concerning cell biology are examples of scientific statements. This is a simplistic format for scientific knowledge, but it is useful for capturing a wide range of generalities obtained from experiments or clinical drug trials, and will serve us for developing the role of logic-based argumentation in science.

A key issue in science is that established scientific knowledge is not without inconsistencies. There are competing theories and interpretations in established scientific knowledge. Furthermore, these conflicts lead to further research and hence new discoveries causing the established scientific knowledge to change over time. This is particularly so in biomedical sciences where even the more established knowledge evolves dramatically with much refinement and some established knowledge being rejected after a relatively short time period. This is manifested by the rate at which new editions of substantially revised standard undergraduate textbooks in biomedical sciences are published. It can also be seen in the rapidly evolving practices in healthcare. Some established practices are rejected in the space of a few years in the light of newly established scientific knowledge. As a result, the process of science routinely involves dealing with uncertain and conflicting information.

Scientists who consider their own experimental results in the context of the established scientific knowledge, as reflected in the scientific literature, need to reason with the conflicts arising, and determine the net results that they should put forward into the public domain, hopefully to become established scientific knowledge. But before scientific knowledge can be regarded as established, it is treated with much caution. We therefore regard findings from research as conditional knowledge, called scientific proposals, of the following form.

Definition 2. *A scientific proposal is a closed formula of first-order logic of the following format where (1) for $0 \leq i \leq n$, μ_i is either a \forall or a \exists quantifier and x_i is a variable; (2) γ is a conjunction of literals; and (3) $\mu_0 x_0, \dots, \mu_n x_n. (\beta \rightarrow \alpha)$ is a scientific statement.*

$$\mu_0 x_0, \dots, \mu_n x_n. (\gamma \rightarrow (\beta \rightarrow \alpha))$$

We call γ the **meta-condition** and $\mu_0 x_0, \dots, \mu_n x_n. \beta \rightarrow \alpha$ the **tentative scientific statement** for the scientific proposal. If f_s is a scientific statement, then $\text{Metacondition}(f_s) = \gamma$ and $\text{Proposal}(f_s) = \mu_0 x_0, \dots, \mu_n x_n. (\beta \rightarrow \alpha)$.

Whilst we do not impose any typing on the language for scientific proposals, it should be clear in the following that we intend meta-conditions to use literals that are not available for scientific statements. In general, we see a number of dimensions that we would want to define qualification (meta-conditions) for a scientific proposal. We briefly consider some examples: (1) the investigators who made the scientific contribution need to have the right qualifications and experience; (2) the methods used in the experiments and the interpretation of the experiments need to be appropriate; and (3) the experimental results from which the tentative contribution is based do justify the tentative contribution.

We assume scientific knowledge is represented by a set of formulae of classical logic and that includes scientific statements, scientific proposals, together with subsidiary information such as details on particular experiments and particular techniques. Later we will define an argument as a minimal set of formulae (called the support) that classically implies a formula (called the consequent).

Example 1. The formula below, denoted f_1 , is a scientific proposal concerning drug trial “trial78” on drug “p237” for “reducing blood cholesterol” .

$$f_1 \forall x. (\text{validDrugTrial}(\text{trial78}) \rightarrow \\ (\text{healthy}(x) \wedge \text{under75}(x) \wedge \text{treatment}(x, \text{p237}, 50\text{mg}, \text{daily}) \\ \rightarrow \text{decreaseBloodCholesterol}(x)))$$

The formulae f_2 and f_3 are subsidiary formulae.

$$f_2 \forall x, y. ((\text{numberOfPatients}(x, y) \wedge y > 1000 \wedge \text{trialAtGoodHospital}(x)) \\ \rightarrow \text{validDrugTrial}(x))$$

$$f_3 \text{numberOfPatients}(\text{trial78}, 2479) \wedge 2479 > 1000 \\ \wedge \text{trialAtGoodHospital}(\text{trial78})$$

Assuming $\{f_1, f_2, f_3\}$ we obtain f_4 by implication.

$$f_4 \forall x. (\text{healthy}(x) \wedge \text{under75}(x) \wedge \text{treatment}(x, \text{p237}, 50\text{mg}, \text{daily}) \\ \rightarrow \text{decreaseBloodCholesterol}(x))$$

This can be summarized by the following argument, where $\{f_1, f_2, f_3\}$ is the support for the argument, and f_4 is the consequent.

$$\langle \{f_1, f_2, f_3\}, f_4 \rangle$$

We now turn to the kinds of counterarguments for arguments. We shall focus on undercuts. An undercut A_j for an argument A_i is an argument with a consequent that negates the support for A_i . By recursion, undercuts may be subject to undercuts. We formalize this in the next section, and then provide a framework for scientific argumentation.

3 First-order Argumentation

In this section, we review a recent proposal for argumentation with first-order classical logic [3]. For a language, the set of formulae \mathcal{L} that can be formed is given by the usual inductive definitions for classical logic. Deduction in classical propositional logic is denoted by the symbol \vdash and deductive closure by Cn so that $\text{Cn}(\Phi) = \{\alpha \mid \Phi \vdash \alpha\}$.

For the following definitions, we first assume a knowledgebase Δ (a finite set of formulae) and use this Δ throughout. We further assume that every subset of Δ is given an enumeration $\langle \alpha_1, \dots, \alpha_n \rangle$ of its elements, which we call its canonical enumeration. This really is not a demanding constraint: In particular, the constraint is satisfied whenever we impose an arbitrary total ordering over Δ . Importantly, the order has no meaning and is not meant to represent any respective importance of formulae in Δ . It is only a convenient way to indicate the order in which we assume the formulae in any subset of Δ are conjoined to make a formula logically equivalent to that subset.

The paradigm for the approach is a large repository of information, represented by Δ , from which arguments can be constructed for and against arbitrary claims. Apart from information being understood as declarative statements, there is no a priori restriction on the contents, and the pieces of information in the repository can be as complex as possible. Therefore, Δ is not expected to be consistent. It need even not be the case that every single formula in Δ is consistent.

The framework adopts a very common intuitive notion of an argument. Essentially, an argument is a set of relevant formulae that can be used to classically prove some claim, together with that claim. Each claim is represented by a formula.

Definition 3. An **argument** is a pair $\langle \Phi, \alpha \rangle$ such that: (1) $\Phi \not\vdash \perp$; (2) $\Phi \vdash \alpha$; and (3) there is no $\Phi' \subset \Phi$ such that $\Phi' \vdash \alpha$. We say that $\langle \Phi, \alpha \rangle$ is an argument for α . We call α the consequent of the argument and Φ the support of the argument (we also say that Φ is a support for α). For an argument $\langle \Phi, \alpha \rangle$, $\text{Support}(\langle \Phi, \alpha \rangle) = \Phi$, and $\text{Consequent}(\langle \Phi, \alpha \rangle) = \alpha$.

Example 2. For $\Delta = \{\forall x.(p(x) \rightarrow q(x)), p(a), \neg\forall x.p(x), \neg\exists x.(p(x) \rightarrow q(x))\}$ some arguments include

$$\begin{aligned} & \langle \{p(a), \forall x.(p(x) \rightarrow q(x))\}, q(a) \rangle \\ & \langle \{\neg\forall x.p(x)\}, \neg\forall x.p(x) \rangle \\ & \langle \{\neg\exists x.(p(x) \rightarrow q(x))\}, \forall x.(p(x) \wedge \neg q(x)) \rangle \end{aligned}$$

Arguments are not independent. In a sense, some encompass others (possibly up to some form of equivalence). To clarify this requires a few definitions as follows.

Definition 4. An argument $\langle \Phi, \alpha \rangle$ is **more conservative** than an argument $\langle \Psi, \beta \rangle$ iff $\Phi \subseteq \Psi$ and $\beta \vdash \alpha$.

Example 3. $\langle \{p(a), \forall x.(p(x) \rightarrow q(x) \vee r(x))\}, q(a) \vee r(a) \rangle$ is more conservative than $\langle \{p(a), \forall x.(p(x) \rightarrow q(x) \vee r(x)), \neg\exists x.r(x)\}, q(a) \rangle$.

Some arguments directly oppose the support of others, which amounts to the notion of an undercut.

Definition 5. An **undercut** for an argument $\langle \Phi, \alpha \rangle$ is an argument $\langle \Psi, \neg(\phi_1 \wedge \dots \wedge \phi_n) \rangle$ where $\{\phi_1, \dots, \phi_n\} \subseteq \Phi$.

Example 4.

$$\begin{aligned} & \langle \{\forall x.p(x)\}, p(a) \rangle \text{ is undercut by } \langle \{\neg\exists x.p(x)\}, \neg\forall x.p(x) \rangle \\ & \langle \{\forall x.p(x)\}, p(a) \rangle \text{ is undercut by } \langle \{\exists x.\neg p(x)\}, \neg\forall x.p(x) \rangle \\ & \langle \{\forall x.p(x)\}, p(a) \rangle \text{ is undercut by } \langle \{\neg p(a)\}, \neg\forall x.p(x) \rangle \\ & \langle \{\forall x.p(x)\}, p(a) \rangle \text{ is undercut by } \langle \{\neg p(b)\}, \neg\forall x.p(x) \rangle \end{aligned}$$

Example 5. Let $\Delta = \{p(a), p(a) \rightarrow q(a), r(a), r(a) \rightarrow \neg p(a)\}$. Then,

$$\langle \{r(a), r(a) \rightarrow \neg p(a)\}, \neg(p(a) \wedge (p(a) \rightarrow q(a))) \rangle$$

is an undercut for

$$\langle \{p(a), p(a) \rightarrow q(a)\}, q(a) \rangle$$

A less conservative undercut for it is

$$\langle \{r(a), r(a) \rightarrow \neg p(a)\}, \neg p(a) \rangle$$

Definition 6. $\langle \Psi, \beta \rangle$ is a **maximally conservative undercut** of $\langle \Phi, \alpha \rangle$ iff $\langle \Psi, \beta \rangle$ is an undercut of $\langle \Phi, \alpha \rangle$ such that no undercuts of $\langle \Phi, \alpha \rangle$ are strictly more conservative than $\langle \Psi, \beta \rangle$ (that is, for all undercuts $\langle \Psi', \beta' \rangle$ of $\langle \Phi, \alpha \rangle$, if $\Psi' \subseteq \Psi$ and $\beta \vdash \beta'$ then $\Psi \subseteq \Psi'$ and $\beta' \vdash \beta$).

The value of the following definition of canonical undercut is that we only need to take the canonical undercuts into account. This means we can justifiably ignore the potentially very large number of non-canonical undercuts.

Definition 7. An argument $\langle \Psi, \neg(\phi_1 \wedge \dots \wedge \phi_n) \rangle$ is a **canonical undercut** for $\langle \Phi, \alpha \rangle$ iff it is a maximally conservative undercut for $\langle \Phi, \alpha \rangle$ and $\langle \phi_1, \dots, \phi_n \rangle$ is the canonical enumeration of Φ .

Proposition 1. Given two different canonical undercuts for the same argument, none is more conservative than the other.

Proposition 2. Any two different canonical undercuts for the same argument have distinct supports whereas they do have the same consequent.

An argument tree describes the various ways an argument can be challenged, as well as how the counterarguments to the initial argument can themselves be challenged, and so on recursively.

Definition 8. An **argument tree** for α is a tree where the nodes are arguments such that

1. The root is an argument for α .
2. For no node $\langle \Phi, \beta \rangle$ with ancestor nodes $\langle \Phi_1, \beta_1 \rangle, \dots, \langle \Phi_n, \beta_n \rangle$ is Φ a subset of $\Phi_1 \cup \dots \cup \Phi_n$.
3. Each child of a node A is an undercut for A that obeys 2.

A **canonical argument tree** is an argument tree where each undercut is a canonical undercut. A **complete argument tree** is a canonical argument tree for each node A , s.t. if A' is a canonical undercut for A , then A' is a child of A . For a tree T , $\text{Nodes}(T)$ is the set of nodes in T and $\text{Depth}(T)$ is the number of arcs on the longest branch of T .

The second condition in Definition 8 ensures that each argument on a branch has to introduce at least one formula in its support that has not already been used by ancestor arguments. As a notational convenience, in examples of argument trees, the \diamond symbol is used to denote the consequent of an argument when that argument is a canonical undercut.

Example 6. Consider the following knowledgebase.

$$\Delta = \{\forall x.(p(x) \vee q(x)), \forall x.(p(x) \rightarrow r(x)), \forall x.\neg r(x), \forall x.\neg q(x), \forall x.(s(x) \leftrightarrow q(x))\}$$

Below is an argument tree from Δ for the consequent $\forall x.(p(x) \vee \neg s(x))$.

$$\begin{array}{c} \langle \{\forall x.(p(x) \vee q(x)), \forall x.\neg q(x)\}, \forall x.(p(x) \vee \neg s(x)) \rangle \\ \uparrow \\ \langle \{\forall x.(p(x) \rightarrow r(x)), \forall x.\neg r(x)\}, \diamond \rangle \end{array}$$

Example 7. Let f_5 and f_6 be the following formulae.

$$f_5 \quad \forall x, y. (\text{irregularitiesDuringTrial}(X) \rightarrow \neg \text{validDrugTrial}(x))$$

$$f_6 \quad \text{irregularitiesDuringTrial}(\text{trial78})$$

Hence we have the argument $\langle \{f_5, f_6\}, \diamond \rangle$ which is an undercut for the argument $\langle \{f_1, f_2, f_3\}, f_4 \rangle$ given in Example 1. This is summarized as follows.

$$\begin{array}{c} \langle \{f_1, f_2, f_3\}, f_4 \rangle \\ \uparrow \\ \langle \{f_5, f_6\}, \diamond \rangle \end{array}$$

A complete argument tree is an efficient representation of all the important arguments and counterarguments.

Proposition 3. *Let $\alpha \in \mathcal{L}$. If Δ is finite, there is a finite number of argument trees with the root being an argument with consequent α that can be formed from Δ , and each of these trees has finite branching and a finite depth.*

4 Scientific Argumentation

After delineating some conflicting (i.e. inconsistent) scientific knowledge, we assume a scientist wants to see if an argument of interest has undercuts, and by recursion, undercuts to undercuts. So when a scientist considers a scientific proposal, undercuts to an argument using the scientific proposal indicate reasons to doubt the proposal, and undercuts to an undercut indicate reasons to doubt that undercut. Argument trees therefore provide a systematic means for representing caution in scientific knowledge. We focus on three types of undercut that arise with a clear aetiology.

One way to reflect caution in a scientific proposal is to consider the meta-conditions for the scientific proposal. This is an important aspect of scientific reasoning, and it may involve considering the reliability of sources, and the believability, plausibility, or quality of information used. The quality of putative scientific knowledge derived from experiments may be questionable in a number of ways based on the quality of the experimental environment, the quality of the starting materials, the nature of any subjects being studied, and the nature of the scientific methodology. The quality may also be questionable in terms of the interpretation of the scientific results, so that cause-effect relationships are claimed for results that should be interpreted as coincidence. Alternatively, incorrect statistical techniques may have been used. So a scientific proposal can be qualified in a number of ways, and arguments against these qualifications can then be represented.

Definition 9. Let A_j be an undercut for A_i . A_j is a **meta-condition violation** of A_i iff there is a scientific proposal $f_i \in \text{Support}(A_i)$ and there is a ground version γ' of $\text{Metacondition}(f_i)$ such that $\text{Support}(A_i) \setminus \{f_i\} \vdash \gamma'$ and $\text{Support}(A_j) \cup \{\gamma'\}$ is inconsistent.

So an argument is subject to a meta-condition violation when the support of the argument includes a scientific proposal and there is an undercut that negates the meta-condition of the scientific proposal. An illustration of a meta-condition violation is given in Example 7.

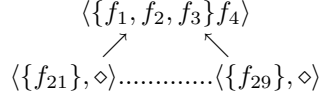
A second way to reflect caution in a scientific proposal is to consider exceptions. As formalized next, an argument is subject to an exception violation when the support of the argument includes a scientific proposal and there is an undercut which has a support that negates the tentative scientific statement for the scientific proposal. As a result, since the consequent is a tentative scientific statement, the ground atoms satisfy the antecedent but negate the consequent of the tentative scientific statement as illustrated in Example 8.

Definition 10. Let A_j be an undercut for A_i . A_j is an **exception violation** of A_i iff $\text{Support}(A_j)$ contains only ground formulae and there is a scientific proposal $f_i \in \text{Support}(A_i)$ such that $\text{Support}(A_j) \cup \{\text{Proposal}(f_i)\}$ is inconsistent.

Example 8. Consider f_4 given in Example 1. Suppose we have nine exceptions f_{21}, \dots, f_{29} as follows.

$$\begin{aligned}
 f_{21} & (\text{healthy}(\text{patient33}) \wedge \text{under75}(\text{patient33}) \\
 & \quad \wedge \text{treatment}(\text{patient33}, \text{p237}, \text{50mg}, \text{daily}) \\
 & \quad \wedge \neg \text{decreaseBloodCholesterol}(\text{patient33})) \\
 & \quad \vdots \\
 f_{29} & (\text{healthy}(\text{patient89}) \wedge \text{under75}(\text{patient89}) \\
 & \quad \wedge \text{treatment}(\text{patient89}, \text{p237}, \text{50mg}, \text{daily}) \\
 & \quad \wedge \neg \text{decreaseBloodCholesterol}(\text{patient89}))
 \end{aligned}$$

Then we have the argument tree



A problem with this type of violation is that there may be a significant number of exceptions of the same form, and so we may wish to abbreviate the information we have about these exceptions. To support this, a useful conservative extension of the first-order language is qualified statements. These allow us to represent a specific set of examples for which a general statement holds.

Definition 11. A **qualified statement** is a formula of the following form $\forall x \in \{t_1, \dots, t_n\}.\alpha$, where $\{t_1, \dots, t_n\}$ is a set of ground terms, and α is a formula.

Definition 12. We extend the \vdash consequence relation with the following holding for all formulae α where $\alpha[x/t_i]$ denotes the grounding of all free occurrences of the x variable by the ground term t_i .

$$\vdash (\forall x \in \{t_1, \dots, t_n\}.\alpha) \leftrightarrow (\alpha[x/t_1] \wedge \dots \wedge \alpha[x/t_n])$$

Example 9. Let $\Delta = \{\forall x \in \{a, b, c\}.\{\forall y \in \{c, d\}.\{p(x, y) \rightarrow q(x, a)\}\}\}$. Hence, $\Delta \vdash p(b, c) \rightarrow q(b, a)$. If Δ' comprises the formulae below, then $\text{Cn}(\Delta) = \text{Cn}(\Delta')$.

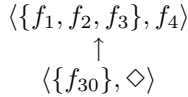
$$\begin{array}{ll} p(a, c) \rightarrow q(a, a) & p(b, c) \rightarrow q(b, a) \\ p(c, c) \rightarrow q(c, a) & p(a, d) \rightarrow q(a, a) \\ p(b, d) \rightarrow q(b, a) & p(c, d) \rightarrow q(c, a) \end{array}$$

Qualified statements are useful shorthand for a set of statements. Also if $\Delta \vdash \forall x \in X.\alpha$ and $X' \subseteq X$, then $\Delta \vdash \forall x \in X'.\alpha$.

Example 10. Let us denote f_{30} by the following formula.

$$\begin{array}{l} \forall x \in \{\text{patient33}, \dots, \text{patient89}\}. \\ (\text{healthy}(x) \wedge \text{under75}(x) \\ \wedge \text{treatment}(x, \text{p237}, 50\text{mg}, \text{daily}) \\ \wedge \neg \text{decreaseBloodCholesterol}(x)) \end{array}$$

Using f_1 , f_2 , and f_3 , from Example 1, with f_{30} , the following is an argument tree for f_4 .



A third way of expressing caution in a scientific proposal is to identify conflicts with the established scientific knowledge used. As discussed earlier, there are numerous inconsistencies of various kinds in the established literature, and so even though a scientific statement may be assumed to be part of the established knowledge, it does not necessarily mean it is absolutely correct, and

such conflicts may need to be highlighted when they are relevant to a scientific proposal under consideration. As formalized next, an argument is subject to presupposition violation when there is a counterargument that negates a scientific statement used in the support of the argument.

Definition 13. *Let A_j be an undercut for A_i . A_j is a **presupposition violation** of A_i iff there is a scientific statement $f_i \in \text{Support}(A_i)$ such that $\text{Support}(A_j) \cup \{f_i\}$ is inconsistent.*

Example 11. For the formula below, h_1 is a scientific proposal, h_2 is an established piece of scientific knowledge, and h_3 and h_4 are subsidiary formulae.

$$h_1 \forall x.(\text{validDrugTrial}(\text{trial990}) \rightarrow \\ (\text{decreaseChronicAnxiety}(x) \rightarrow \text{increasedLifeExpectancy}(x)))$$

$$h_2 \forall x.(\text{treatment}(x, \text{daloxopin}, 4\text{mg}, \text{daily}) \rightarrow \text{decreaseChronicAnxiety}(x))$$

$$h_3 \text{validDrugTrial}(\text{trial990})$$

$$h_4 \forall x \in \{\text{patient1}, \dots, \text{patient241}\}.(\text{treatment}(x, \text{daloxopin}, 4\text{mg}, \text{daily}))$$

Assuming $\{h_1, h_2, h_3, h_4\}$ we obtain h_5 by implication.

$$h_5 \forall x.(\text{decreaseChronicAnxiety}(x) \rightarrow \text{increasedLifeExpectancy}(x))$$

In addition, assume we have the formula h_6 that says that it is not the case, for any patient, any dose, or any frequency of treatment, that daloxopin decreases chronic anxiety. This formula is therefore negating some established scientific knowledge used above.

$$h_6 \forall x, y, z. \neg(\text{treatment}(x, \text{daloxopin}, y, z) \rightarrow \text{decreaseChronicAnxiety}(x))$$

From this, we get the following argument tree that reflects the presupposition violation.

$$\begin{array}{c} \langle \{h_1, h_2, h_3, h_4\}, h_5 \rangle \\ \uparrow \\ \langle \{h_6\}, \diamond \rangle \end{array}$$

As stated earlier, undercuts can also be undercut by recursion. These undercuts may also include circumstantial undercuts which are undercuts based on special circumstances arising when undertaking the trial or experiment or when drawing up the scientific proposal. For example, an exception may be undercut because it arises from a possibly faulty observation or an incorrect experimental set-up.

5 Degree of Undercut

An argument conflicts with each of its undercuts, by the very definition of an undercut. Now, some may conflict more than others, and some may conflict a little while others conflict a lot. To illustrate, consider the following trees.

$$\begin{array}{ccc}
T_1 & T_2 & T_3 \\
\langle \{P(a)\}, P(a) \rangle & \langle \{\forall x.P(x)\}, P(a) \rangle & \langle \{\forall x.P(x)\}, P(a) \rangle \\
\uparrow & \uparrow & \uparrow \\
\langle \{\neg P(a)\}, \diamond \rangle & \langle \{\neg P(a)\}, \diamond \rangle & \langle \{\neg P(b)\}, \diamond \rangle
\end{array}$$

All of T_1, \dots, T_3 have $P(a)$ as the conclusion. In T_1 , the support for root is $\{P(a)\}$ and the support for the undercut is $\{\neg P(a)\}$. This can be described as a propositional conflict where $P(a)$ is against $\neg P(a)$. In T_2 , the support for root is $\{\forall x.P(x)\}$ and the support for the undercut is $\{\neg P(a)\}$. This can be described as equivalent to T_1 since the conflict is only with respect to one grounding of x , viz. the grounding by a . In T_3 , the support for the root is $\{\forall x.P(x)\}$ but the support for the undercut is $\{\neg P(b)\}$. This can also be described as equivalent to T_1 since the conflict is only with respect to one grounding of x , viz. the grounding by b .

$$\begin{array}{ccc}
T_4 & T_5 & T_6 \\
\langle \{\forall x.P(x)\}, \forall x.P(x) \rangle & \langle \{\forall x.P(x)\}, \forall x.P(x) \rangle & \langle \{\forall x.P(x)\}, \forall x.P(x) \rangle \\
\uparrow & \uparrow & \uparrow \\
\langle \{\neg P(a)\}, \diamond \rangle & \langle \{\neg \forall x.P(x)\}, \diamond \rangle & \langle \{\forall x.\neg P(x)\}, \diamond \rangle
\end{array}$$

All of T_4, \dots, T_6 have $\forall x.P(x)$ as the conclusion. In T_4 , the support for the root is $\{\forall x.P(x)\}$ and the support for the undercut is $\{\neg P(a)\}$. So this can be described as having the same degree of conflict as T_2 . In T_5 , the support for the root is $\{\forall x.P(x)\}$ and the support for the undercut is $\{\neg \forall x.P(x)\}$. Since $\neg \forall x.P(x)$ is logically equivalent to $\exists x.\neg P(x)$, the conflict only necessarily involves one grounding for x . Hence, this can also be described as having the same degree of conflict as T_2 . In T_6 , the support for the root is $\{\forall x.P(x)\}$ and the support for the undercut is $\{\forall x.\neg P(x)\}$. Here, the conflict is much more substantial since it involves all possible groundings for x .

By these simple examples, we see there is an intuitive difference in the degree of conflict between supports, and hence an intuitive starting point for defining the degree of undercut that an argument has against its parent. This degree of undercut depends on the logical nature of the supports involved. Above we have considered this informally for some examples of monadic literals. In the following, we review a formal conceptualization of this for formulae involving n -predicates and involving logical connectives [3], and then consider how it can be used for analysing scientific arguments. For this, the conflict of an argument with each of its undercuts is reflected by a position in an ordering (possibly a partial one) but not necessarily a numerical value in some interval (i.e., orders of magnitude are not necessarily needed).

Definition 14. A **degree of undercut** is a mapping $\text{Degree} : \Omega \times \Omega \rightarrow O$ where $\langle O, \leq \rangle$ is some poset such that for $A_i = \langle \Phi, \alpha \rangle$ and $A_j = \langle \Psi, \beta \rangle$ in Ω ,

- (1) $\text{Degree}(A_j, A) \leq \text{Degree}(A_i, A)$ for all $A \in \Omega$ if $\Phi \vdash \Psi$
- (2) $\text{Degree}(A_i, A_j)$ is minimal iff $\Phi \cup \Psi \not\vdash \perp$

The last clause in Definition 14 means that $\text{Degree}(A, A')$ is minimal when A and A' are two arguments which do not conflict with each other (so, none is an undercut of the other, as Degree is rather a degree of conflict but it is called degree of undercut here because we are only interested in its value when A' is an undercut of A). Definition 14 allows for many possibilities, leaving you to choose a suitable mapping.

We now introduce labelled argument trees. I.e., we label each arc with the degree of undercut. In the rest of the paper, we assume that O is the interval $[0, 1]$.

Definition 15. A **labelled argument tree** is an argument tree such that if A_j is a child of A_i in the argument tree, then the arc from A_j to A_i is labelled with $\text{Degree}(A_i, A_j)$.

Example 12. A labelled argument tree for $\forall x.\alpha[x]$ is:

$$\begin{array}{ccc} & \langle \{\forall x.\alpha[x]\}, \forall x.\alpha[x] \rangle & \\ & \nearrow 1/n \quad \nwarrow m/n & \\ \langle \{\neg\alpha[a]\}, \diamond \rangle & & \langle \{\neg\alpha[b_1] \wedge \dots \wedge \neg\alpha[b_m]\}, \diamond \rangle \end{array}$$

From now on, n is some reasonable upper bound for the size of the universe of discourse (it is supposed to be finite).

One conceptualization for degree of undercut is based on Herbrand Interpretation. For the rest of the paper, we assume that the non-logical language for Δ is restricted to predicate, variable, and constant symbols, and so function symbols are not used. We also assume that Δ includes at least one constant symbol, and normally, numerous constant symbols. Note, there are other conceptualizations of degree of undercut where we do not restrict ourselves to a finite universe, and can use an unrestricted first-order classical language [4].

Definition 16. Let Π be the set of ground atoms that can be formed from the predicate symbols and constant symbols used in Δ . Π is the **base** for Δ . Each $w \subseteq \Pi$ is an **interpretation** s.t. each atom in w is assigned true and each atom in $\Pi \setminus w$ is assigned false. For a set of formulae X , let $M(X, \Pi)$ be the set of **models** of X that are in $\wp(\Pi)$. So $M(X, \Pi) = \{w \models \wedge X \mid w \in \wp(\Pi)\}$ where \models is classical satisfaction.

Example 13. Let $X = \{q(b) \wedge q(c), \neg r(c), \forall x.p(x), \exists x.(r(x) \wedge q(x))\} \subseteq \Delta$ and so $\Pi = \{p(a), p(b), p(c), q(a), q(b), q(c), r(a), r(b), r(c)\}$. Hence $M(X, \Pi)$ contains

exactly the following models.

$$\begin{aligned} & \{p(a), p(b), p(c), q(a), q(b), q(c), r(a), r(b)\} \\ & \{p(a), p(b), p(c), q(b), q(c), r(a), r(b)\} \\ & \{p(a), p(b), p(c), q(a), q(b), q(c), r(a)\} \\ & \{p(a), p(b), p(c), q(a), q(b), q(c), r(b)\} \\ & \{p(a), p(b), p(c), q(b), q(c), r(b)\} \end{aligned}$$

We now recall the definition for Dalal distance for comparing a pair of models which is the Hamming distance between the two models [8].

Definition 17. Let $w_i, w_j \in \wp(\Pi)$. The **Dalal distance** between w_i and w_j , denoted $\text{Dalal}(w_i, w_j)$, is the difference in the number of atoms assigned true:

$$\text{Dalal}(w_i, w_j) = |w_i - w_j| + |w_j - w_i|$$

To evaluate the conflict between two theories, we take a pair of models, one for each theory, such that the Dalal distance is minimized. The degree of conflict is this distance divided by the maximum possible Dalal distance between a pair of models (i.e. \log_2 of the total number of models in $\wp(\Pi)$ which is $|\Pi|$).

Definition 18. For $X, Y \subseteq \Delta$ s.t. $X \not\vdash \perp$ and $Y \not\vdash \perp$, let $\text{Distances}(X, Y, \Pi)$ be

$$\{\text{Dalal}(w_x, w_y) \mid w_x \in M(X, \Pi) \text{ and } w_y \in M(Y, \Pi)\}$$

The **degree of conflict**, denoted $\text{Conflict}(X, Y, \Pi)$, is:

$$\text{Conflict}(X, Y, \Pi) = \frac{\text{Min}(\text{Distances}(X, Y, \Pi))}{|\Pi|}$$

Example 14. Let $\Pi = \{p(a), p(b), p(c), q(a), q(b), q(c), r(a), r(b), r(c)\}$.

$$\begin{aligned} \text{Conflict}(\{\forall x.p(x)\}, \{\exists x.\neg p(x)\}, \Pi) &= 1/9 \\ \text{Conflict}(\{\forall x.p(x)\}, \{\neg(p(a) \vee p(b))\}, \Pi) &= 2/9 \\ \text{Conflict}(\{\forall x.p(x)\}, \{\forall x.\neg p(x)\}, \Pi) &= 3/9 \end{aligned}$$

For $X, Y \subseteq \Delta$, such that $X \not\vdash \perp$ and $Y \not\vdash \perp$, we can show the following: (1) $0 \leq \text{Conflict}(X, Y, \Pi) \leq 1$; (2) $\text{Conflict}(X, Y, \Pi) = \text{Conflict}(Y, X, \Pi)$; and (3) $\text{Conflict}(X, Y, \Pi) = 0$ iff $X \cup Y \not\vdash \perp$.

Definition 19. Let $A_i = \langle \Phi, \alpha \rangle$ and let $A_j = \langle \Psi, \beta \rangle$ be arguments. The **Dalal-Herbrand degree of undercut** by A_j for A_i , denoted $\text{Degree}_{dh}(A_i, A_j, \Pi)$, is $\text{Conflict}(\Phi, \Psi, \Pi)$.

Clearly, if A_i is an undercut for A_j , then $\text{Degree}_{dh}(A_i, A_j, \Pi) > 0$.

Example 15. Let $A_1 = \langle \{\neg\exists x.p(x)\}, \neg\forall x.p(x) \rangle$, $A_2 = \langle \{\exists x.\neg p(x)\}, \neg\forall x.p(x) \rangle$, $A_3 = \langle \{\neg p(a_1)\}, \neg\forall x.p(x) \rangle$, $A_4 = \langle \{\forall x.p(x)\}, p(a_1) \rangle$, and $\Pi = \{p(a_1), \dots, p(a_n)\}$.

$$\begin{aligned} \text{Degree}_{dh}(A_4, A_1, \Pi) &= n/n \\ \text{Degree}_{dh}(A_4, A_2, \Pi) &= 1/n \\ \text{Degree}_{dh}(A_4, A_3, \Pi) &= 1/n \end{aligned}$$

A scientist can use the degree of undercut to compare arguments and counterarguments. We can regard each argument in a tree as either an **attacking argument** or a **defending argument**. The root is a defending argument. If an argument A_i is a defending argument, then any child A_j of A_i is an attacking argument. If an argument A_j is an attacking argument, then any child A_k of A_j is a defending argument. For a scientific proposal used in the root, a scientist could publish a scientific proposal in the public domain with more confidence, if the undercuts to defending arguments have a low degree of undercut, and the undercuts to attacking arguments have a high degree of undercut.

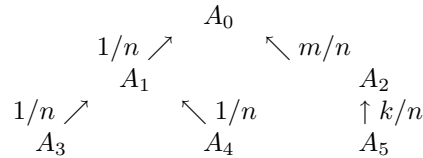
Example 16. Consider the argument tree given in Example 10. Suppose the knowledgebase from which the tree is constructed contains just the formulae f_1, f_2, f_3, f_{30} , together with the following 2479 formulae.

$$\begin{array}{ll}
g_1 & (\text{healthy}(\text{patient1}) \wedge \text{under75}(\text{patient1})) \\
g_2 & (\text{healthy}(\text{patient2}) \wedge \text{under75}(\text{patient2})) \\
g_3 & (\text{healthy}(\text{patient3}) \wedge \text{under75}(\text{patient3})) \\
& \vdots \\
g_{2479} & (\text{healthy}(\text{patient2479}) \wedge \text{under75}(\text{patient2479}))
\end{array}$$

Hence the Dalal-Herbrand degree of undercut by $\langle \{f_{30}\}, \diamond \rangle$, for $\langle \{f_1, f_2, f_3\}, f_4 \rangle$ is $9/2479$.

Labelled argument trees provide extra information that leads to a useful abstraction of the original argument tree.

Example 17. Let $A_0, A_1, A_2, \dots, A_5$ be arguments, and let $k < n$ and $m < n$ hold. For this, the following is a labelled argument tree.



In the above labelled argument tree, if n is significantly greater than 1, then it may be natural to ignore the left subtree rooted at A_1 and to concentrate on the right-most branch of the tree. If m is close to n , then A_2 is an important undercut of A_0 , whereas if it is close to 1, then it may be natural to ignore this branch also.

The tension of an argument tree is the cumulative conflict obtained from all the undercuts in the tree. As tension rises, the more the scientist has to be careful how a new scientific proposal is presented.

Definition 20. Let T be an argument tree, and let A_r be the root node. The degree of tension in T , denoted $\text{Tension}(T)$, is given by the value of $\text{Retension}(A_r)$,

where for any node A_i in the tree, if A_i is a leaf, then $\text{Retension}(A_i) = 0$ otherwise $\text{Retension}(A_i)$ is

$$\sum_{A_j \text{ s.t. } A_j \text{ undercuts } A_i} \text{Retension}(A_j) + \text{Degree}(A_i, A_j, \Pi)$$

Clearly, $\text{Tension}(T) < |\text{Nodes}(T)|$. Furthermore, $|\text{Nodes}(T)| = 1$ if and only if $\text{Tension}(T) = 0$. Tension is maximized when each formula in Δ has to be inconsistent with every other formula, such as $\{\alpha \wedge \beta, \alpha \wedge \neg\beta, \neg\alpha \wedge \beta, \neg\alpha \wedge \neg\beta\}$, so that every argument is an undercut to every other argument.

We conclude this section by sketching another conceptualization of degree of undercut. Here, we assume \mathcal{O} is $\mathbb{N} \cup \{\infty\} \times \mathbb{N} \cup \{\infty\}$ (and so for this paragraph we suspend our general assumption in this section of \mathcal{O} being $[0, 1]$). Informally, for arguments A_i and A_j , the degree of undercut of A_j for A_i is a pair (n, k) where n is the number of situations where the support of A_j is regarded as holding (and thereby justifying the support), and k is the number of situations where the support of A_i is regarded as holding (and thereby justifying the support). Now, consider an argument tree about clinical drug trials, the number of situations where a support holds can be defined in terms of the number of patients involved in the trial. If we consider Example 1, for the argument $\langle \{f_1, f_2, f_3\}, f_4 \rangle$, the support is justified by 2479 patients, and if we consider Example 10, for the argument $\langle \{f_{30}\}, \diamond \rangle$ the support is justified by 9 patients. So the degree of undercut is $(9, 2479)$. For supports that use only established scientific knowledge, we use the value ∞ to denote the understanding that the support uses only established scientific knowledge. So an argument with support containing knowledge from a trial involving a 1000 patients that undercuts an argument that uses only established scientific knowledge, the degree of undercut is $(1000, \infty)$. Similarly, for an argument that uses only established scientific knowledge undercutting an argument with support containing knowledge from a trial involving a 1000 patients, the degree of undercut is $(\infty, 1000)$. Finally, for an argument that uses only established scientific knowledge undercutting an argument that uses only established scientific knowledge, the degree of undercut is (∞, ∞) .

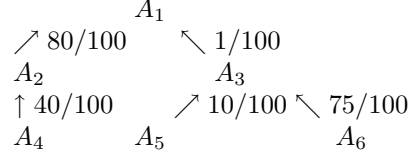
6 Editing Argument Trees

Even for small first-order knowledgebases, the number of arguments generated may be overwhelming for a scientist to be able to study at any one time. To address this problem, we review some proposals for rationalization of argument trees [3, 4] including (1) Pruning arguments that have a degree of undercut that is below a certain threshold; and (2) Merging arguments to create fewer undercuts but without losing vital information. Rationalization is part of a process of editing a set of arguments and counterarguments to allow a scientist to focus on key issues.

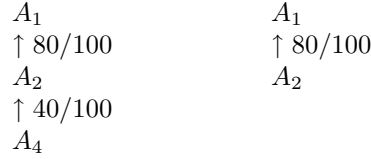
For pruning, we choose a threshold for a minimum degree of undercut. If an undercut has a degree of undercut below the threshold, then the undercut is dropped, together with any offspring of that undercut.

Definition 21. A threshold, denoted τ , is a value in $[0, 1]$ such that if T is an argument tree, $\text{Prune}(T, \tau)$ is the **pruned argument tree** obtained from T by removing every undercut A_j for an argument A_i if $\text{Degree}(A_i, A_j, \Pi) \leq \tau$ and for any undercut removed, all the offspring of that undercut are also removed.

Example 18. Let T be the following labelled argument tree.



Below, the left argument tree is $\text{Prune}(T, 0.3)$ and the right one is $\text{Prune}(T, 0.5)$.



So pruning of argument trees allows us to focus our attention on the most conflicting undercuts.

Proposition 4. For $i \in [0, 1]$, if $T' = \text{Prune}(T, i)$ then $\text{Tension}(T') \leq \text{Tension}(T)$ and $|\text{Nodes}(T')| \leq |\text{Nodes}(T)|$ and $\text{Depth}(T') \leq \text{Depth}(T)$.

Also, $\text{Prune}(T, 0) = T$ and $\text{Prune}(T, 1)$ returns a tree containing just the root of T . For all $i \in [0, 1]$, if T is a canonical argument tree, then $\text{Prune}(T, i)$ is a canonical argument tree. However, if T is a complete argument tree, then $\text{Prune}(T, i)$ is not necessarily a complete argument tree.

For merging, we use the following notion of compression which combines arguments without loss of essential information. Compression merges siblings in order to reduce the number of arguments and to reduce the “redundancy” arising by having numerous similar arguments or logically equivalent arguments, and to make appropriate “simplifications” of the syntax of some arguments.

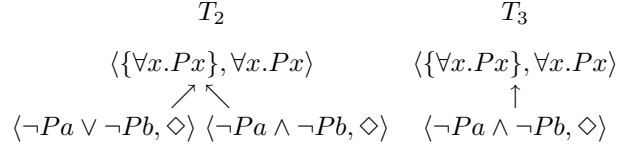
Definition 22. Let T_1 and T_2 be argument trees. T_2 is a **compression** of T_1 iff there is a surjection $G: \text{Nodes}(T_1) \rightarrow \text{Nodes}(T_2)$ such that for all $B \in \text{Nodes}(T_2)$,

$$\text{Cn}(\text{Support}(B)) = \text{Cn}\left(\bigcup_{A \in G^{-1}(B)} \text{Support}(A)\right)$$

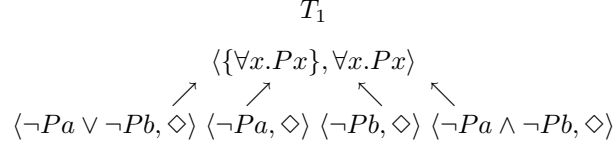
We call G the *compression function*.

The argument tree in Example 10 is a compression of the argument tree in Example 8. Logical simplification of supports of arguments, as illustrated in the example below, may also be useful in some circumstances. Such simplifications may be important in focussing on the main issues, and removal of less relevant concepts.

Example 19. T_3 is a compression of T_2 :



while each of T_2 and T_3 is a compression of T_1 :



Proposition 5. *If T' is a compression of T , then $\text{Tension}(T') \leq \text{Tension}(T)$ and $|\text{Nodes}(T')| \leq |\text{Nodes}(T)|$ and $\text{Depth}(T') = \text{Depth}(T)$.*

Compression is not necessarily unique, and there are limits to compression, for example when an argument tree is a chain, and when all pairs of siblings have supports that are mutually contradictory. If compression is restricted to just replacing ground formulae with qualified formulae, then the tension is constant. Alternatively, we may choose to just use compressions that do not change the tension. For more details on compression, and for alternatives, see [3, 4].

A presentation of arguments and counterarguments can also be edited in order to improve the impact of the argumentation [12], and/or to increase the believability of the argumentation [11], from the perspective of the intended audience of argumentation.

For increasing the impact of argumentation, we have developed an evaluation of arguments in terms of how the arguments resonate with the intended audience of the arguments. For example, if a scientist wants to present results from a research project, the arguments used would depend on what is important to the audience: Arguments based on the potential economic benefits of the research would resonate better with an audience from the business community and from the funding agencies, whereas arguments based on the scientific results would resonate better with an audience of fellow scientists. By analysing the resonance of arguments, we can prune argument trees to raise their impact for an audience.

For increasing the believability of argumentation, we have developed a model-theoretic evaluation of the believability of arguments. This extension assumes that the beliefs of a typical member of the audience for argumentation can be represented by a set of classical formulae (a beliefbase). We compare a beliefbase with each argument to evaluate the empathy (or similarly the antipathy) that an agent has for the argument. On the basis of believability, a scientist may wish to ignore arguments for which the audience has antipathy.

The use of pruning, of rationalization, and of selectivity based on raising impact and optimizing believability, is part of a trend to consider the audience in argumentation, and present constellations of arguments and counterarguments

that are appropriate for the audience. For example, formalising persuasion has a role in modelling legal reasoning [2].

7 Discussion

The primary aim of this paper has been to provide a framework for presenting scientific arguments and counterarguments based on first-order predicate logic. We can view the framework in this paper as a specification for a decision-support system for scientists to evaluate new scientific proposals. To use it, a scientist would be responsible for adding the relevant scientific knowledge together with the scientific proposal of interest. The decision-support system would then construct the labelled argument trees. Scientists are a user community who may be highly amenable to learning and using predicate logic to use this system. Alternatively, we may need to look towards developments in natural language processing for translating free text into logical formulae.

One of the key advantages of undertaking meta-analysis of scientific knowledge using logic-based argumentation is that when we do not have access to all the original data, we need to deal with the arguments that can be constructed from the publically available information. Consider for example comparing clinical trials undertaken at different hospitals where it may be difficult to have access to all the primary data and/or there may be heterogeneity arising from differing protocols or differing usages of language.

Another way of looking at this is that often the results of an experiment can be captured by a conditional probability statement $P(\alpha | \beta)$. This says that the proportion of examples that meet condition β also meet condition α . So a conditional probability statement also captures the proportion of counterexamples which is given by $P(\neg\alpha | \beta)$. However, dealing with conditional probabilities cannot be easily extended to dealing with established scientific knowledge, to dealing with exceptions to exceptions, or to dealing with conflicting information, without recourse to a much more comprehensive knowledge of the total probability distribution. This is often impractical or impossible. Scientists do not normally have access to the full experimental data for established scientific knowledge. They normally only have access to the universal statements as an abstraction. So representing conditional probability statements of the form $P(\alpha | \beta)$ by statements of the form $\beta \rightarrow \alpha$ when the probability value is greater than say 0.9, is an efficient format. We can reason with the logical formulae using argumentation and represent exceptions by counterarguments. Moreover, we can directly represent inconsistencies in the established scientific knowledge.

Scientific knowledge can also be compared with the commonly considered usage of a default (or defeasible) knowledge. It is noteworthy that human practical reasoning relies much more on exploiting default information than on a myriad of individual facts. Default knowledge tends to be less than 100% accurate, and so has exceptions [5]. Nevertheless it is intuitive and advantageous to resort to such defaults and therefore allow the inference of useful conclusions, even if it does entail making some mistakes as not all exceptions to these defaults are

necessarily known. Furthermore, it is often necessary to use default knowledge when we do not have sufficient information to allow us to specify or use universal laws that are always valid. This paper raise an opportunity to revisit the notion of default knowledge, and consider its relevance to scientific knowledge.

The secondary aim of the paper has been to extend logic-based proposals for argumentation with techniques for first-order argumentation. Degree of undercut, labelled argument trees/graphs, and pruning and compressing arguments, could be adapted for other logic-based proposals such as [10, 1, 9].

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