Essays on Genetic Programming Applied to Financial Modelling

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Abstract

This thesis empirically analyses the effect of market condition variables on modelling realised volatility and high frequency delta hedging strategies. The methodology used, Genetic programming (GP), allows the incorporation of time-varying nonlinear relationships in empirical studies. The analysis demonstrates that variables related to the condition of the market such as liquidity, bid-ask spread and volatility indicators have a dynamic impact on realised volatility forecasting and delta hedging strategies, which are two fundamental risk management issues in the financial derivative markets.

In the study of realised volatility (RV) modelling, RV is constructed using five-minute futures logarithmic returns. One-day horizon RV is forecasted using lagged RVs and market condition factors as dynamic and possible nonlinear explanatory factors. The out-of-sample forecasting results from the proposed model are compared with benchmark models including GARCH, ARMA, HAR and stepwise linear regression models. The model outperforms the benchmark models when compared across a number of different diagnostic measures. The study illustrates the importance of allowing market condition factors to have a dynamic and nonlinear impact on RV when modelling RV out-of-sample.

The delta hedging study examines the portfolio of an option writer who delta hedges their exposure on a high frequency basis using futures contracts. Three different deterministic rules are used to indicate when the option writer should rebalance the hedged portfolio. Rebalancing is conducted at uniform time intervals, when the underlying asset moves by a fixed number of ticks and based on a change in the delta of the option. The main contribution of this study is to propose a rebalancing trigger based on the output from an optimal hedging strategy that rebalances the portfolio based on dynamic nonlinear factors related to the condition of the market including a number of liquidity and volatility factors. The proposed optimal hedging strategy outperforms the other deterministic hedging methods with a significantly lower risk than the other strategies.

Statement of Original Authorship

I hereby certify that the submitted work is my own work, was completed while registered as a candidate for the degree stated on the Title Page, and I have not obtained a degree elsewhere on the basis of the research presented in this submitted work.

Acknowledgements

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Chapter 1

Introduction

A financial derivative is a financial instrument whose value depends on or derives from the values of other, more basic underlying assets. A big portion of the derivative markets comes from the exchange-traded futures and options. The size of this market is significant. The world GDP in nominal value was $US 71.7 trillion in 2012 according to the International Monetary Fund. The notional principal turnover of futures and options traded on organised exchanges was $US 1,568.5 trillion in 2012 including interest rate, currency and equity index markets according to the Bank for International Settlements. The size of the derivative market suggests the utilities of this market. Financial derivatives are used for a number of purposes including hedging or risks insurance, arbitrage between markets, speculation and changing the nature of an investment without incurring the transaction costs.

Risk management in derivative markets is very important and challenging. The derivative markets are leveraged financial markets. The investors only need a small amount of money to take a derivative position. However, the potential loss could be unlimited. The relationship between the underlying financial instrument and the derivative itself is complex. The price of the derivative depends on the price of the underlying asset. The risk and reward pattern of the derivative may be linear or nonlinear of the underlying price changes depending on the particular derivative product. Further the product structure in the financial derivatives markets is completely different to conventional financial markets. There are multiple affecting factors in the derivative markets besides the underlying price. They include among others, the underlying volatility, interest rate, credit rating, terms in derivative contract and alternative products.

This thesis has studied two risk management problems in the exchange-traded futures and options market. The first is to accurately forecast the measure of the futures price fluctuation, the realised volatility (RV). The second is to find a dynamic optimal delta hedging strategy for the options market makers who need to hedge the risk, which is transferred from the options investor. These two problems are fundamental topics in risk management in financial derivative markets related to the underlying price risk, which is from the uncertainty of the futures price. It is usually measured by the underlying return volatility, which provides important risk indicators for financial market participants. Investors use this measure to pick up derivative products to suit their risk appetite. Regulators use this measure to make policies. It is also an important input for derivative security valuation. Risk management tries to reduce the portfolio return volatility. Delta hedging is the standard way to hedge the risk from the underlying price move. In extreme events, risk management can reduce the cost of financial distress [2].

Following the global financial crisis of 2007-2009, financial investors face financial risks that are greater than ever. One of the lessons from the past crisis is that risk modelling should be conducted by taking market conditions into account [1]. In practice, it is hard to take market conditions into account through conventional parametric methodologies, such as linear regression where the linear dependence relation is assumed as fixed. This thesis applies a novel approach, Genetic Programming (GP) to model risk issues based on market conditions.

Forecasting daily return volatility is crucial in finance. Traditionally, volatility is modelled by its own lagged information only. The relationship of market information variables and volatility has been studied for two decades. However, there is no theory to tell us which are the suitable function forms to be used on these exogenous market information variables when forecasting volatility. The question of how to utilise market information variables in volatility forecasts is a challenging task. This thesis forecasts one-day ahead realised volatility (RV) using a methodology of GP that incorporates information on market conditions including trading volume, number of transactions, bid-ask spread, average trading duration and implied volatility. The forecasting result from GP is significantly better than the benchmark models including the autoregressive and moving average model (ARMA), the generalised autoregressive conditional heteroscedastic model (GARCH), the heterogeneous autoregressive model (HAR) and a stepwise regression. Relations between market information variables and RV are examined through the best individual from GP.

Effective hedging of derivative securities is of paramount importance to derivatives investors and market makers. Delta hedging is the standard approach used to hedge derivative securities. The theory of delta hedging in the presence of transaction costs, market microstructure effects and in a discrete time setting has been well developed in theoretical literature. However, there has been very little empirical analysis of this issue. In practice, the market is incomplete and the assumptions under the theoretical works may not hold. This thesis seeks to address that deficit of empirical analysis by examining delta hedging strategies using one year FTSE 100 futures and options dataset. There are multiple goals in this empirical analysis. The first is to assess the Black-Scholes delta hedging theory in an intraday setting, the second is to assess the bid-ask transaction cost impact on delta hedging performance and the third is to propose an adaptive intraday delta hedging strategy through the methodology of GP. The tests in this thesis show that option writers who hedge their positions on a high frequency basis do generate returns greater than the risk-free rate even after transaction costs, although their hedged portfolios also bear some risk. As the hedging frequency increases the risk and return of the hedged portfolio reduces as expected. However, the optimal GP based hedging strategy outperforms the other naive hedging methods with a significantly lower risk as it successfully includes recent information on market conditions such as bid-ask spreads, volume and volatility, to determine when the option writer should optimally rebalance their portfolio.

The rest of this chapter is organised as follows. The research aims and objective of this thesis are outlined next. A short overview of the research framework will then follow, along with the contribution of the thesis and its scope limitations. An overview of the structure of the remainder of this thesis then completes the chapter.

1.1 Research Aims

In this thesis, two very important risk management issues in financial derivative markets have been addressed through an evolutionary perspective to view the financial market and the participants as proposed in the study of [1], which concludes that the risk reward relation is not fixed in the investment management context. In this thesis this idea is understood to be that the dependent variable and the independent variables do not necessarily have a rigid fixed relation. Their relations adapt to the environment. The surrounding environment should be considered dynamically in financial decision-making. A good strategy is the one that is best adapted to the environment and one that will wax and wane. This research will aim to answer the following questions;

1. What are the characteristics of the derivative product and what are the most important factors in the financial derivative market environment? Answers to these questions will determine the most efficient way to do the modelling. The best strategy should be adapted to the environment. In practice, the environment should be identified, measured and used in the modelling. An assessment of the environment must be carried out prior to starting the risk modelling.

2. How to apply GP efficiently to two financial problems? One of the difficulties of GP is parameter tuning. A good result only comes from suitable parameters setting.

3. How to forecast realised volatility by considering all related market conditions? The realised volatility is a prevailing volatility measurement. It has potential utilities in policy-making and derivative trading. Traditionally, it is modelled by its lagged information. In this thesis, all relevant factors are considered.

4. How to get an optimal intraday delta hedging strategy by considering market conditions? Black-Scholes theory provides a formula to calculate the delta hedge ratio with some strong assumptions and requires continuous rebalancing. In reality, this is impossible. Practitioners need more realistic rebalancing frequencies that take into account transaction costs and the rebalancing decision should be triggered by the changes in market conditions.

1.1.1 Research Objectives

The following objectives need to be achieved in order to fulfil the research aims of this thesis.

1. Critically assess the derivative market trading environment by analysing the complex futures and options dataset.

2. Critically review GP methodology and its current applications in finance.
3. Use experiments to set GP parameters to suit a given problem.
4. Critically review the current literature in volatility modelling.
5. Calculate the realised volatility for FTSE 100 index futures.
6. Assess the current prevailing RV forecasting methods.
7. Design an RV forecasting method that adapts to market conditions.
8. Draw conclusions from the results for RV modelling.
9. Critically review the current literature in delta hedging.
10. Assess delta hedging theory in a discrete time intraday setting.
11. Assess the bid-ask transaction cost impacts to delta hedging.
12. Design an adaptive intraday delta hedging method.
13. Draw conclusions from the results for delta hedging.

1.2 Framework of Research

The thesis uses a one year FTSE 100 index futures and options intraday dataset to model the realised volatility for futures and investigate the optimal delta hedging strategy for options. The first research aim proposed in Section 1.1, is to assess what are the characteristics of options and futures and what are the most important factors in the financial derivative market environment. It is undertaken using high-frequency dataset from Euronext-Liffe. This one year data set includes all trading transactions and quotations for FTSE 100 index futures and options. The characters of futures contracts and options contracts are analysed. The prevailing derivative market theories are assessed against real data. The dynamic trading environments for futures and options are reviewed initially.

To address the second research aim, GP methodology is firstly systematically introduced and its current applications in finance are reviewed. GP is one type of Evolutionary Computation (EC), which belongs to population-based and biologically inspired (survival of the fittest) algorithms. The common logic behind EC is: given a population of individuals, according to different objectives, fitter/better individuals survive/are selected to become parents of child solutions in the next generation, with the elements of those child solutions stemming from the application of pseudo-genetic operators such as crossover and mutation. After genetic iterations under environmental pressures optimum/sub-optimum solutions are returned. EC's basic utility is optimisation.

Another particularly interesting aspect of GP is that both the solution form and associated parameters are co-evolved. This is particularly useful in financial modelling because in financial markets the data is rich but the theory is poor. Typically, many plausible explanatory variables exist but the interrelationship among the relevant variables may be poorly understood, although some domain knowledge may exist. This suggests that model induction methodologies will have particular utility.

Unlike methodologies such as neural networks, GP offers the potential to generate human-readable rules. This is of particular importance in, for example, a financial trading system where human decision-makers want to have an insight into the trading rationale. Another advantage of GP is that it permits the incorporation of domain knowledge, and the generation of "solutions" of a particular form. This allows the financial user to seed the evolutionary process with their current trading strategies or basic statistical relations, in order to see what improvements GP can uncover. More generally, all evolutionary algorithms allow the incorporation of complex fitness functions, which is of particular importance in finance as fitness is generally a complex amalgam of return and risk. Recent years have also seen an explosion in the quantity and quality of electronic financial information that is available, hence, the practicality of applying EC-type methodologies in finance has increased.

Parameter tuning is one of the challenging tasks in order to have GP applied efficiently. There are 11 types of parameters that need to be prespecified before the application. Experiments are designed to investigate the quantitative values of key parameters. A demonstration is given to illustrate the application of a new form of GP, where the probabilities of crossover and mutation are adapted dynamically during the GP run, to the important real-world problem of options pricing. The tests are carried out using market option price data and the results illustrate that the new method yields better results than those obtained from GP with fixed crossover and mutation rates. The developed methodology has potential for implementation across a range of dynamic problem environments.

Based on the results of the first two research aims, GP is applied to achieve the third and fourth research aims. The volatility concepts used in finance and volatility modelling approaches are firstly reviewed. Traditionally volatility is modelled parametrically as a latent variable. Realised volatility is a measure of volatility, which bridges the high frequency data and the daily volatility. With this concept volatility can be modelled directly. The realised volatility is then estimated based on FTSE 100 index futures data. One-day-ahead RV is forecasted based on market conditions by GP. The out-of-sample model performance from GP is compared with benchmark models. Market conditions are analysed by the best individual from a GP. The results confirm the implication of the Adaptive Market Hypothesis (AMH) from [1] that the best strategy will wax and wane.

In the final research aim the literature on delta hedging in the presence of market microstructure effects is reviewed. The empirical analysis gap of the optimal rehedging frequency is identified. Theory-based delta hedging strategies are analysed empirically based on the one year intraday futures and options data. Delta hedging is implemented by different rebalancing strategies. Rehedging is triggered by uniform time change, a certain amount of underlying tick changes and a percentage amount of delta change. Firstly, this delta hedging analysis is conducted on traded prices where transaction costs are not considered. The impact of bid-ask spread transaction costs on delta hedging is assessed by the same delta hedging tests except an ask price is used when buying, and a bid price is used when selling. An optimal GP based delta hedging strategy is proposed by considering relevant market conditions based on the theory of delta hedging. In the proposed delta hedging strategy, rebalancing frequency and the new hedge ratio are driven by market conditions. At the end, the market conditions are analysed by the best individual solution from GP. The result again confirms the implications from AMH that the best strategy is the one that adapts to the environment.

1.3 Contributions of Thesis

The work presented in this thesis has given rise to a number of contributions, which are summarised in the following sections:

1.3.1 Literature Review Contributions

This thesis provides up-to-date literature reviews in different fields. GP's applications in financial modelling are reviewed and summarised. GP has been selected as the methodology to examine the modelling issues in derivative markets. These reviewed financial applications give references regarding how to efficiently apply GP in this thesis.

In the chapter regarding RV modelling, the volatility concepts and up-to-date traditional volatility modelling approaches are reviewed. The relationships of volatility and market conditions including trading volume, number of transactions, bid-ask spread and price range are then covered. Empirical volatility modelling works are also reviewed and the research gap in RV modelling is identified.

In the chapter on delta hedging modelling, the historical and up-to-date theoretical and empirical delta hedging works are reviewed. Firstly, the theoretical works of delta hedging under the discrete hedging and transaction costs are reviewed. Then the different methodologies measuring the statistics of the discrete hedging errors are summarised. Theoretical works of model misspecification on hedging are reviewed. Studies of the hedging error in the presence of microstructure are covered. Empirical analyses under the controlled environment and by using real data are also summarised. The research gap in delta hedging is revealed.

1.3.2 Methodology Contribution

A new form of GP is designed where the probabilities of crossover and mutation are adapted dynamically during the GP run. Compared with GP with a static parameter setting, the proposed GP is able to provide an improved result, which is demonstrated in an option pricing example. This adaptive form of GP is suitable for complex problems where local optimum exists. This study demonstrates the potential ability of GP in solving complex problems.

1.3.3 Empirical Analysis Contributions

This thesis has made a number of empirical analysis contributions below. The first two are from Chapter 3. The third one is from Chapter 4, in which the first modelling topic, RV forecasting is conducted. The other three are from Chapter 5, in which the second modelling topic, option delta hedging is carried out.

- **Analysis of the characteristics and the intraday trading environment of financial derivative products including the stock index futures and stock index options.**

  FTSE 100 index futures and options are financial derivatives written on the same underlying stock index. An option contract is characterised by option types, strike price, time to maturity. A futures contract is referred by its delivery date. On a daily basis, the number of active option contracts is much higher than that of futures contracts. The analysis in this thesis gives the statistical summary of the traded price, volume, quotation price and quoted depth for both futures and options. In theory, futures and options written on the same underlying security should be perfectly correlated. An intraday price plot of the selected most active futures and call option contract shows that this perfect correlation does not always hold in the selected data sample. This indicates that market conditions are important in financial decision-making besides fundamental financial theories.

- **Examine the theoretical relations in the high frequency dataset**

  The put-call-parity, spot-futures-parity and futures and options co-movements are examined in a selected sample of the one year FTSE 100 index futures and options dataset. The analysis shows that under the microstructure effects, financial theories do not work perfectly in the refined time intervals although they do hold in longer time periods.

- **The relationships of volatility and market conditions are examined empirically.**

  The outstanding one-day RV forecasting results from the proposed model indicate that the relations between RV and market conditions are nonlinear. The relationships of market conditions and Realised Volatility are further analysed dynamically. The factor of average trading duration has been shown to have a consistent nonlinear relation with RV.

- **Empirically assessing Black-Scholes delta hedging by high frequency data.**

According to the Black-Scholes option pricing model, as long as the hedged portfolio composed by an option and a certain amount of underlying assets indicated by the delta ratio are rebalanced continuously, the portfolio earns the risk-free rate. The availability of high frequency data makes this assessment possible. This dissertation has made the first attemp to empirically assess delta hedging theory using high frequency data. The result demonstrates that for at-the-money options and out-of-the-money options the option writers who hedge their options positions on high frequency bases do generate returns greater than the risk-free rate although the hedge portfolio also bears some risk. When the rehedging frequency increases, the risk and the return decrease. The risk-rewarding relationship is not the same across different option segments. This analysis again provides evidence for the AMH that a perfect theory scenario does not hold in reality.

- **Empirically assessing the bid-ask spread transaction cost impact on delta hedging**

  Designed deterministic delta hedging methods, where the rebalancing decisions are not based on market conditions are carried out on the futures transaction price as well as on the bid and ask prices. In the second case, the ask price is used when buying the futures, and the bid price is used when selling. The hedging results without a transaction cost and with a transaction cost are compared. The bid-ask spread transaction cost has impacted on the hedging result differently for different option segments.

- **Empirically assessing the relationships of the delta hedging and market conditions**

  In the proposed GP optimal delta hedging strategy, rebalancing is triggered by the sensitive market conditions through a control variable, which also provides the rehedging ratio. The relationships of the delta hedging and the selected market conditions are analysed dynamically for at-the-money options and out-of-the-money options through the fitted function form of the control variable.

1.3.4 Modelling Contributions

This thesis models the realised volatility and option delta hedging strategies by taking market conditions into account. There are two specific modelling contributions in these two topics and also one general modelling contribution.

- **Forecasting one day horizon realised volatility by market conditions.**

  RV is adaptively forecasted by GP with a number of market conditions considered. The one day out-of-sample forecasting result is compared with benchmark models including ARMA, GARCH, HAR and a stepwise linear regression. In the first three benchmark models, only RV lagged information is used. All potential market condition factors provided to GP are also used in the stepwise regression. The difference is that the stepwise regression only assumes the linear relations between RV and market conditions and GP can use nonlinear functions to link RV and market conditions. GP's forecasting results are significantly better than the benchmark models.

- **An optimal delta hedging strategy is proposed by taking market conditions into account**

  An optimal GP delta hedging strategy is proposed by considering all relevant market conditions including bid-ask spread, trading volume, average trading duration and implied volatility. In this strategy, rebalancing frequency and the new hedge ratio are driven by market conditions. The out-of-sample results show that this optimal GP delta strategy can outperform the other strategies with a significant lower risk when minimum risk is required.

- **A demonstration of risk modelling by taking market conditions into account.**

  This thesis demonstrates that by using relevant market conditions, improved modelling results are achieved based on theoretical domain knowledge. In a normal time, financial theories are expected to be a good proximation to reality. In reality, financial modelling that considers relevant market conditions could be potentially highly rewarded.

1.3.5 Conference Contributions

Parts of this thesis have been presented in the following conferences.

1. Z. Yin, A. Brabazon and C. O'Sulivan. "Genetic Programming and Option Pricing," presented in the 2006 Annual Irish Accounting & Financial Association Conference, Dublin, 2006.

2. Z. Yin, A. Brabazon and C. O'Sullivan. "Adaptive Genetic Programming for Option Pricing." in *Proceedings of the 2007 GECCO Conference Companion on Genetic and Evolutionary Computation*, 2007, pp.2588-2594.

3. Z. Yin, A. Brabazon, C. O'Sullivan and M. O'Neill. "Genetic Programming for Dynamic Environments." in *Proceedings of the International Multi-conference on Computer Science and Information Technology*, 2007, pp. 437-446.

4. Z. Yin, A. Brabazon and C. O'Sullivan. "Genetic Programming Applications in Financial Modelling: A Brief Survey." Presented in the Workshop/Summer School on Evolutionary Computing Lecture Series by Pioneers, Londonderry, Aug. 2008.

5. Z. Yin, C. O'Sullivan and A. Brabazon. "Empirical Analysis of Delta Hedging." presented at the 2013 Annual Irish Accounting & Financial Association Conference, Dublin, 2013.

1.4 Scope Limitations

Risk management in derivative markets is a very broad area. Take an option for example, the different dimensions of related risk can be summarised by Greek letters. The common Greeks that traders would consider include the first-order Greeks, *delta*, *vega*, *theta*, *rho* and the second-order Greek, *gamma*. In practice, it is not possible to maintain all Greeks at zero. Usually traders zero out *delta* and keep the other Greeks monitored. This thesis only looks at one of these Greeks, *delta*. A risk management to balance all Greeks falls outside the scope of this thesis.

This thesis examines underlying price change related risk management issues in derivative markets. In detail, it makes contributions in RV forecasting and option delta hedging in the presence of market microstructure effects. There are a number of areas worthy of further research that fall outside the scope of this work. They are discussed below.

Realised volatility in this thesis is estimated by an approach from [60]. There is also an increasing interest to control microstructure effects to construct RV by ultra high frequency data [126] [127]. The modelling approach in this thesis can be extended to a RV constructed on ultra high frequency data.

Due to time limitations, the put option contracts are not used in the delta hedging strategy tests. This limitation should be considered in future work.

The delta hedging strategy considered here is for an option life time rehedging. In reality, practitioners also open and close an option position on a daily base. It will be useful to extend the current study to an intraday exposed option position.

GP is a promising modelling methodology for the finance area. Researchers have devoted themselves to the development of advanced forms of GP, such as automatic defined functions (ADF), multi-objective GP and adaptive forms of GP. In this thesis only basic GP is applied. Modelling financial problems by advanced GP should be considered in the future work.

1.5 Structure of Thesis

The remainder of this thesis is structured as follows.

- **Chapter 2 Genetic Programming (GP)**

  This chapter firstly provides an introduction to GP. Next, a new form GP is proposed and an illustration example to apply GP in option pricing is given. GP's applications in finance are reviewed at the end.

- **Chapter 3 Derivative Data Analysis**

  One year FTSE 100 index futures and options dataset is described. The trading structures of futures and options are given. Data statistics of transaction price, trading volume, quotation price and quoted depths are provided. Joint Intraday trading patterns of futures and options are analysed. Important indications are drawn from this data analysis for the empirical analysis conducted in the following chapters.

- **Chapter 4 Realised Volatility Estimation and Forecasting**

  In this chapter, GP is applied to volatility modelling. A review of the literature on the volatility concept, and volatility modelling as well as the relationship between volatility and different market conditions is provided. The application motivation is explained. The experiment design is also outlined. The testing results and the discussion of the relationships between volatility and market conditions are given. The conclusions are then provided in the summary.

- **Chapter 5 Empirical Analysis of Option Delta Hedging**

  In this chapter, GP is applied to delta hedging. First, a review of the literature on delta hedge theoretical work and empirical work is given. The gap of the deficit in empirical analysis is recognised. The application motivations are explained. Experiment design of a two-step testing, with and without the transaction costs, is outlined. An optimal GP delta hedging strategy is proposed. The BSM delta hedging is empirically analysed through three rehedging strategies. The performances from different hedging strategies are compared. The transaction costs impact on delta hedging is analysed. The reason for the success of GP optimal delta hedging strategy is investigated. The summary is then given to conclude the chapter.

- **Chapter 6 Conclusions and Future Work**

  A summary of the thesis is given in this chapter, along with an overview of its contributions and limitations. Opportunities for future works are also outlined.

Part I

Genetic Programming(GP)

Chapter 2

GP

In this thesis, two detailed financial modelling studies are conducted on an ultra-high frequency dataset of FTSE 100 index futures and options by a novel approach GP. In this chapter, the methodology, GP is introduced in Section 2.1, where the GP background knowledge, basic concepts, work process and parameters are explained. A demonstration example of using GP in option pricing is given in Section 2.2. Financial applications of GP are reviewed in Section 2.3. Section 2.4 concludes the chapter.

2.1 Introduction of GP

2.1.1 GP Background

Recent years have seen the application of multiple biologically-inspired algorithms (BIA) for the purposes of financial modelling [6]. Main components of BIA are; Neural Networks whose inspiration arises loosely from a simplified model of the working of the human brain; Social System, some of which are drawn from a swarm metaphor; Immune System, which draw inspiration from the working of the natural immune system to develop algorithms for optimisation and classification; and Evolutionary Computation (EC), which draw metaphorical inspiration from processes of natural evolution. Genetic Programming (GP) belongs to EC. Figure 2.1 provides a broad taxonomy of the primary methodologies in BIA [6].

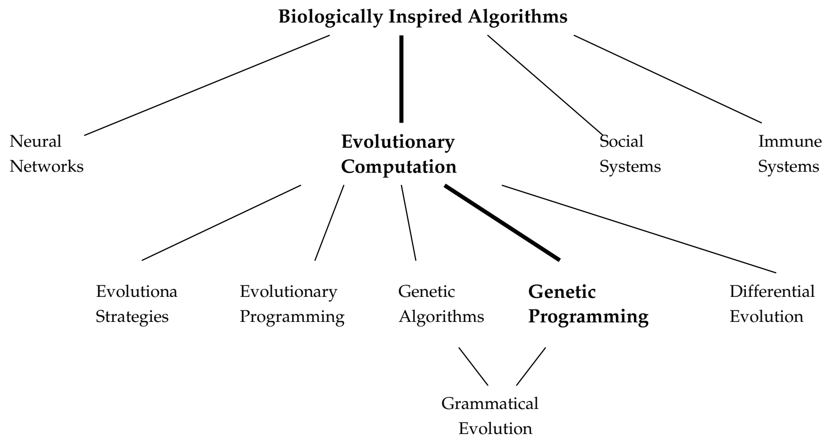


Figure 2.1: GP in Biologically-Inspired Algorithms

Charles Darwin was the first to formulate a scientific argument for the theory of evolution by means of natural selection. In biology, evolution is the change in the inherited traits of a population from one generation to the next. These traits are the expression of genes that are copied and passed on to offspring during reproduction. Mutations in these genes can produce new or altered traits, resulting in heritable differences between organisms. New traits can also come from transfer of genes between populations, as in migration, or between species, in horizontal gene transfer. Evolution occurs when these heritable differences become more common or rare in a population, either non-randomly through natural selection or randomly through genetic drift.

EC draws inspiration from the processes of biological evolution to breed solutions to problems. Main components in EC are Differential Evolution (DE), Evolution Strategies (ES), Evolutionary Programming (EP), Genetic Algorithm (GA), Genetic programming (GP) and Grammatical Evolution (GE).

Like natural evolution, the common logic behind EC is: given a population of individuals, according to different objectives, fitter/better individuals survive/are selected to become parents of child solutions in the next generation, with the elements of those child solutions stemming from the application of pseudo-genetic operators such as crossover and mutation. After genetic iterations under environmental pressures optimum/sub-optimum solutions are returned. EC's basic utility is optimisation. They can be used to search optimum solutions through exploration and exploitation in potential solution space for the problem at hand.

The workflow of a general EC process is shown in Figure 2.2. The first step is usually done off-line. In this step, a proper representation needs to be chosen, which maps the potential solution space into the EC's searching space. This step is also called encoding, which transfers the phenotype, which is the nature form of the real world problem, into genotype, the form, which EC works with. There are some common representations, such as binary string, real number and syntax tree. The best individual returned from the final generation needs to transfer back to the phenotype, which is called decoding. The second step is to initialise a population of individuals in the form of chosen representation. The common way to initialise population is to randomly create a predefined number of individuals. Each individual is evaluated by the predefined fitness function to get their fitness quantity in the third step. In the fourth step, the stop conditions will be checked first. If any one of the stop conditions is matched the process terminates. Otherwise it goes to the next step. The stop condition could be that a number of generations have been achieved or a predefined quality measured by the fitness has been reached by the best individual in the population. In step five, parents are selected from the current population. They are used to create new individuals in the next generation. There are several selection methods available, such as roulette selection or tournament selection. In step six, the selected parents are used to implement a recombination operation, by which the information from both parents is exchanged to create new individuals. Different EC algorithms have different recombination operators and different recombination operators have different ways to change and utilise the information from the parents. Some EC algorithms might omit this step and go directly to the next step. In step seven, mutation is applied to individuals that have just been created by recombination in step six. Some EC algorithms might omit this step. In the last step individuals to form the new population are selected from either newly created individuals or individuals from the current population by the same selection criteria. After this step it goes back to the third step and another generation starts. EC will exit until a stop condition is matched in step four. Different EC components have different algorithms to implement these steps. The process order may also vary.

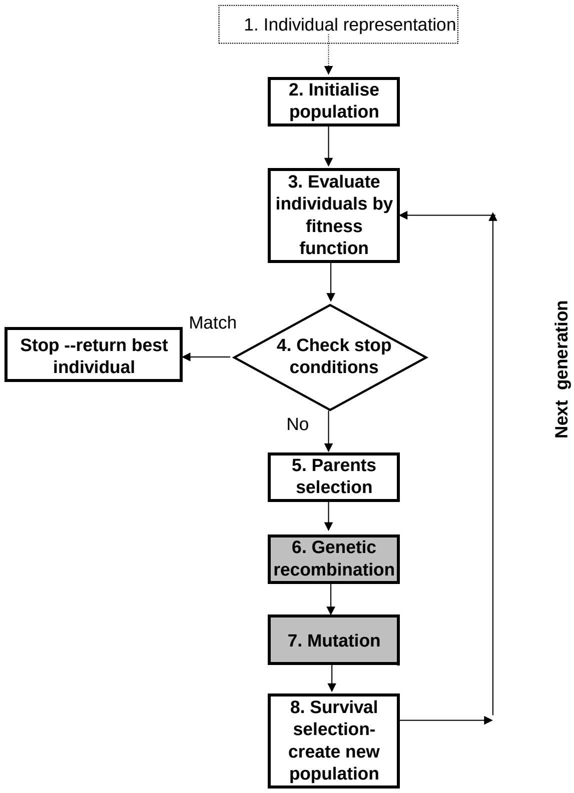


Figure 2.2: Evolutionary Computation Workflow

2.1.2 Genetic Programming Process

Genetic Programming (GP), one of the youngest branches of EC and started in the early 90s [8]. GP is very similar to Genetic Algorithms (GA) in the EC family as the genetic recombination operators in both of them are called crossover. Also both of them have mutation operators. The main difference between GP and GA is the representation of the solution. GA operates on a representation (a binary or real-valued genotype) of the solution. A mapping relation exists between the solution and the coding string for an encoding method. In contrast, GP's evolutionary search is applied to the solution directly and the solution is in a special tree representation. Most EC algorithms, including ES, EP, GA and DE are used for the parameters optimisation. GP evolves the function form besides optimising the coefficients of the function.

GP was initially developed to allow the automatic creation of a computer program from a high-level statement of a problem's requirements, by means of an evolutionary process. In GP, a computer program to solve a defined task evolves from an initial population of random computer programs. An iterative evolutionary process is employed by GP, where better (fitter) programs for the task at hand are allowed to 'reproduce' using recombination processes to recombine components of existing programs. The reproduction process is supplemented by incremental trial-and-error development, and both variety-generating mechanisms act to generate variants of existing good programs. Over time, the utility of the programs in the population improves as poorer solutions to the problem are replaced by better solutions.

Basic Concepts in GP

Tree representation In the form of GP popularised by Koza [8], these solutions take the form of Lisp S-expressions. In Lisp, operators precede their arguments (known as a pre-fix notation), so the expression *2+1* is written as *(+ 2 1)*. Similarly, $9*((2-1)+4)$ is written as *(* + - 2 1 4 9)*. More generally, Lisp can be any standard programming operators/functions which adopt a pre-fix notation, for example *(setf X 5)* assigns the value 5 to the variable X, where the *setf* is just a function that assigns one of its arguments to another. S-expressions can be naturally represented as a syntax tree. One example is given in Figure 2.3. The element in the circle is called node. The left tree has 6 nodes and the right one has 5 nodes. It is while working on these trees that the evolutionary search operators such as crossover and mutation are applied.

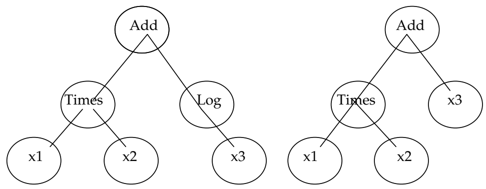


Figure 2.3: GP Syntax Tree

GP syntax tree, the S-expression for the left tree is $(+(\times\ x1\ x2)(logx3))$, its normal expression is $x1 \times x2 + log(x3)$, this tree has a nodes size 6 and a depth level size 3. The s-expression for the right one is $(+(\times\ x1\ x2)(x3))$ the normal expression for the right one is $x1 \times x2 + x3$ this tree has a nodes size 5 and a depth level size 3.

Function Set and Terminal Sets The function set is also called the non-terminal set. The individuals in GP, which are potential solutions to the problem at hand, are generated using elements from the function/non-terminal set and terminal set. This means that the syntax tree in GP is composed by the elements from the function set and terminal set.

The function set contains mathematic or logic functions that have an arity greater than zero, while the terminal set contains functions that have an arity of zero, or constant parameters. The arity of a function refers to the number of arguments it can take. For example the operator $+$ has an arity of two and functions *log* has an arity of one. Constants and variables have an arity of zero as they do not take any arguments [6]. Inside a syntax tree, if an element is from the terminal set then there will be nothing following it; if the element is from the function set then there must be at least one node below it. In Figure 2.3, *x1*, *x2* and *x3* are from a terminal set as there is nothing following them. This can be compared with *Add*, *times* and *log*, which are from the function set as there is at least one node following them.

Breeding Operators Crossover, mutation and reproduction are basic genetic breeding operators in GP, through which children are reproduced. GP could take different forms to implement the breeding operation. Different from the general EC form, where the recombination (crossover) and mutation are implemented in two steps in Figure 2.2, the GP example given in this section as shown in Figure 2.6, implements crossover and mutation in a parallel way in one step. Crossover and mutation are independent operators to create new individuals.

Crossover is operated on two individuals represented as syntax trees. Take Figure 2.4 for example, two individuals are selected as parents (the top two trees) in order to do a crossover. In the crossover process one random node is selected from each of the parents. The node *x2* was chosen from *tree1* and the node *Exp* was chosen from *tree2*. The sub-trees below and including the nodes chosen are swapped over to create two new trees. In the case of *tree1*, the sub-tree is only node *X2* itself as there is nothing below it. In the case of *tree2*, the sub-tree is composed by nodes *Exp* and *10*. Two children, *tree3* and *tree4* are created after the crossover.

Mutation is operated on one parent. Take Figure 2.5 for example, the top tree, *tree1*, is the individual chosen to be mutated. In the mutation process one random node is chosen from *tree1*. In this case, the node *divide* is selected. The sub-tree below and including this node chosen was cut. Instead of the old sub-tree, a new randomly created sub-tree composed by nodes *divide*, *x1*, *4*, *minus*, *sin* and *10* grows in the same place. This new randomly created sub-tree in Figure 2.5 is labelled *Sub-tree1*. The child reproduced through the mutation process is called a *mutant*.

In the reproduction process, an exact same child is copied from the selected parent in the current population.

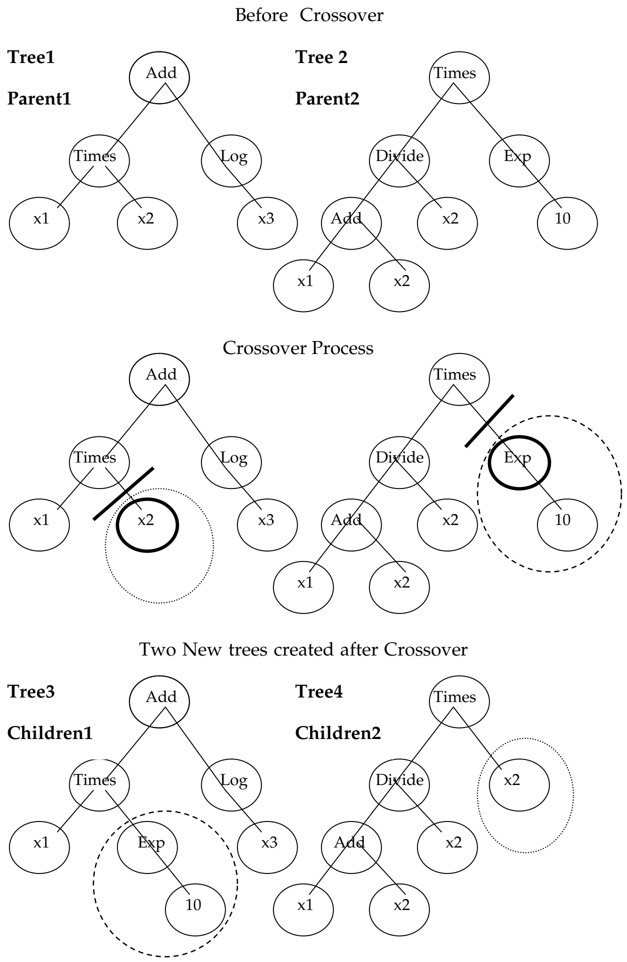


Figure 2.4: GP Crossover Process

tree1's S-expression is $(+(\times\ x1\ x2)(logx3))$, its normal expression is $x1 \times x2 + log(x3)$; *tree2*'s S-expression is $(\times(divide(+(\ x1\ x2)x2))(Exp(10)))$, its normal expression is $((x1+x2)/x2) \times e^{10}$; *tree3*'s S-expression is $(+(\times\ x1\ (Exp(10)))(logx3))$, its normal expression is $x1 \times e^{10} + log(x3)$; *tree4*'s S-expression is $(\times(divide(+(\ x1\ x2)x2)x2))$, its normal expression is $((x1+x2)/x2) \times x2$.

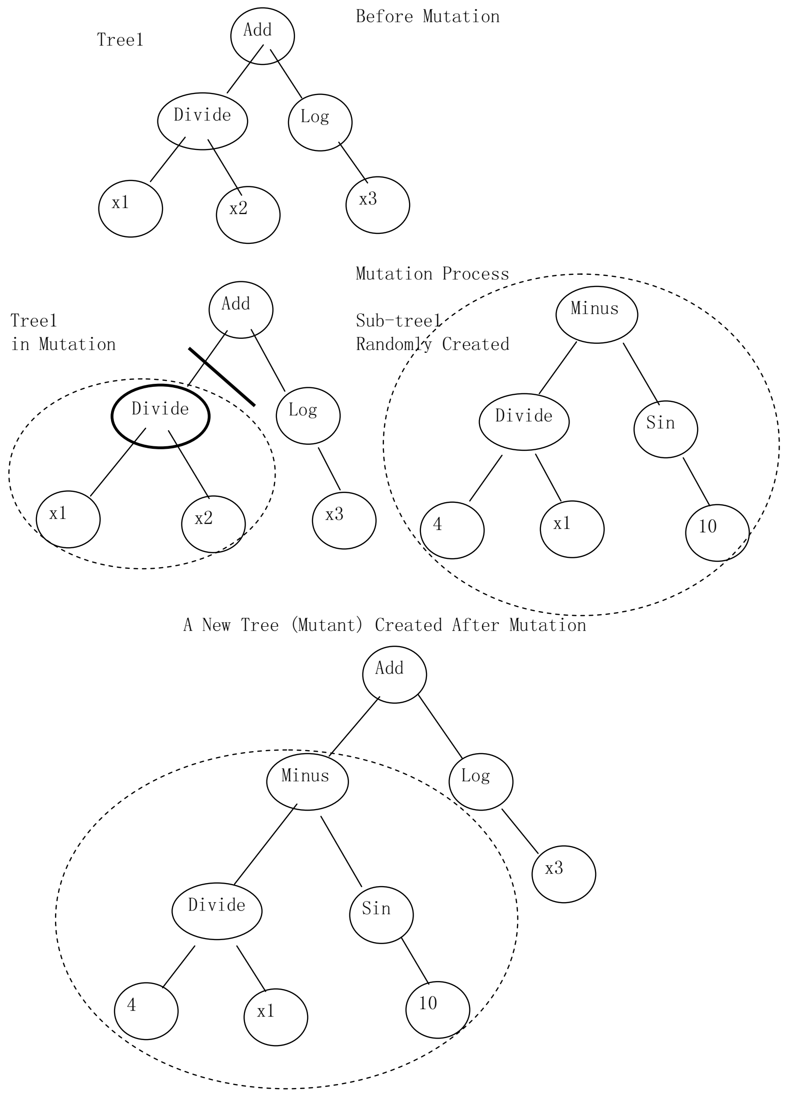


Figure 2.5: GP Mutation Process

The example of *tree1*'s S-expression is $(+(/\ x1\ x2)(logx3))$. Its normal expression is $x1/x2 + log(x3)$; the mutant's (newly created tree) S-expression is $(+(-(/(4\ x1)\ Sin(10)))(logx3))$. Its normal expression is $4/x1 - Sin(10) + log(x3)$

Fitness Each of the individual programmes in the form of a syntax tree is a potential solution to the problem to be solved. The fitness is a numerical value, showing how good the individual is. It is calculated by evaluating the individual solution against the problem given through the fitness function, which is designed to measure how well each individual programme solves the problem.

GP Workflow

The GP workflow is shown in Figure 2.6. Most steps in the GP workflow are the same as the steps in the EC workflow in Figure 2.2 except that the genetic recombination step and the mutation step are combined in one step in the GP example given in this section.

In the first step, all candidate solutions are encoded to a syntax tree, which is the typical representation for GP. All solutions should be able to represent into a tree structure. In the second step, a population of individuals(trees) is randomly created. In the third step, each individual in the population is evaluated by fitness function and assigned a fitness value. The stop conditions will be checked in step 4. If any individual in the current population matches the defined criteria, then GP exits. Otherwise it goes to the next step, where parents are selected from the current population based on fitness measurement. There are three parallel processes in step 6, crossover, mutation and reproduction to produce children individuals for the population in the next generation. How many children will be produced from each of these operators is decided by the probability set by the user for each operator. The probability of copying a parent to create a child may be set to zero. In step 7, individuals are selected from newly created children to form the new population, which replaces the old population. The selection in this step is called survival selection. When this step finishes one generation is completed. The following generations repeat the process from step three to step seven until one of the stop conditions is matched. The best one in the current population is then returned as the solution to the problem given.

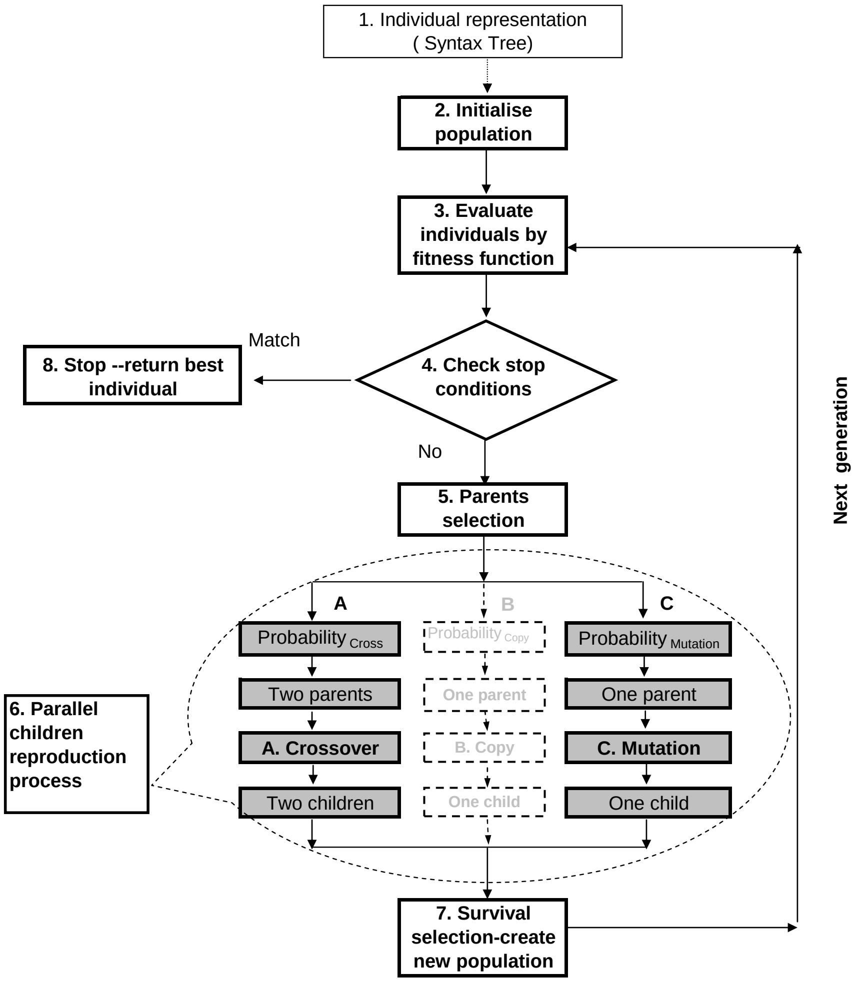


Figure 2.6: GP Workflow Chart

GP Parameters

The most attractive aspect of GP is that it can automatically generate computer programs to solve problems according to the data presented to it. In other words we can say that GP automatically produces methods tailored to particular problems we are interested in. However, to run GP successfully and efficiently, parameters to control steps discussed above have to be set properly.

There are 11 types of parameters that need to be set by a GP user in Table 2.1. The performance of GP and how good it is depends on these settings. Typical choices for each parameter are given in this section. Most of these parameter options are available in a GP toolbox, which is written in different computer languages. For example, *ECJ* [46] is written in JAVA and *GPLAB* [47] is written in MATLAB. The parameter setting is more critical for a complex problem than for an easy task. **Tree Size** Tree size controls the size of each individual. There are two ways to measure the tree size. One is by the nodes number in the tree and the other is to count the depth level of the tree. The tree depth shows how many layers of parents nodes a terminal node has. Individuals in the first generation are normally small. The average tree size in a population gradually grows up as the generation number increases. There are two tree size parameters needed in a GP run, which are listed in Table 2.1. The starting tree size is the tree size used in the population initialisation. Individuals in the first population are created by random selection according to this starting tree size. The maximum tree size is used to validate individuals created by breeding methods.

The complexity of syntax trees produced by GP can easily grow to over hundreds of nodes or more. Solutions from complex trees may potentially suffer from an overfitting problem, which is common for many machine learning techniques. That is, the complex tree can perform overly well in the training sample while the performance does not generalise to the out-of-sample data. Also for a financial decision problem, an over complex solution is not comprehensible to human decision-makers as discussed in [22].

Parameters	Definitions	Typical Choices	GP Workflow
Starting tree size	Tree size in population initialisation	Integer	Step2
Max tree size	Valid tree size for GP individuals	Integer	Step6
Function Set	A collection of functions with arity>0	f(y,z), g(x)	Step2 Step6
Terminal Set	Constants and Variables	x,y,z and constants	Step2 Step6
Population Size	Number of individuals in a population	Integer	Step2
Initialisation Method	Method to create individuals in the first population	Full Method, Grow Method, Ramped Half-and-Half	Step2
Fitness Function	Function used to calculate individual fitness	AE, MSE, MPE, Profit	Step3
Stop Conditions	Criteria that GP exists	Fitness is better than a defined threshold	Step4
Generation Number	Maximum number of generations	Integer	Step4
Probability of Crossover	The probability to create an individual by crossover	The sum of 3 probabilities should be one	Step6
Probability of Mutation	The probability to create an individual by mutation		Step6
Probability of Reproduction	The probability to create an individual by reproduction		Step6
Parents Selection Method	Methods to select parents for breeding	Roulette Method, Tournament Selection	Step5
Survival Selection Method	Methods to select individuals to form the new population	Generational GP/Total Replacement, Steady GP/Keep Best	Step7

Table 2.1: GP Parameters

This table includes parameters, which a GP user needs to specify. Column *GP Workflow* outlines which step in the GP workflow (Figure 2.6) the corresponding parameter is used.

However, if the problem to be solved is very complex while a small tree size is defined, in this case GP cannot return a good solution as the tree size does not allow it to investigate a proper solution. The maximum tree size should be selected properly according to the problem to be solved.

Function Set Function set includes all functions taking at least one operand needed to solve a problem. Function $f(x,y)$ listed in Table 2.1 can be any mathematical function, such as addition, multiplication, or any special function defined by the user and taking two operands. There is no limit to the number of functions that should be given to GP. It depends on the problem to be solved.

Terminal Set Terminal set includes all the explaining variables and constants contributing to the problem. Elements in this set have no operand. The different combinations of elements from the function set and the terminal set compose the searching space of the problem given. The terminal set and function set should be set precisely as the problem requires. A GP search with irrelevant elements in these two sets takes an unnecessary amount of time to implement.

Population Size This parameter defines how many individuals are in a population. GP does a parallel searching. The population size determines how many searching directions there are in a run. For an easy task the population size does not need to be large. However, for a complex problem, a large population size will help to solve the problem more quickly. More importantly, a large population can keep the population diversified and the final result is less likely to be sub-optimal.

Initialisation Method The initialisation method specifies how the individuals in the first generation are created. An individual created by the full method is illustrated by *Tree1* in Figure 2.4. *Tree1* is a fully balanced tree, in which the depth level of each terminal node is the same. There are three terminal nodes, *x1*, *x2* and *x3* in *Tree1*. The depth level is 3 for all of them.

Tree4 in Figure 2.4 is an example of individuals created by a grow method. There are three terminal nodes created by variable *x2*. The one circled has a depth level, 2. The other two terminal nodes by *x2* without a circle have a depth level, 3 and 4.

Through the method, ramped half-and-half, half of the population is created by a full method and the other half is created by a grow method. This method is expected to keep the population fully diversified.

Fitness Function Fitness Function is used to calculate the individual's fitness, i.e. to evaluate how good the individual is for the problem to be solved. The error measures, such as absolute error (AE), mean squared error (MSE) and mean percentage error (MPE) are usually used as the fitness function. Through these error measures, the solution from each GP individual is compared with the target values. When the error measure is used as the fitness function, the smaller the fitness the better the solution. A fitness function may also be designed to be maximised. Such as, when GP is applied in finance trading, the fitness can be the profit from each individual. In this case the individual with the maximum fitness is the best individual from GP. In a more complex fitness function, the tree size could be taken into account as a penalty to the fitness value as applied in [22]. In a multi-objective GP more than one fitness function is required, such as where one is designed to minimise the risk and the other one is designed to maximise the profit.

Stop Conditions In each generation, GP will check the stop condition to decide if one more generation is needed. A typical stop condition can be the fitness reaching a pre-defined value, such as MSE being zero.

Generation Number Generation number is the maximum number of iterations needed in a GP run. If GP cannot find an ideal individual, but the maximum generation number has been reached, GP terminates and the best individual in the current population is returned. Therefore the generation number is actually one of the stop conditions that GP will check. It should be set according to the problem and the combination of other parameter values. If the searching space defined by the elements of the terminal set and function set is big and the population is small, a big generation number is more appropriate compared with a small one. Generally, it should be set by experimentation. In the tests, it is normally set as the number of the generation, after which there is no significant improvement observed in population fitness, after a certain number of generations.

Probability of Breeding Methods The breeding method is also called genetic operator, which GP used to produce offspring in the next generation. Basic breeding methods in GP include crossover, reproduction and mutation as discussed in Section 2.1.2. The probability of a breeding method specifies the chance that a child is from the associated breeding method. The probabilities of breeding methods are important settings to control the population diversity level, which is crucial to avoid the population being premature. Premature convergence means that a population of GP has converged too early and the result is sub-optimal.

Parent Selection Method Parent selection method controls how the parents employed in the genetic operation will be selected from the current population. Roulette selection is a type of fitness proportionate selection method. Each individual has a chance to be selected as parents according to the rank of its fitness in the current population. Tournament selection chooses each parent by randomly drawing a number of individuals from the current population and the one with the best fitness is the winner.

Survival Selection Method Survival selection controls the way in which the new population is created in the next generation. Generational GP method, also called total replacement, is a method in which individuals in the current population do not go to the next generation. The new population is only filled by the newly created children. Steady-state GP method is also called keep best, in which a proportion of the new population is filled by the best individuals from the current population and the rest of the new population is filled by children.

Parameter Tuning

GP is a population-based search methodology. The mechanism of it is to maintain a diversified population in the search space for the problem given, the quality of the population improves after a number of generations during which information is communicated and transferred. GP's performance is dependent on the cooperation of its parameters, such as the parents' selection strategy, genetic operation and population replacement strategy. Different parameter combinations give different levels of performance for the same problem. Therefore parameter tuning is a fundamental task in order to apply GP efficiently.

There are two ways to set the parameters; one is to set parameters before a GP run and GP parameters are kept constant during the run. In this parameter tuning method, empirical tests are necessary to get the best combination for a specific problem. The other way is to control parameters during a GP run, in which the parameters adapt to the problem dynamically.

2.1.3 Section Summary

In this section the background of GP is first given and then GP itself is introduced. The basic concepts in GP including tree representation, function set, terminal set, breeding operators and fitness functions are explained. Steps in a GP workflow are listed. GP's key parameters are explained in detail and GP parameter tuning is outlined. In the next section a demonstration example of the application of GP in finance is provided.

2.2 A Demonstration Example: GP for Option Pricing

2.2.1 Introduction

The background and work process of GP are provided in Section 2.1. This section gives an example and demonstrates how to apply GP in finance. Instead of giving an example of basic GP, this section proposes a new form of GP, where the probability of crossover and mutation is dynamically altered during the GP run. The utility of the new GP system is tested by applying it to the finance problem of options pricing and benchmarked against a fixed parameter GP system. The developed method has the potential for implementation across a range of dynamic problem environments.

In the next section, an introduction to options pricing and a review of previous work in applying GP to options pricing are given. The following subsection provides the basic GP settings used in this application. Experimental design and key results are outlined. A summary for this section is also provided.

2.2.2 Exchange-Traded Options

Exchange-traded options have been actively traded on stocks, stock indices, foreign currencies and futures contracts by hedgers, speculators and arbitrageurs since the 1970s [7]. An option can be defined as the right but not the obligation to buy or sell a financial asset at a stated price at or during a specified time window. Option prices are affected by multiple factors. The first and most well-known option pricing model is the Black-Scholes model. This model assumes the underlying asset returns follow an arithmetic Brownian motion which results in normally distributed returns at every horizon. The Black-Scholes model is a simplified version of reality and when used to fit market options data it performs relatively poorly because of its underlying assumptions. These assumptions include the following; that asset prices follow geometric Brownian motion; that asset prices cannot experience discontinuous jumps; that the volatility of asset prices is constant; and that one can trade the financial asset continuously and therefore options can be perfectly hedged by trading in the underlying asset and a risk-free bond.

The flaws in the Black-Scholes model have encouraged the development of multiple new approaches to options pricing. Due to the complexity in developing closed form theoretical models for options pricing, the domain is particularly amenable to techniques such as GP.

Motivation of Applying GP in Option Pricing

GP is useful in data-rich environments; where the search space is large and highly complex; where conventional mathematical analysis cannot provide analytic solutions; and where the interrelationships among the relevant variables are poorly understood. In options pricing there are multiple factors that affect the option price, therefore the search space is large. The interrelationships of these factors are also complex and nonlinear. Current option pricing theory has obvious biases that usually lead to model option prices that can differ systematically from market option prices. Certain options such as exotic options have no exact pricing formula, thus GP has obvious utility in these situations.

One of the early applications of GP to the option pricing problem is provided by [88]. Since then, there have been many improvements, such as seeding the initial population with elements drawn from the Black-Scholes option pricing formula, and the combination of other domain knowledge into the GP set of terminals / functions[108].

Compared with recent applications of GP to options pricing, this section focuses on how to apply GP in a more efficient way not only by embedding domain knowledge of option price characteristics in the GP, but also by allowing parameter settings to adapt during the GP run.

2.2.3 Dynamic GP for Options Pricing

In this demonstration GP is used in options pricing. The objective is to uncover the Black-Scholes option pricing formula, using market data. The utility of the formula is typically tested by comparing the quality of its predictions against real market option prices.

In this study, data is drawn from market option prices on the FTSE 100 on the 17th of March 2006. The options used are call options on the FTSE 100 futures index. The data supplied was in the form of implied volatilities as in Figure 2.7. There are 187 different end-of-day settlement implied volatilities quoted for various strike prices and maturities. The option *moneyness* (defined as the underlying asset price divided by the strike price) in our 187 data points varies from 0.77 to 1.4 and the time-to-maturity varies from 35 days to 5754 days. Market call option prices are calculated by substituting the implied volatilities into the Black-Scholes formula. The object of the GP application therefore, is to try and recover the Black-Scholes formula for call options that will reproduce the entire set of the 187 option prices given the explanatory variables. Other data including underlying stock index and risk-free rate are observed from the market.

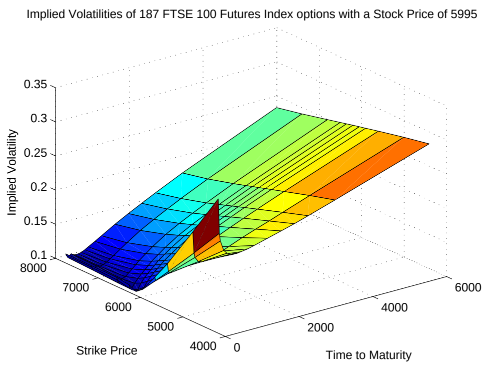


Figure 2.7: Data used in GP Applied in Option Pricing

Terminal and Function Set Selection

In selecting terminals and functions for the experiments, explanatory variables that might affect the option prices should be included in the terminal set. The function set should include all the potential mathematical or logic functions that could be used in options pricing formula. In selecting variables for inclusion as terminals, we used domain knowledge [108] to include option moneyness, the risk-free rate scaled by maturity (defined as the time to maturity multiplied by the risk-free rate of return) and implied volatility during the life of the option (see table 2.2). The non-terminal set is as listed in table 2.3.

ExpressionSign	*Expression*	*Definiton*
X1	S_0/K	Asset price / Strike price
X2	$r*T$	time to maturity * risk-free rate
X3	$\sigma*\sqrt{T}$	implied asset price volatility during option's life

Table 2.2: GP in Option Pricing-Terminal set

Expression Sign	*Definiton*
+	Addition
-	Subtraction
*	Multiplication
x/y	Protected division, if y=0 then x/y= x, if y $\neq$ 0 then x/y=x/y
x^2	Square
$N(x)$	Accumulated normal distribution
e^x	Exponential Function

Table 2.3: GP in Option Pricing-Function Set

Fitness Function

The evolved models are designed to predict the ratio of market option price divided by its strike price across a range of strike prices and time-to-maturity periods. The most common fitness functions for options pricing are the mean squared error, average absolute error, and percentage error. It should be noted that when the option is deep in-the-money, the absolute error is large compared with the percentage error, but when the option is out-of-the-money, the option value is very small and near zero. The absolute error is typically much smaller compared with the percentage error. The fitness function in Equation 2.1 ensures that the model works well for both in-the-money and out-of-the-money options. Where R_{GP} is the result evaluated from the function tree (each individual in a population) created in GP, R_M is the market option price divided by its corresponding strike price, and N is the number of data observations, which in this case is 187.

$$fitness = \sqrt{\frac{\sum(R_{GP} - R_M)^2 + \sum(\frac{R_{GP}-R_M}{R_M})^2}{2 \times N}} \quad (2.1)$$

Error Measurement

Our target set here is the Black-Scholes call option price divided by the strike price. Ultimately with a view to trading options, it is hard to directly infer the financial utility of a GP evolved formula with the fitness metric used in the GP application. To make the results more intuitive we calculate and report two error measures for the GP runs, namely average absolute error in Equations 2.2 and percentage absolute error in Equations 2.3, Where $P_{GP} = R_{GP} * K$, $P_M = R_M * K$ and K is the strike price, i.e. P_{GP} is the GP predicted option price and P_M is the market option price.

1. **Absolute Error Measurement**

$$A.E. = \frac{\sum \sqrt{(P_{GP} - P_M)^2}}{N} \quad (2.2)$$

2. **Percentage Error Measurement**

$$P.E. = \frac{\sum \frac{\sqrt{(P_{GP}-P_M)^2}}{P_M}}{N} \quad (2.3)$$

Proposed Dynamic GP

The parameter choices for crossover and mutation are clearly critical in ensuring a successful GP application. It impacts on populational diversity and the ability of GP to escape from local optima. Optimal parameter setting is also linked to the complexity of the problem and to the size of the population. If the search space is large and/or the population size is relatively small, then the mutation rate will need to increase. In previous applications of GP to options pricing, the probabilities of crossover and mutation were typically kept constant. For example, in [88] mutation is applied at a rate of 0.0033, mutation with a level of 0.001 being applied by [9] and [10]. The work of [97] investigates the utility of various mutation rates between 0.1 and 0.5 (each of which was constant in a single run) with a population from 100 to 50,000.

This section investigates a component of GP's parameter adaptation problem, through the application of a dynamic form of GP in which the probability of crossover and mutation adapts during the GP run. This permits GP to adapt its diversity-generating process during a run in response to feedback from the fitness function. This is supplemented by considering how long it has been since a new best solution was last uncovered; that is, how many generations have been elapsed since a new best solution was last uncovered. We term this period *the generation gap* in this study. Compared with other adaptations this method does not involve extra calculation and memory cost though it returns improved results.

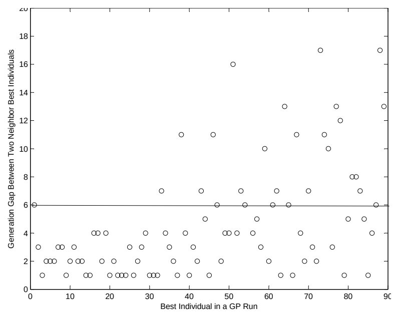
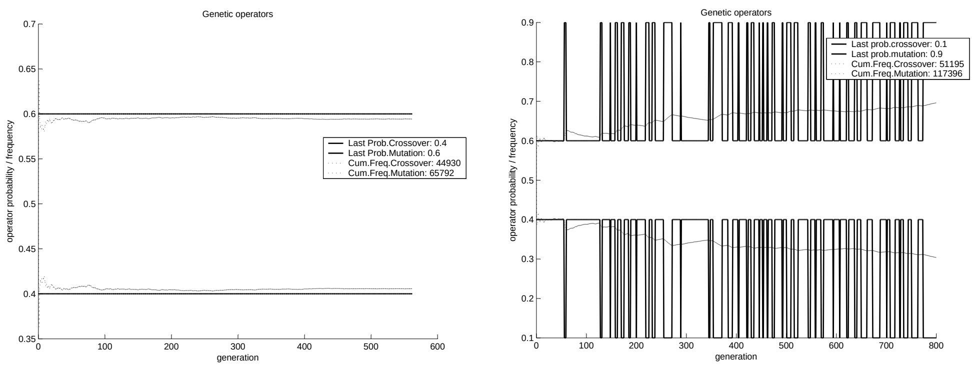


Figure 2.8: The *Generation Gaps* Between Neighbour Best Individuals

This figure shows all different best individuals in the evolution order in the horizontal axis covered from a GP run in different generations. An individual that has the best fitness in a population in GP is called the best individual in that population. In this example, there are around 90 different best individuals in the run. The one in the first generation is excluded. The second best individual covered from GP is on the left and the last best individual covered is on the right in the plot. The vertical axis measures the *generation gap* for best individuals. Take the first point for example, it shows the second best individual is covered after another 6 generations after the first best individual covered. The best individuals in populations from the first generation to the sixth generations are identical.

As already noted, in this study we employ three explanatory variables, stock price divided by strike price, risk-free rate times options' time to maturity, underlying variance, and there are seven functions in the non-terminal set. Based on initial experiments, it was found that the best individual changes frequently in early generations, with the rate of change slowing down later in the run. At the same time the population's fitness only increases significantly in the first few generations. The final result is not the optimal solution. This indicates that GP has converged too early. The mutation rate should increase to let GP escape from local optimal solutions, when the best individual changing frequency slows down.

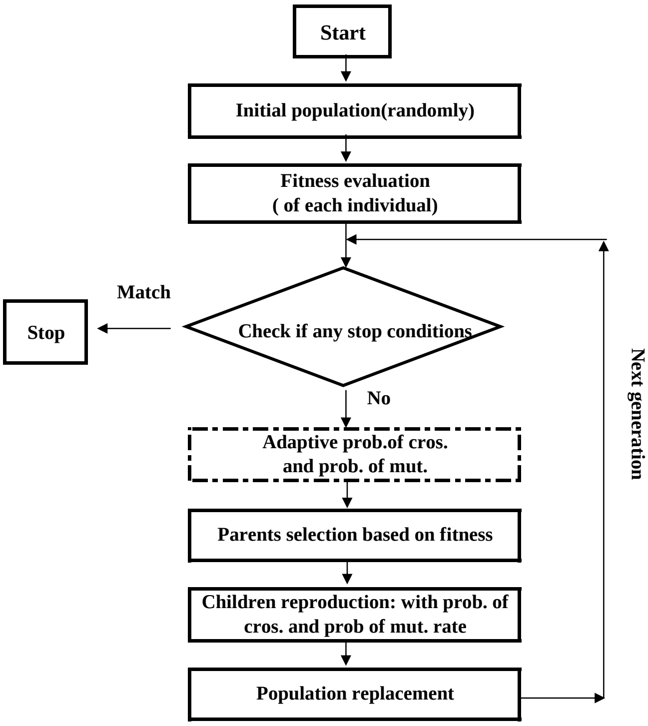
Generation gaps between new best individuals are plotted in Figure 2.8. It is observed that in early generations the *generation gap* is small and it gets bigger when generation number increases. The points above the line in the graph indicate the *generation gap* between subsequent best individuals is more than six generations. Based on these results we should set a window size to six generations. However, to let GP fully explore local search, the window size is set to ten generations. In future applications, the window size could be determined dynamically during the GP run.



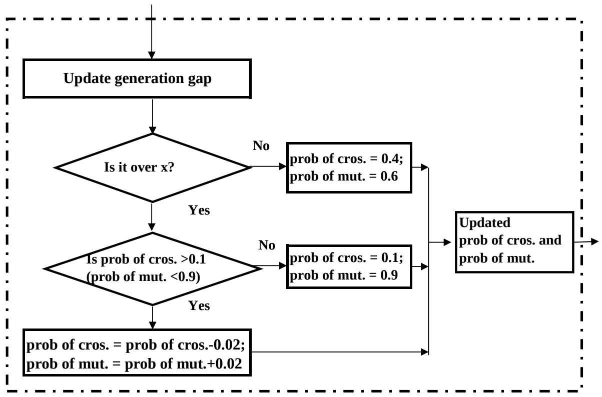
(a) Constant parameter GP, Run 11 (b) Adaptive parameter GP, Run 1

Figure 2.9: Crossover and Mutation Parameters from Run 1 and 11

In adapting the mutation parameter, we steadily increase its value whenever the best individual is unchanged over several generations. The parameter then reverts to its initial level, a new best individual is uncovered by GP. This allows the search process to adapt to escape local optima, whilst permitting local improvement around 'just discovered' new solutions. In the experiments, the width of the window is set at ten generations. In other words, the mutation parameter is fixed for the ten generations following the uncovering of a new best solution. After ten generations without finding a new best individual, the mutation probability increases until it either reaches 0.9 or alternatively, a new best solution is uncovered. The workflow of the proposed adaptive GP is in Figure 2.10a and Figure 2.10b. Figure 2.9b (Figure 2.9a) illustrates the adaption (constant proportion) of crossover and mutation rates during a sample GP run.



(a) Adaptive GP Workflow



(b) Adapting Prob. of Cros. and Prob of Mut.

Figure 2.10: Proposed Adaptive GP

The sub-figure 2.10a gives the workflow of the proposed adaptive GP. There is one extra step compared with a basic GP in Figure 2.6. The sub-figure 2.10b is the extra step in sub-figure 2.10a to adapt the probabilities of crossover and mutation. x in sub-figure 2.10b is the threshold value. It is 10 in this application.

2.2.4 Experimental design and results

A total of twenty GP runs were undertaken, ten of which were fixed parameter GPs and ten of which were dynamic parameter GPs. The fixed parameters for crossover and mutation are 0.4 and 0.6 respectively, set after some initial trial and error experiments. In the adaptive experiments, the parameters for crossover and mutation are initially set to 0.4 and 0.6. If ten generations have elapsed and the best individual has not changed, this means the population is perhaps too concentrated, hence the mutation rate is increased by 0.02 per generation, with crossover decreasing by 0.02 each time until limits of 0.9 and 0.1 are reached. Once a new best individual appears the mutation and crossover rates are put back to their initial values of 0.6 and 0.4.

For all experimental runs, ramped half-half initialisation is employed. A roulette parental selection with a replacement strategy of half elitism, which means half of the new population will be filled by the best from both parent and children and the remaining places will be left to the best children, is also employed. The population size is fixed at 300. The GP run is terminated either when there has been no performance improvement for 40 generations, or when the maximum number, 800 generations has been reached.

Run	1	2	3	4	5	6	7	8	9	10	Average
P.E. (%)	18.7	22	144	28.3	35.1	48.5	32.7	29.1	48.2	32.3	43.9
A.E.	87.8	99	151	170	178	178	186	189	197	204	163.9

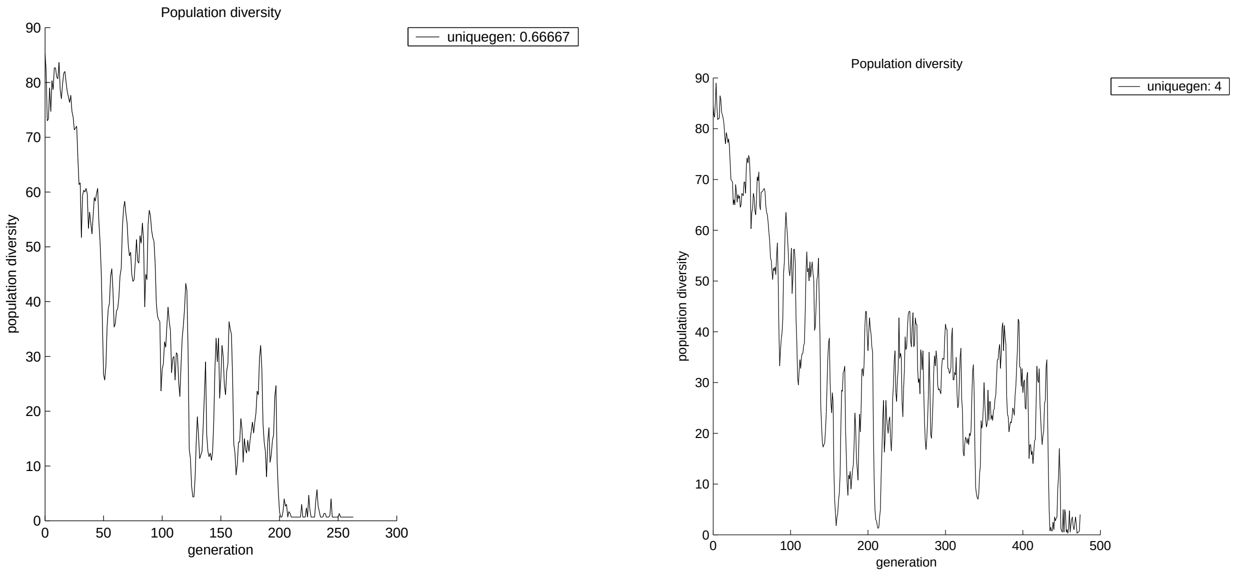
Table 2.4: The Error Results from Constant Parameter GP

Run	11	12	13	14	15	16	17	18	19	20	Average
P.E. (%)	14.2	16.9	21.1	17.1	22.3	42.9	34.7	24.4	28.1	31.8	25.3
A.E.	42.9	55.5	84.7	95.1	99	133	137	147	157	189	114

Table 2.5: The Error Results from Adaptive Parameter GP

The results from the twenty GP runs are provided in Tables 2.4 and 2.5. The average absolute error from the constant parameter GP is 44 percent higher than the dynamic parameter GP counterpart, and the average percentage error is 73 percent higher. These results suggest that significant improvements are made with the use of a dynamic parameter GP.

It can be seen from the population diversity graphs, Figures 2.11a and 2.11a, generated in the GP run, that the population in the dynamic parameter GP is quite diversified in the middle of the run compared to the constant parameter GP where the population setting decreases monotonically.



(a) Constant Parameter GP, Run 1

(b) Adaptive Parameter GP, Run 11

Figure 2.11: Population Diversity from Run 1 and 11

It is noted that this dynamic GP method improves the performance without extra calculation costs. This method has the potential for implementation across a wide range of dynamic problem environments In future application, window size could be determined dynamically during the GP run.

2.2.5 Section Summary

This paper illustrates the application of a novel adaptive form of GP to the financial problem of option pricing, where the probability of crossover and mutation is adapted dynamically during the GP run. The tests are carried out using market option price data and the results illustrate that the new method yields better results than those obtained from GP with fixed crossover and mutation rates.

This section serves a few purposes in this thesis. Firstly, it illustrates how GP is applied to a finance problem. Secondly, it shows the importance of GP parameter settings. Also, it demonstrates how to improve GP work efficiency. In the next section GP's applications in finance are reviewed.

2.3 A Review of GP's Financial Economics Applications

2.3.1 Introduction

This thesis examines two empirical financial modelling issues in derivative markets. The GP methodology is introduced in Section 2.1. It is a population-based search algorithm. It starts from a high-level statement of what is required and automatically creates a computer program to solve the problem. An practical example of GP's application to option pricing is given in Section 2.2. Its flexibility and efficiency in symbolic regression and model induction areas have been picked up in financial modelling. This section reviews GP's applications in different finance areas.

In the next section, GP's applications in financial time series prediction, technical trading and other financial modelling areas are then reviewed. The summary of this section is given at the end.

2.3.2 GP in Asset Price/Return Prediction

Traditional econometric analysis often requires the assumption of stationarity. However, financial time series and their transformations, such as log difference, increments and residuals can hardly satisfy this requirement especially when the internal relation among explain variables is nonlinear. Beside this, high frequency, tick data has been challenging traditional parametric methods which is used to analyse daily/weekly data. There is no uniform theory to show how prices are decided by all these affecting factors. Using data to test their relationship sounds like the best choice. GP is a method that allows data speak for itself.

A number of explanatory factors that have economic and financial significance on the dependent variable are given to GP. The best individual returned from the GP is the functional form indicated by real data, which links the financial asset price/return and their explanatory factors. GP has been applied to stock markets in the studies of [32]–[35] and in option pricing in [88], [97] and [108]. There is also GP's application in economic time series forecasting [11]. How powerful GP is depends on its settings include function set, terminal set, fitness function, operators settings and other parameters settings. The first three options should come from the area where it applied to. The GP parameter settings of the above applications are summarised in Tables 2.7 and 2.6.

Terminal Set The data from financial market has noisy property. Original time series do not tell GP too much. In most of these applications (Table 2.6), GP is fed with preprocessed data, such as lagged time series and normalised data. In [35] two process steps, data transformation and embedding are implemented before data is fed to GP. A wavelet analysis has been used to extract indicators from the raw index data in [32]. With these wavelet based indicators, GP returns improved results.

App.	Terminal Set	Function Set
[34]	Different frequency intraday data	+,-,×,÷,sin,cos,exponential
[31]	Lagged return and explanatory variables	+,-,×,÷,sin,cos,exponential,Sqrt,logarithmic
[35]	Preprocessed time series including volume	polynomials
[33]	Original time series and differed, average time series	+,-,÷,power, Sqrt,Logarithmic, Exponential
[32]	Wavelets based indicators, Prediction terminals(0 and 1)	I-T-E, And, Or, Not,<,>,≥,≤,=
[88]	$S/E, \tau, R_f, Randomnumber$	+,-,×,÷,sin,cos, Real Logarithmic, Exponential
[97]	S,E,S/E,τ, max(S-E,0)	+,-,×,÷,Natural Logarithmic, Exponential,Sqrt, NCDF
[108]	S/E, $R_f \times \tau$, $\sigma \times \sqrt{\tau}$	+,-,×,÷,Square, Exponential, NCDF
[11]	Economic indicators	+,-,×,÷,Sqrt, Exponential, sin,cos, logarithmic

I-F-E: if-then-else. C_M:Option daily closing price. S:Underlying stock index daily closing price on the trading day. E: The strike price of the index option.τ, The option's time to maturity in year. R_f, The 3-month treasure-bill annual rate. NCDF: Normal cumulative distribution function. σ: Implied volatility. Economic indicators: 29 economic indicators including employment financial, survey, production and sales and others for U.S. GDP; Unemployment rate and past values of the monthly inflation rate for U.S. CPI Inflation

Table 2.6: GP Time Series Forecasting Summary 2-1

Fitness Function GP is an objective driven random search. How good the solution is depends on the fitness measure. Mean Squared Error (MSE), Sum of Squared Error(SSE), Mean Absolute Percentage Error (MAPE) are common fitness functions for time series prediction. The logic here is to minimise the distance of predicted points and realised points.

GP applications in [97] and [108] have picked up the fitness function of the combination of absolute error and percentage error decided by the options special property. When the option is in-the-money (out-of-the-money), the option value is high (low) and the absolute error (percentage error) is more important.

The fitness function of Rational Average Error(RAT) in [35], not only minimises the above distance but also takes into account the size of the individual from GP, which is designed to relieve the overfitting problem. The fitness function in [32] looks at for a set tolerance region, how many points have fallen within the tolerance region and how many points are outside. The points outside the tolerance region represent missed opportunities in financial trading.

Part of the fitness function, Mean Absolute Deviation(MAD) in [11] is from the argument that the outliers from either noise or pattern shift tend to influence MSE more than they influence MAD. For series in which outlier data represent noise, MAD might be a more effective measure.

App.	Times Series	Fitness Function	Performance Measures	Benchmark
[34]	Nikkei225	*MSE*	MSE, HIT, Profit gain	ANNs
[31]	Dow Jones Stocks	*SSE*	*SSE*,STD,η,θ,R^2	-
[35]	Nikkei225	*RAT*	MSE,RSE,GCV,HIT,Profit Gain	Standard GP
[33]	DJIA	MSE/MAPE	Weighted Sum of Absolute Errors	-
[32]	DJIA	w1*RC-w2*RMC-w3*RF	RP, RMC, RC, TP, FP, TN, FN(p-value)	GP with different terminals
[88]	S&P 500 index options	SSE	MSE	Linear Regression, B-S, ANNs
[97]	Simulated Option Price	SAP	Hedging effects, AE, PE	Linear Regression, B-S, ANNs
[108]	FTSE 100 index futures options	Sqrt of average AE and PE	AE, PE	standard GP
[11]	U.S. GDP,CPI Inflation	MSE/MAD/CF	RMSE	Benchmark models

DJIA: Dow Jones Industrial Average index. η:The predictability measure, a percentage hypothetical reversed entropy when a shuffled series is put back to its original order. STD: Standard deviation. θ:A complexity statistic measure. *PC*: price-change *MSE*: Mean square error of predicted value and target data. ANNs: Artificial neural networks. *SSE*: Sum of squared errors *RAT*: Rational average error, a combination of an accuracy component reflecting the degree of fitting wildly varying series and a complexity measurement. RSE: Ratio of squared errors. HIT: the Hit percentage. GCV: the generalised cross validation.MAPE: Mean absolute percentage error. RC: Rate of Correctness. RMC: The rate of missing chance. RF:The rate of failure. w1,w2 and w3 are weights for RC, RMC and RF. RP: The rate of precision. TP:True positive. FP:False positive. TN:True negative. FN:False negative. B-S: Black-Scholes option pricing model. SAP: Sum of absolute dollar error and percentage errors.AE: Absolute error. PE: Percentage error. Sqrt: Square root. GDP: Gross Domestic Product. CPI Inflation:Consumer price index inflation rate. MAD: The mean absolute deviation. CF: A combination of MSE and MAD by a user specified threshold value. RMSE: Root mean squared error

Table 2.7: GP Time Series Forecasting Summary 2-2

Two applications, [11], [108] have been specially designed for dynamic financial markets where the environments change over time but not completely. In [108] the probabilities of crossover and mutation are adapted during the GP searching process. In [11], the size of data window fed to GP is sliding “on-the-fly”and past knowledge is retained by a “Dormant solution” where subtrees from a past search are kept and go alive when necessary. It shares a similar logic as Automatically Defined Functions (ADFs).

2.3.3 GP in Financial Trading

Technical analysis is not new in financial markets. It may provide a buy or sell indicator, however, it could not tell how much to buy or sell and it could not take into account the round trip cost and the whole portfolio optimisation. People also expect better rules from combining different technical indicators through mathematic and logic functions. An optimisation and model induction algorithm is needed to make the whole process automated in an efficient way. GP can combine existing domain knowledge into its searching, such as the traditional technical trading rules and fundamental information which can be embedded into the function set and terminal set. Trading and time series prediction have a significant difference as the most accurately predicted price point may not be the best buy/sell point. Trading needs to take into account the transaction costs and also the timing problem. Most GP applications in financial trading are in stock markets and foreign exchange markets. There are also GP applications in commodity/futures markets, such as [39].

Stock Markets

In the studies of [12], [21]–[24] and [26]–[30] GP has been applied in stock market trading. These applications cover US, Canadian and Asian markets.

Terminal Set The traditional technical indicators in financial trading have been used as the element in the terminal set, such as moving average rules, filter rules and trade range break rules in [30]. Sometime the technical indicators are embedded in the function set. Relative Strength Index (RSI) and Rate of Change (ROC) are included in the function set in [23]. There is some evidence from [21] and [22] that GP's searching efficiency can be improved through reducing operators in the function set and at the same time including more derived technical indicators in the terminal set.

App.	Terminal Set	Function Set
[12]	past prices, true, false, constant	Avg, maximum, minimum,+,-,$\times$,$\div$,Norm,I-T-E,and,or,not,$<,>$,lag
[26]	MA, L_{max},L_{min},lag of Stock Index,Stock Index	+,-,$\times$,$\div$,Norm,C(0,2), if-then,and,or,$<,>$, not, true, false
[24]	Positive, Negative, constant, 6-Indi.	if-then,and,or,not,$<,>$
[30]	Positive, Negative, constant, 6-Indi.	I-T-E,and,or,not,$<,>$
[21][22]	Tech-Indi.	AND,OR,NOT,$<,>$
[23]	price,volume,true,false,real in[0,250]	+,-,$\times$,$\div$,and,or,not,$<,>$,I-T-E,Norm, Avg, Max,Min,lag,volatility,RSI,ROC
[27][28]	Price,real constant in [-1,1] and integer constant in [0,1000]	+,-,$\times$,$\div$, norm, Avg, Max,Min,lag, and, or, not,$<,>$,I-T-E, true, false

MA: Moving Average. L_{max}:Local maximum. L_{min}: Local minimum 6-Indi.: 12-day Moving average, 50-day Moving average,5-day filter,63-day filter, 5-day Trading Range Breakout rule, 50-day Trading Range Breakout rule. Tech-Indi.: Prices(opening, closing, high, low of the current month), Moving Average(2,3,6 and 10 month),Rate of change(3 and 12 month),Price resistance markers: two previous 3-month moving average minima and two previous 3-month moving average maxima. Trend line indicators: a lower resistance line based on the slope of the 2 previous minima and a upper resistance line based on the slope of the 2 previous maxima.RSI:Relative strength index. ROC: Rate of change

Table 2.8: GP in Stock Market Trading Summary 3-2

Fitness Function The purpose of trading is making profit. Cumulative return, excess return over buy-and-hold strategy and portfolio value at the end of a trading period have been the most popular forms for fitness function shown in Table 2.9. Risk appetite has also materialised as the fitness function in [26] where different risk adjusted return measures and cumulative excess returns were used as fitness functions individually and compared with the effects from different fitness functions. Overfitting has been taken into account in the fitness consideration in [21] and [22]. There is also a study [30] using the linear combination of rate of correctness, the rate of missing chance and the rate of failure as fitness function. Different weights effects have been tested.

App.	Trading Markets	Fitness Function	Performance Measures	Benchmark
[12]	S&P 500 (1928-1995)	ER over B-A-H	Fitness, Ntr	B-A-H
[26]	S&P 500 Index(1929-1995)	Cum.ER,S-R,X^*,X_{eff}, X_{eff},Jensen's α	Fitness	B-A-H
[24]	DJIA(1969-1980)	–	PA and ARR	Technical trading rules (6-Indi.)
[30]	DJIA(1969-1981)	RC,$f_{(1)}, f_{(2)}$	RC, RMC, RF,AARR,RPR	3 types ANNs, linear classifier
[21][22]	S&P 500 (1954-2002)	PV,PV with C-P,Consist-f	Avg PV	B-A-H
[23]	14 stocks in TSE 300 index	ER over B-A-H	Fitness	B-A-H
[29]	S&P 500, S&P auto,S&P banks(1990-1999)	Max-diff	Gross%Returns, Risk adjusted % returns	B-A-H
[27][28]	TSE 300 index, Nikkei Dow Jones(Japan),TAIEX	Accumulated return	Avg total return	B-A-H

ER:Excess Return. Cum.:Cumulative S-R: Sharpe ratio. $S-R, X^* statistic, X_{eff} measure/Jensen's \alpha$: Different measures of risk adjusted returns. DJIA: Dow Jones Industrial Average index. PA: Prediction accuracy, the percentage of correct predictions. ARR: annualised rate of return. AARR:Average annualised Rate of Return. RPR: Ratio of positive returns.$f_{(1)}$:w1*RC-w2*RMC-w3*RF, w1,w2,w3 are weights for RC,RMC and RF.$f_{(2)}$: constrained $f_{(1)}$ with $R = [P_{min}, P_{max}]$, R defines the minimum and maximum percentage of recommendations that GP has to make. Weights approached by trial and error. 6-Indi.: 6 trading indicators include 12 days Moving Average, 50 days moving average, 5 days filter rule, 63 days filter rules, 5 days trading range breakout rule(the difference of today's price and previous 5 days maximum price), 50 days trading range breakout rule. Avg:Average. Ntr: Number of Trades. PV: Portfolio value produced by the rule at the end of the in-sample period.C-P: complexity-penalising factor. Consist-f: Consistency of performance fitness function, calculated as the number of periods, modified by the factor with 12 month. Well performing if it beat or equal to buy-and-hold return and the risk-free interest rate. B-H: Buy-and-hold. Max-diff:maximise the difference between buying and selling price. TAIEX:the Capitalisation Weighted Stock Index(Taiwan)

Table 2.9: GP in Stock Market Trading Summary 3-1

In stock market trading, GP's performance has been compared with individual technical indicators and buy-and-hold strategy which are traditionally benchmarks in this area. According to [26] [29] there has been no evidence that GP deduced rules can outperform buy-and-hold on a risk-adjusted basis. There are positive results from [24], where the evolved GP strategies can achieve better performance which cannot be obtained by individual technical trading rules. In [23], the GP trading rules generate excess returns over the buy-and-hold approach for the scenario where the buy-and-hold approach generates small positive returns close to zero or negative returns. Not all trading rules evolved by GP bring excess returns and some of them could not beat the simple buy-and-hold strategy. The buy-and-hold strategy may not be the right *benchmark* as it works in bull markets but not in bear markets. This fact is considered by [27] and [28] who have proposed the use of "pretest" information to answer two questions when applying GP to technical trading. The first is whether profitable trading strategies exist (whether there is anything to learn). The second question is whether or not GP machinery is working properly. Experiments in the Canadian market and Asian market show when there is nothing to learn, GP performs very poorly. On the contrary when the pretest indicator suggests that there is something to learn, GP is more efficient.

Foreign Exchange(FX) Markets

The FX Market is the largest and most liquid financial market in the world. Its low margin compared with other markets means that it is inevitable to pick up good optimisation tools when trading currencies. According to [20], more than 90 percent of surveyed foreign exchange dealers in London use some form of technical analysis to help with their trading decisions. GP has been applied in FX market trading in a number of studies. A sample of them includes [14], [15], [19] and [13].

Terminal Set Similar to the applications reviewed in stock market trading, technical indicators have been used as the terminals in FX market trading. Besides moving average, relative strength index used there, price channel breakout, the commodity channel index, the stochastic have been included in the terminal set [13], [19]. The FX market has a special participant compared with the stock market, central bank intervention. The US authorities' intervention data has been included in the terminal set in the study of [18] to explore the relation of currency rate change and authority intervention. The impartibility of interest rate and currency exchange rate has decided that the interest rate is a good explaining variable for FX trading [17], [16].

App.	Terminal Set	Function Set
[14]	open/closing price,highest/lowest price during the day	lag, fuzzy operators(and,or,not)
[15]	NER, constants	$+,-,\times,\div$,norm, average, Max,Min, lag, and, or, not,$<,>$,if-then, I-T-E, true, false
[25]	Pre-optimised momentum indicators,RAND in [-2,2]	$+,-,\times,\div$,ABS, Max,Min,and, or, not,$<,>$,if
[17]	NER, interest rate data , constants	$+,-,\times,\div$,norm, average, Max,Min, lag, and, or, not,$<,>$,if-then, I-T-E
[19]	Technic indicators(SMA,AMA,PCB,The stochastic, RSI, CCI)	and, or, xor
[18]	NER, US authorities intervention data, constants between(0,6)	$+,-,\times,\div$,norm,MA,L_{Max} ,L_{Min}, lag of data, and, or, not,$<,>$,if-then,true, false
[13]	AMA,CCI,RSI	–
[16]	NER, interest differential, the hour of day	–

NER: normalised exchange rate time series. SMA:Simple moving average crossover. AMA: adaptive moving averages. PCB:Price channel breakout. RSI: the relative strength index, CCI:The commodity channel index.

Table 2.10: GP in FX Market Trading Summary 3-1

Fitness Function Excess returns and risk adjusted returns are popular fitness functions as they are the trading objects no matter in which financial markets. However, the return from long position, percentage of long positions in total trades and break even traction cost have been used more often to valuate the performance of GP returned trading rules in the FX market. Maximum cash drawdown has been used as risk criteria in [19], [13]. GP performance is benchmarked with a traditional econometric model, GARCH and ARMA in [15].

Advanced GP A parallel GP is applied by [25] consisting of multiple master-slave instances to evolve trading models in the FX market. By allowing the subpopulation to exchange individuals asynchronously through the ring topology the premature convergence problem has been reduced. Another parallel optimisation is tested in [19]. It simultaneously starts on a number of identical machines. This parallel implementation speeds up without compromising the nature of the results.

App.	Trading Markets	Fitness	Performance Measures	Benchmark
[14]	US T.Bond, Nikkei, FTSE, S&P, DM	MPLE	annualised profit	–
[15]	(DM,JPY,GBP,CHF)/USD, DM/JPY,GBP/CHF(1975-1995)	ER	mean return,monthly Std, S-R, Ntr, PtgL	Random walk, ARMA(2,2), GRACH(1,1)-ARMA(2,2)
[25]	(GBP,DM,ITL,JPY,CHF,FRF,NLG) / USD(1987-1994)	X'_{eff}	Avg return, X_{eff},Avg complexity	–
[17]	DEM/(USD,NLG,FRF,ITL,GBP) (1979-1996)	ER	mean return,monthly Std, S-R, Ntr, PtgL, Market timing test	MA, filter rules
[19]	GBP/USD tick data(1993-1997)	MSR	Cum. profit,Stirling Ratio	–
[18]	(DM,JPY,GBP,CHF)/USD(1975-1998)	ER	Mean annual ER, Std, S-R, Ntr, PtgL	–
[13]	AUS/USD(1993-2001)	MSR	Trading activity, trading profit	AMA,CCI,RSI
[16]	(DEM,JPY,GBP,CHF)/USD(1996 intraday data)	ER	Annual return, Ntr, PtgL, log return break-even transaction cost	linear forecasting models

MPLE:A mixture of final profit and linearity of the equity curve X_{eff}:risk adjusted return by taking account of variance of total return over time. PtgL:percentage of long position. X'_{eff}:is a transformed form of X_{eff}.Std: standard deviation. Break-even transaction cost is the one-way transaction cost which reduces the annual excess return during the test period to zero. Tick data: A new data point or tick, is recorded with every change in price. MSR:Modified Stirling ratio, the ratio of return against modified max drawdown.

Table 2.11: GP in FX Market Trading Summary 3-2

2.3.4 GP in Other Financial modelling area

This section reviews GP applications in other financial modelling areas including discovering arbitrage opportunity and portfolio management. Its application in volatility modelling is reviewed in Section 4.3.4.

Arbitrage trading seeks to make profits by exploiting price differences of identical or similar financial instruments, on different markets or in different forms. GP has been used to discover arbitrage opportunities in call, put options and futures by [37]. The challenge is to not only spot profitable opportunities, but also to discover them ahead of other arbitrageurs. This study showed that GP is a promising interactive tool that brings human users and the computer program together to discover arbitrage opportunities.

One of the classic multiple objective optimisation problems in finance is portfolio selection, where the object is to invest a fixed amount of money in a diverse set of assets so as to maximise return while minimising a risk measure. The portfolio should also meet the investor's risk preference. The study of [38] has tried to use GP as a robust stock selection method in the Malaysian stock market. GP is used to evolve a nonlinear factor model for stock selection, coupled with an investment simulator that models a long-short, market neutral and sector neutral hedge fund trading contracts for difference (CFDs). A special fitness function has been designed to assess the robustness of an individual when it is exposed to multiple training scenarios.

2.3.5 Section Summary

The widespread application of GP in finance is not surprising given the nature of financial modelling, being typically undertaken in a data-rich, theory poor environment. It is also notable that the availability of data and the increasing power of computers alter the relative costs of theory vs inductive modelling methodologies, which helps to explain the growth of GP type applications in finance. The financial returns to even small edges in asset pricing / trading systems also explain the industry's enthusiasm for the application of GP methods.

From the literature review, we can see that GP has special utilities for the financial area. It is a stochastic optimisation model induction tool. Compared with other non-parametric methods, its advantage is model induction which gives a function shape among factors. Financial modelling is a suitable environment. Firstly, there are circumstances to explore the interrelationship among different factors where traditional analytical hardly work, such as derivative security pricing. Secondly, an approximate solution is acceptable as long as the profit is above a threshold. Thirdly, small improvements in performance will be highly prized. In leveraged trading environments, a small improvement in return can potentially bring huge profits. Last but not least, GP's special tree format allows for keeping part of the tree for future use. This allows for the storing of information for future use, which can be valuable in a complex, dynamic environment such as a financial market, where circumstances change but not completely.

2.4 Chapter Summary

In this chapter, GP is firstly introduced in Section 2.1, where its background, working process and parameter setting are explained. A demonstration of how to apply GP in the finance problem of option pricing is given in Section 2.2, where GP is in a dynamic form and the probabilities of crossover and mutation adapt during the run. GP's applications in the finance area are reviewed in Section 2.3. The focus of this review is to provide an experienced reference regarding how to efficiently apply GP in finance problems.

This chapter has made two contributions. Firstly, GP's applications in financial modelling are reviewed. GP parameter settings used in the reviewed applications are summarised, which provide references to efficiently apply GP in this thesis. Secondly, a new form of GP is designed where the probabilities of crossover and mutation are adapted dynamically during the GP run. Besides the demonstration example given in option pricing, this new form of GP is also suitable for other financial complex problems, where local optimum exist.

Part II

Empirical Work

Chapter 3

Derivative Data Analysis

3.1 Introduction

In this thesis, two detailed financial modelling studies are conducted on two ultra-high frequency datasets of FTSE 100 index futures and options by a novel approach GP. The methodology, GP is introduced in Chapter 2. This chapter is devoted to explaining and analysing this one year high frequency dataset before we undertake the empirical modelling works. This is the first time that such detailed description has been given to a derivative market dataset. The background of the markets is given in Section 3.2. The FTSE 100 index options dataset is described in Section 3.3 and FTSE 100 futures dataset is introduced in Section 3.4. Their co-movements are examined in Section 3.5. Section 3.6 concludes the chapter.

3.2 Market Background

The London International Financial Futures and Options Exchange (Liffe) was established in 1982. It was taken over by Euronext in January 2002 to form a market called Euronext-Liffe. Euronext merged with the NYSE group in April 2007 to form NYSE-Euronext. Since 2000, all trading in financial contracts on Liffe takes place on an electronic limit order book system, which is called the Liffe Connect platform. All derivatives trading on the different Euronext satellite markets transferred to the Liffe Connect trading system from 2003.

In this market, orders are submitted electronically to a central limit order book. Incoming market orders are automatically matched with orders in the order book to produce trades. Orders are given priority according to price. Orders at the same price are filled in a pro-rata manner according to the order size. Only exchange members can submit orders directly to Liffe Connect. There are no designated market makers with special quoting obligations or privileges in the FTSE 100 index options. Any member firm can use software, known as automated price injection models that automatically generate order submissions or cancel outstanding orders. Information on the best quotes and depths as well as quotes and depths away from the best quotes is distributed in real-time via Liffe Connect to the members. Trading is anonymous both before and after a trade.

Our data is from Euronext-Liffe. This dataset consists of the records of all quotes and trades for all European-style FTSE 100 index option contracts and FTSE 100 index futures contracts in 2004.

Global trading of exchange-traded futures and options is growing fast. According to data provided by the US-based Futures Industry Association, global exchange derivatives volume rose from 2.4 billion contracts in 1999 to 9.9 billion contracts in 2005, representing an annual growth rate of 27 percent. The growth rate is much higher than the 8 percent in the global equity market over the same period according to the statistics of the World Federation of Exchanges (WFE). Euronext is ranked 4th by contract volume of financial derivatives based on the data of 2005. In the first half of 2006, Euronext-Liffe is ranked 3rd in the WFE and the International Options Market Association (IOMA) exchanges based on the notional value of FTSE 100 index futures [3].

3.3 FTSE 100 Index Option Data Description

3.3.1 Option Dataset Overview

In this one year intraday option dataset there are in total 75,755,106 records, in which 41,794,081 are from call options and 33,961,025 are from put options as in Table 3.1. There are different quotation types, ask quotation, bid quotation, trade and another two wholesale trading types K and S, which have higher trading sizes as shown in Table 3.1. The call option quotations are updated more frequently than the put option quotations, though put options have been traded more frequently than call options. There are around 21 million bid/ask quotation updates for call options and around 17 million for put options. However, there are around 78 thousand trading records for put options, which is 12 percent more than that of call options. The average quoted depth and trading volume of put options is also higher than those of call options. The average traded price for both call (171.5) and put (116.8) options is significantly below the average ask quotation (413.5 for calls and 251.5 for puts) and bid quotation (404.0 for calls and 243.5 for puts). This is because only a portion of quoted contracts are traded, as can be seen from the large number of quotation records. There are around 41.7 million updated quotation records for calls and 33.8 million updated quotations for puts. However, the number of trading records for both call and put options is only around 0.15 million. This information is illustrated in Figure 3.1, where the daily numbers of different quoted and traded contracts are given. The trading hours for FTSE 100 index options are from 8:00 to 16:30. Within the 75,755,106 traded quotes, 22,221 are outside of this trading time period and are therefore deleted. Also, the 69,235 records from the wholesale quotations “S” and “K” are also excluded in the following data analysis.

Quote	Records		Mean Q-D/T-V		Std. Q-D/T-V		Mean Quot./Price		Std. Quot./Price	
	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
Ask	20,837,796	16,969,620	22.0	25.0	23.9	26.2	413.5	251.5	414.9	195.8
Bid	20,852,385	16,878,053	22.3	26.1	24.2	28.0	404.0	243.5	411.3	190.3
Trade	69,926	78,091	17.8	22.5	50.8	62.2	171.5	116.8	244.2	148.6
K	404	615	2,748.4	2,917.9	1,655.4	2,194.7	187.5	153.3	285.9	397.6
S	33,570	34,646	144.4	190.2	350.9	456.6	131.7	109.9	248.1	173.6
Total	41,794,081	33,961,025								

Table 3.1: Option Raw Dataset Quotation and Trading Information Summary

This table gives the number of quotation/trading records, mean of *Q-D/T-V*, standard deviation(*Std.*) of *Q-D/T-V*, mean of *Quot./Price* and standard deviation(*Std.*) of *Quot./Price* for different quotation types for call options and put options. *Q-D/T-V* is Quoted depth for *ask* and *bid* quotation type and trading size for *Trade*. The number of records counts quotation updates for *bid* and *ask* and trades for *Trade* and other wholesale trading type(K, S).

3.3.2 Option Contracts Characteristics

Option contracts are characterised by contract maturity, option type and option strike price. At any time in the market, there are call option contracts and put option contracts written on different maturity dates with different strike prices. Table 3.2 lists all available option contracts in this one year option dataset, which have been actively quoted or traded. There are 23 contract maturity months listed in the first column. For one maturity time there are multiple strikes available. To save space, not all available strikes are listed for each maturity under both call and put options. Instead, the maximum, minimum, mean and the total number of strikes available for each maturity date for both call and put options are listed. There are altogether 1,667 contracts, in which 794 are call options and 873 are put options. Take the maturity Jan-04 for example, there are 26 different strike prices for call options with maturity in Jan-2004 and 34 different strike prices for put options with maturity in Jan-2004. In total, there are 60 different option contracts with the same maturity date Jan-04. Within the call option contracts with the same maturity in Jan-2004, the maximum strike is 4875, the minimum strike is 2235 and the mean of strikes is 4381.6

From the last two columns in Table 3.2, it is observed that for the same maturity date there are in general more strikes quoted for put option contracts than there are for call option contracts. There are only six exceptions. Within these six cases, for contract maturity dates Oct-04 and Jun-05, call options and put options have the same number of strikes quoted; for another four contract maturity dates Dec-04, Sep-05, Mar-06 and Jun-06, call options have marginally more strikes than put options. The range of available strike prices for both call options and put options are the same in the majority of short-term contracts. It should be noted that option contracts that are not actively quoted or traded are not listed in this table. Therefore the exception cases may be due to deep in-the-money (ITM) options that are less actively quoted/traded, therefore the information is not in this one year dataset. According to the put-call-parity, call option prices that are deep in-the-money have a one-to-one relationship with put option prices of the same strike price and maturity that are deep out-of-the-money (OTM). However, the liquidity for in-the-money options and for out-of-the-money option contracts is not symmetrical, therefore the contracts quotation and trading information for call options and put options are not fully symmetrical.

Contract	Max Strike Price		Min Strike Price		Mean Strike Price		Diff. Strike No	
Maturity	Call	Put	Call	Put	Call	Put	Call	Put
Jan-04	4875	5275	3325	3325	4381.6	4532.2	26	34
Feb-04	5325	5325	3325	3325	4402.2	4469.6	36	39
Mar-04	5525	5525	2325	2325	4410.1	4507.4	45	63
Apr-04	5325	5325	3325	3325	4316.3	4521.1	38	41
May-04	5425	5425	3425	3425	4201.3	4610.6	40	41
Jun-04	5525	5525	2525	2525	4276.5	4534.5	44	59
Jul-04	5425	5425	3425	3425	4241.1	4541.5	35	37
Aug-04	5525	5525	3025	3025	4186.4	4441.8	41	43
Sep-04	5525	5525	2525	2525	4275.0	4382.2	46	55
Oct-04	5525	5525	3325	3325	4365.8	4552.5	45	45
Nov-04	5325	5425	3025	3025	4527.9	4605.0	43	47
Dec-04	5625	5625	2525	2525	4506.2	4616.1	61	58
Jan-05	5525	5525	3525	3525	4711.5	4669.1	39	40
Feb-05	5425	5525	3625	3625	4772.6	4713.3	32	35
Mar-05	5825	5825	2525	2525	4720.4	4476.0	46	50
Apr-05	5425	4925	4525	3725	4806.7	4682.3	19	25
Jun-05	5825	5625	2525	2525	4732.8	4233.0	32	32
Sep-05	5825	5525	2525	2525	4813.7	4151.8	32	31
Dec-05	5825	5825	2525	2525	4802.5	4059.9	32	34
Mar-06	6225	6025	3725	3625	5137.0	4129.1	26	24
Jun-06	5625	5325	3725	3525	5000.8	4105.3	19	18
Sep-06	5725	5425	4625	3725	5263.2	4065.2	11	15
Dec-06	5825	4725	4725	3825	5672.2	4109.0	6	7
The Number of Contracts for Calls and Puts							794	873

Table 3.2: Option Contracts in Raw Dataset

3.3.3 Daily Information

On each trading day all available contracts may get updated quotations. However, not all available contracts are traded as indicated by the large quotation numbers in Table 3.1. Graph 3.1 gives the number of different contracts that have been actively quoted from the ask side and the bid side as well as the number of different contracts that have been traded on each trading day in this one year dataset. In the majority of days, only less than half of the quoted contracts have been traded. Take the first day 2-Jan-2004 for example; there are 187 different option contracts with an updated ask quotation, 145 different option contracts with an updated bid quotation and 87 different option contracts traded.

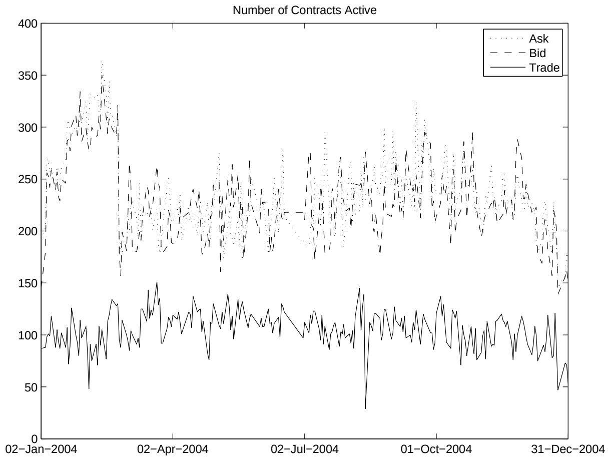


Figure 3.1: The Number of Active Option Contracts by Asks, Bids and Trades

This plot gives the number of actively quoted/traded option contracts.

Sample Day Options Characteristics

It is not possible to give the full picture of all options with different terms in the full year options dataset. Therefore the first day, 02-Jan-2004, is used as a sample to review the quotation/trading information for all traded contracts. The options' moneyness, time-to-expiry, quotation and trading information for call options and put options traded on the first day of this dataset are given in Table 3.3 and Table 3.4 separately. The option moneyness is the average underlying price (4484.99) on 02-Jan-2004 divided by the strike price. The underlying prices on 02-Jan are implied from the FTSE 100 index futures prices (which will be explained later). The time to expiration is the time between the current date and the contract expiration date. Take the contract maturity Jan-04 for example, it is the days between 02-Jan-2004 and the contract expiration date, which is the third Friday (16-Jan-2004) in the expiration month. The quotation and trading information given includes the number of trading transactions /quotations updates, total volume/quoted depth and average trading/ask/bid prices. The indicated bid-ask spread is just the difference between the average ask quotation and bid quotation. It is a very rough calculation and is only used for indication purposes. For example, the call option contract with a maturity time Jan-04 and a strike price 4225 has 5 trading transaction records, 633 asking quotation updates and 609 bid quotation updates on 02-Jan-2004 in Table 3.3. The total trading volume is 22, the total ask quoted depth is 4,039 and the total bid quoted depth is 3,757. Its average traded price is 563.4, the average ask price is 570.8 and the average bid price is 564.1 on this day. The indicated bid-ask spread is 6.7.

Call Options				Trades			Ask Quotes			Bid Quotes			Ind.
Maturity	Strike	S/K	DTE	No.	T. Volume	Avg P	No	T. Q-D	Avg.P	No.	T. Q-D	Avg. P	SP
Jan-04	4225	1.06	14	5	22	563.4	633	4,039	570.8	609	3,757	564.1	6.7
Jan-04	4275	1.05	14	4	12	468.3	2,118	19,856	479.6	2,750	23,628	470.1	9.6
Jan-04	4325	1.04	14	8	107	366.4	2,625	19,753	378.1	2,798	23,162	373.1	5.0
Jan-04	4375	1.03	14	9	88	282.4	2,579	26,818	285.7	2,872	27,541	278.8	6.9
Jan-04	4425	1.01	14	21	109	186.9	2,519	34,550	201.9	2,801	31,852	195.6	6.3
Jan-04	4475	1.00	14	24	181	119.6	2,199	39,268	127.9	2,699	48,047	123.4	4.5
Jan-04	**4525**	0.99	14	40	293	71.3	1,374	27,581	74.3	1,914	41,944	69.5	4.8
Jan-04	4575	0.98	14	21	109	36.9	1,006	30,264	38.9	974	26,337	34.5	4.4
Jan-04	4625	0.97	14	15	352	18.7	285	11,687	19.4	383	10,348	16.0	3.4
Jan-04	4675	0.96	14	6	152	9.0	93	2,399	8.8	97	2,978	5.3	3.4
Jan-04	4725	0.95	14	3	7	4.0	84	895	15.0	11	56	2.8	12.2
Jan-04	4775	0.94	14	10	30	2.2	10	193	5.4	22	287	1.5	3.9
Feb-04	4325	1.04	49	2	9	412.0	1,768	18,220	442.3	1,731	17,562	428.2	14.1
Feb-04	4375	1.03	49	2	20	347.0	1,487	31,250	358.7	1,477	21,630	352.2	6.5
Feb-04	4425	1.01	49	1	1	277.0	1,537	33,250	285.1	1,552	29,151	278.2	6.9
Feb-04	4475	1.00	49	3	17	213.0	904	19,545	222.6	915	19,146	216.0	6.6
Feb-04	4525	0.99	49	7	34	167.4	738	18,886	165.3	781	19,261	156.8	8.5
Feb-04	4575	0.98	49	2	4	110.0	139	6,758	117.8	123	5,757	110.4	7.3
Feb-04	4625	0.97	49	13	86	77.8	470	10,267	81.8	490	14,848	75.2	6.6
Feb-04	4675	0.96	49	5	21	50.6	160	4,920	55.5	169	7,543	48.1	7.3
Feb-04	4725	0.95	49	10	143	33.8	179	6,017	36.6	326	12,327	31.3	5.4
Feb-04	4775	0.94	49	3	46	20.3	20	309	25.6	41	712	8.7	16.9
Feb-04	4825	0.93	49	1	8	10.0	15	225	23.8	12	110	2.7	21.1
Feb-04	4925	0.91	49	1	100	5.0	4	55	12.0	1	100	5.0	7.0
Mar-04	4425	1.01	77	2	7	292.5	1,318	14,406	310.3	1,451	23,010	300.7	9.6
Mar-04	4475	1.00	77	1	4	240.0	1,184	18,109	249.0	1,296	15,410	241.0	8.1
Mar-04	4525	0.99	77	6	17	189.0	1,190	20,188	194.8	1,076	20,592	187.3	7.5
Mar-04	4575	0.98	77	7	150	138.9	934	27,169	150.6	773	30,268	143.4	7.2
Mar-04	4625	0.97	77	2	9	106.0	630	9,201	113.8	607	10,023	106.8	7.1
Mar-04	4675	0.96	77	1	2	81.0	152	7,862	82.1	131	6,550	75.8	6.3
Mar-04	4725	0.95	77	6	35	60.2	89	1,305	66.0	73	711	52.6	13.4
Mar-04	4825	0.93	77	2	16	24.5	10	310	32.0	6	56	12.8	19.2
Apr-04	4525	0.99	105	1	2	224.0	88	4,172	246.4	102	4,920	228.3	18.1
Apr-04	4725	0.95	105	2	20	88.5	4	49	111.0	5	54	68.6	42.4
Apr-04	4825	0.93	105	1	15	44.0	-	-	-	1	15	44.0	-
Jun-04	4225	1.06	168	1	1	736.0	3	39	767.3	5	69	707.6	59.7
Jun-04	4525	0.99	168	26	550	325.1	131	4,430	330.5	122	4,635	312.7	17.9
Jun-04	4625	0.97	168	1	1	232.0	89	3,951	236.4	57	2,400	208.5	27.8
Jun-04	4725	0.95	168	1	10	150.0	21	680	167.5	97	4,226	143.4	24.0
Sep-04	4425	1.01	259	1	2	536.0	10	47	613.3	8	40	449.3	164.1
Dec-04	5025	0.89	350	1	5	146.0	1	5	146.0	-	-	-	-

Table 3.3: Call Option Contracts Traded on 02-Jan-2004

This table gives aggregated trading and quotation information for each call option contract traded on 02-Jan-2004. *S/K* is the option's moneyness. *DTE* is days to expiration. *No* is the number of transactions under the *trades* section; and it is the number of quotation updates under the section of *ask quotes* and *bid quotes* for each contract. *T volume* is the total trading volume. *T Q-D* is the total quoted depth for bid and ask quotations for each contract. *Avg. P* is the average trading/ask/bid price for each contract on this trading day. *Ind. SP* is the indicated bid-ask spread.

Put Options				Trades			Ask Quotes			Bid Quotes			Ind.
Maturity	Strike	S/K	DTE	No.	T. Volume	Avg P	No	T. Q-D	Avg.P	No.	T. Q-D	Avg. P	SP
Jan-04	3875	1.16	14	1	30	3.0	18	665	5.8	6	144	1.8	4.0
Jan-04	3925	1.14	14	1	30	3.0	27	644	40.4	9	360	2.0	38.4
Jan-04	3975	1.13	14	1	70	3.0	19	735	6.4	8	235	2.3	4.2
Jan-04	4025	1.11	14	1	5	4.0	14	560	6.7	56	1,101	3.1	3.7
Jan-04	4075	1.10	14	1	100	4.0	22	248	64.1	52	649	3.2	60.9
Jan-04	4125	1.09	14	6	22	5.5	34	391	9.2	56	1,654	4.4	4.8
Jan-04	4175	1.07	14	1	1	7.0	19	247	11.3	52	2,776	5.8	5.5
Jan-04	4225	1.06	14	7	202	8.1	157	2,397	9.6	180	7,120	6.3	3.3
Jan-04	4275	1.05	14	9	311	11.1	264	4,079	13.2	195	4,380	8.9	4.3
Jan-04	4325	1.04	14	10	80	15.6	241	7,622	17.6	267	7,161	13.0	4.6
Jan-04	4375	1.03	14	8	30	23.5	549	17,655	25.0	518	13,975	21.0	4.0
Jan-04	4425	1.01	14	16	105	37.8	885	26,025	40.1	889	26,841	37.0	3.0
Jan-04	**4475**	1.00	14	18	88	63.9	1,344	36,449	67.8	1,417	37,981	64.6	3.3
Jan-04	4525	0.99	14	14	61	115.4	1,826	36,749	113.5	2,362	43,481	110.2	3.3
Jan-04	4575	0.98	14	11	47	172.9	1,948	35,483	173.4	2,554	44,255	170.7	2.7
Jan-04	4625	0.97	14	5	31	258.0	2,382	26,621	258.7	2,743	24,774	251.3	7.4
Jan-04	4675	0.96	14	1	3	331.0	2,066	14,291	343.5	2,490	16,083	336.7	6.8
Feb-04	3625	1.24	49	1	1	5.0	12	92	9.2	-	-	-	-
Feb-04	3825	1.17	49	2	14	9.5	16	148	14.6	8	140	6.5	8.1
Feb-04	3875	1.16	49	1	1	10.0	3	30	17.0	6	55	6.8	10.2
Feb-04	4025	1.11	49	3	25	19.3	31	382	23.4	27	590	9.6	13.8
Feb-04	4125	1.09	49	1	3	24.0	9	90	39.0	13	288	15.1	23.9
Feb-04	4225	1.06	49	1	2	42.0	18	307	49.6	29	1,008	29.2	20.4
Feb-04	4275	1.05	49	3	12	51.0	411	4,112	52.7	315	3,269	45.4	7.3
Feb-04	4325	1.04	49	5	26	59.6	410	6,300	65.9	245	8,564	59.5	6.4
Feb-04	4375	1.03	49	3	11	87.0	737	18,925	85.8	547	21,590	79.4	6.4
Feb-04	4425	1.01	49	6	13	106.2	802	14,592	112.0	773	25,958	105.0	7.1
Feb-04	4475	1.00	49	5	16	145.2	145	6,426	146.0	113	5,593	137.3	8.7
Feb-04	4525	0.99	49	6	17	183.2	179	8,151	186.7	244	10,621	179.3	7.4
Feb-04	4625	0.97	49	1	2	311.0	111	5,071	306.7	108	5,326	291.9	14.8
Mar-04	3625	1.24	77	8	591	12.0	5	470	12.0	17	1,055	11.1	0.9
Mar-04	4125	1.09	77	6	25	52.3	27	270	70.0	34	352	42.6	27.4
Mar-04	4225	1.06	77	4	11	81.5	139	924	86.4	144	1,221	71.2	15.2
Mar-04	4325	1.04	77	2	6	112.5	78	2,797	125.3	163	7,603	109.6	15.7
Mar-04	4375	1.03	77	6	129	145.7	981	12,206	143.4	658	24,390	136.0	7.4
Mar-04	4425	1.01	77	4	109	176.8	877	8,918	175.2	877	27,901	167.1	8.1
Mar-04	4525	0.99	77	1	4	268.0	1,310	25,987	262.3	1,164	17,850	251.8	10.5
Mar-04	4625	0.97	77	1	2	358.0	22	160	379.7	4	59	358.8	20.9
Apr-04	4125	1.09	105	1	4	82.0	7	82	99.0	4	44	59.3	39.8
Jun-04	2525	1.78	168	4	110	6.0	12	650	8.8	1	50	4.0	4.8
Jun-04	2625	1.71	168	1	50	7.0	8	310	9.6	1	-	-	-
Jun-04	3725	1.20	168	2	20	58.5	2	20	59.0	1	10	59.0	0.0
Dec-04	2925	1.53	350	1	50	44.0	1	50	44.0	1	-	-	-
Dec-04	3025	1.48	350	1	50	52.0	2	100	52.5	1	-	-	-
Dec-04	3525	1.27	350	1	1	110.0	3	3	114.7	1	-	-	-
Sep-05	3025	1.48	623	1	42	98.0	1	-	-	6	258	97.7	-

Table 3.4: Put Option Contracts Traded on 02-Jan-2004

This table gives aggregated trading and quotation information for each put option contract traded on 02-Jan-2004. Column names refer Table 3.3.

Options Sample Analysis

Most Actively Traded Options

ATM options have been traded more actively. The number of trading records, the number of bid and ask quotation updates, the total trading volume and bid and ask quoted depth summarise the activity for an option contract. The relatively large numbers of trading records and bid and ask quotation updates in these columns are concentrated in the shortest time-to-expiry option contracts in the top part of the table, which have 14 days to expiry for both call options and put options. Within these 14 days to expiry options, options having a moneyness near one are traded more often than others. For call options, when moneyness decreases from 1.06 to 1, the number of transactions increases, while when moneyness increases from 0.94 to 1, the number of transactions increases. There are two exceptions at strike prices 4225 and 4775. Because this is only from one day in the dataset, exceptions are expected. The same trend is also observed for put options contracts although OTM contracts are more active than ITM.

Option Moneyness Distribution

Call (put) options with moneyness less than one are OTM (ITM) options; greater than one are ITM (OTM) options; around one are ATM options. The call options' moneyness in Table 3.3 are in the range of 0.89 to 1.06. For put options, moneyness in Table 3.4 are in the range of 0.96 to 1.78. In both cases, moneyness is skewed to OTM options. This indicates that the most actively traded options are OTM and ATM options for both calls and puts.

Option Price by Option Moneyness and Time-to-expiry

The options moneyness indicates the options intrinsic value. Therefore the option traded price is expected to be consistent with the option moneyness. For call options with the same time-to-expiry call options, as moneyness increases, the average traded price increases as seen in Table 3.3. For put options when moneyness decreases the average traded price increases as seen in Table 3.4. For options with the same moneyness, options with a longer time to expiration are expected to be more expensive than the shorter ones. In Table 3.3 the average prices for three call options with the same moneyness of one, and different maturity Jan-04, Feb-04 and Mar-04 are 119.6, 213.0 and 240.0. The data is consistent with expectations.

Option Traded Prices and Bid-ask Quotes

The average traded price is not always in between the average ask price and bid price as seen in Table 3.3 and Table 3.4, although these quotations are for the same traded contracts. For example, the average traded price of 563.4 for call option contracts with maturity Jan-04 and strike price 4225, is less than its average bid price of 564.1. This is because the calculation includes not only the bid quotations at the time when the trading transaction happens, but also includes bid quotations during a no trading time period. There are only five transactions, which happened for this contract, while there are 633 ask quotation updates and 609 bid quotation updates during the first trading day.

Regularities in Bid-ask Spread

The last column in Table 3.3 and Table 3.4 is the indicated bid-ask spread which is the difference of the average ask price and average bid price for each contract. There are a number of regularities recognised from the indicated bid-ask spread, although it is a very rough calculation. When the option moneyness is the same, the longer time-to-expiry option has a broader bid-ask spread and the shorter one has a narrower bid-ask spread. For example, the spreads for call options, which all have the option moneyness equal to one but different maturity times, Jan-04, Feb-04 and Mar-04 are 4.5, 6.6 and 8.1. Within options having the same time-to-expiry, at-the-money (ATM) options have the smallest bid-ask spread. There are a few very large indicated bid-ask spreads in Table 3.4 for put options with 14 days to expiry. The largest bid-ask spread 60.9 is from a put option contract with maturity Jan-04 and strike 4075. This maybe due to the same reason discussed above that the calculation of average ask and bid quotation includes all quotations during the day and it is not only limited to the trading time period. Also this indicated bid-ask spread calculation is only a very rough one.

Unusual Option Trades

It is noticed in Table 3.3 that the largest total trading volume 550 is from the call option contract with maturity Jun-04 and strike 4525. This volume is much higher than that of the most active contact having the same strike 4525 and 14 days to expiry. It is also noticed that for this option, the number of trading records is 26 while the other call options with the same maturity only have one trading transaction. The number of ask quotation updates, 131 for this contract, is much higher than the number from the next active call option with the same maturity date, which is 89. The number of bid quotation updates, 122, is more than two times the number from the next being actively quoted contract, which is 57. The put option contract with the same maturity and strike was not traded on 02-Jan-2004. In Table 3.3, the put option contract with maturity Mar-04 and strike 3625 also has a high total trading volume 591. To confirm whether such a high volume number is due to a data mistake or just a special trading behavior, the call option contract with total volume 550 and the most active call option contract with maturity Jan-04 and strike 4525 as well as the most active put option with maturity Jan-04 and strike 4475 are selected to do further intraday analysis in the next section.

3.3.4 Intraday Information

The intraday information on 02-Jan-2004 for the three selected options is given in this section. Their aggregated hourly trading and quotation information are provided in Table 3.5. The number of total transactions, average trading volume, average traded price, the number of quotation updates, the average quoted depth and the average quotation price in each trading hour from 8:00 to 16:30 are given. All trades and quotation update are plotted in Figure 3.2. For each selected option contract, its intraday traded price, ask price and bid price are in one sub-figure while its trading volume, ask quote depth and bid quote depth are in a separate one.

Transactions and Quotations

The bid and ask quotations for the selected most active call and put options are updated frequently. However, the trading transaction only happens a few times per trading hour, though they are the most actively traded among all option contracts. In Table 3.5 there is a one hour interval from 8:00 to 9:00, when the selected most actively traded call option is not traded. For the most active put options on the same day there are two such intervals, which are at the beginning of the day from 8:00 to 10:00. The bid and ask quotations are updated very frequently in these intervals where trading has not occurred. This is also displayed in Figure 3.2a and Figure 3.2c. The quotation prices are dense all the time, while the transactions only scatter during the day.

Traded Price Range

The price of the selected most active call option closed higher than the opening price as shown in Figure 3.2a on 02-Jan-2004. The average traded price in the hourly interval of 8:00 to 9:00 is 59.3 and the average traded price in the closing hour interval of 16:00 to 16:30 is 76.8 in the top panel of Table 3.5. The first trade in the put option happens just before 11:00 and the price is 70.0 in Figure 3.2c. The average price in the closing hour is 61.3. Therefore the put option closed at a lower price than at its opening.

Intraday Trading Patterns

In Figure 3.2a, during the day the price is quite volatile. Starting from the beginning up to 11:00 the price steadily increases and is nearly flat. There are few trades that happen in the meantime. In the next two hours up to 13:00, 20 percent of the total transactions for the day happen and the price increases sharply and achieves the maximum price around 86 at 13:15. The bid-ask spread is very wide at the start in Figure 3.2a; then it decreases as the price increases. The bid-ask spread is very narrow at 11:00. As the price sharply increases the bid-ask spread increases slightly and then it narrows down again. A nearly zero bid-ask spread happens around 13:00, just before the maximum price is achieved. After that the price remains steady and slightly deceases in the following hour. At the same time the bid-ask spread becomes wider. Starting from 14:30 the price sharply deceases and the bid quotation achieves the minimum price during the day just before 15:00. In the following few minutes the price sharply increases and then deceases again. At around 15:30 the price starts to increase. There are no transactions happening in this volatile period from 13:30 to 15:30. One trade comes at 15:40 and around that point the bid-ask spread is very narrow and nearly zero. After that the price sharply increases and the bid-ask spread slightly widens. The last four trades happen just a few minutes before the market closes. The price slightly decreases and the market closes. From Table 3.5 the narrowest indicated bid-ask spread happens in the intervals of 10:00-11:00 and 13:00-14:00. This is consistent with what we observed from Figure 3.2a.

Traded Prices and Bid-ask Quotes

The traded price is always between the ask price and the bid price at the same time stamp. It is either very close to the ask price or very close to the bid. In Table 3.5, there are two exceptions for the put option. In the middle panel, at the intervals of 12:00-13:00 and 14:00-15:00, the average traded price is lower than the average bid price. To clarify this, the trading and quotation information of the put option for the time 12:00-15:00 is provided in Figure 3.3a. In this time period there are four trading transactions. For the first transaction, the traded price is nearly the same as the bid price. For the other three cases, the traded prices are the same as the bid prices. The reason for the average traded price being lower than the average bid price in those two intervals in Table 3.5 is that the calculation of the average bid price not only includes quotations updated during the trading time, but also quotations updated when no trades occur.

Call and Put Option Comparison

From the put-call-parity when the price call increases, the put moves in the opposite direction. This is confirmed in Figure 3.2c. There is one trade that happens for the put option around 12:40, which is in the time period of 13:30-15:30 when there is no trading happening for the call option. Around that point when the put trading happens, the bid-ask spread for the put option is very narrow and nearly zero. It is noted that around the same time, the call option bid-ask spread also narrows.

In general, the selected call option is more actively quoted and traded than the selected put option. This is indicated from the hourly number of transactions, the number of quotation updates and the indicated bid-ask spread for the call option and the put option in Table 3.5. This is consistent with the daily total number of transactions and quotations provided in Table 3.3 and Table 3.4. It appears from Figure 3.2a and Figure 3.2c that the bid-ask spread for the put option is much wider than for the call option. However, this may be only due to the graph scales used, as the bid-ask spread at the opening for the call option is very wide at around 70. The mean of the hourly average indicated bid-ask spreads in the top panel of Table 3.5 is 4.4 for the selected most active call option. The mean of the hourly average indicated bid-ask spreads in the middle panel of Table 3.5 is 4.5 for the most active put option. This indicates that the put option has a wider bid-ask spread than the call option. Compared with this, the average bid-ask spread is 3.3 for the most active put option in Table 3.4 and it is 4.8 for the selected call option in Table 3.3, which gives a different result. It should be noted that the first case calculated from the hourly intervals should be more accurate than that from the daily information. This is because the first case is calculated in more granular time intervals.

Bid and Ask Quote Comparison

It is indicated that on average the trading size is much smaller than the quotation size from Figures 3.2b and 3.2d as well as the average quote depth and the average trading volume in Table 3.5. For the selected most active call option, the number of bid quotation updates is more than that of the ask quotation in all nine hourly intervals. Also, for more than half of the hourly intervals the bid quoted size is bigger than the ask quoted size. It is interesting to note that the average traded price is closer to the average bid price than the average ask price except in the interval from 13:00-14:00. The average traded price, bid and ask are compared in Table 3.6, where the difference between the average traded price and the average bid and the difference of the average traded and the average ask as well as these two difference comparisons are provided. If the traded price is at the middle of bid and ask we would expect the *Diff* column to be zero. In Table 3.6, if the traded price is closer to the bid price then we expect that the *Diff* column will have a positive value. If the traded price is closer to the ask price then we expect that the

Diff column to have a negative value.

This pattern is also true for the selected put option. In six out of nine trading hour intervals the number of bid quotation updates is more than that of the ask quotation. Also, in six out of nine trading hour intervals, the average bid quoted size is bigger than the ask quotation size. In the majority of hourly intervals, the average traded price is closer to the average bid than the average ask price except for the hour intervals 11:00 to 12:00 and 16:00 to 16:30.

Unusual Option Trades

The trading and quotations of the selected call option with a total trading volume 550 on 02-Jan-2004, are in Figures 3.2e and 3.2f. The day trading volume 550 is from 26 trading transactions. Note that in Figure 3.2e eight of them happen between 13:19 and 13:37; one transaction is around 14:30; another eight happen from 15:41-15:59 and the other nine happen during 16:10 to 16:26. From Table 3.5, the average hourly trading volume is quite even and ranges from 18.1 to 25.6. From Figure 3.2f, there are 15 transactions with a trading size of 20, 5 transactions with a size of 15, 25 transactions with a trading size of 25 and one trade with a size of 50. The trading sizes of these 26 transactions are comparably even. The hourly average trading volume is bigger than that of the most actively traded call and put option except the one from the hourly interval of 15:00-16:00 when the average trading size is 30 for the most active call option in Table 3.5.

Different to the most active call and put options, there is no sign that the bid quotations are updated more actively than the ask quotations. Actually since the trading starts for this option the number of ask quotation updates in the hourly interval is more than that of the bid quotation except for the last hourly interval. It is indicated in Table 3.6 that there are two hourly intervals where the traded price for this contract is closer to the ask price.

This high trading volume does not like mistakes and it rather appears to be part of a trading strategy. The research in [5] indicates that the trading strategy in the FTSE 100 option market outside the exchange's special strategy trade facility, i.e. trading each leg as a regular option trade, could expose the trade to execution risk and a higher trading fee. The trading type for this selected option are just regular trades. However, it may be part of a strategy which is not listed in the exchange's special strategy trade facility. The current dataset does not include this level of detail for strategy trades, therefore it does not allow one to carry out further investigation.

Time	Trades			Ask Quotes			Bid Quotes			Ind.
H	No.	Vol.	P	No.	Q-D	P	No.	Q-D	P	SP
Call Option with Maturity Jan-04 and Strike Price 4525										
8	4	3.8	59.3	65	11.5	63.3	189	6.9	56.3	7.0
9	2	27.5	60.5	158	10.2	62.8	184	28.8	59.0	3.7
10	4	2.3	62.0	79	15.7	64.2	84	21.3	61.4	2.8
11	11	6.0	69.8	123	17.0	72.3	140	29.5	68.5	3.8
12	11	4.5	77.5	170	23.1	80.6	225	20.4	76.7	3.8
13	3	5.3	84.3	104	24.3	85.5	205	30.1	82.0	3.6
14				249	18.2	78.3	294	24.4	73.7	4.5
15	1	30.0	66.0	229	26.6	71.2	367	21.9	65.8	5.4
16	4	13.0	76.8	197	24.4	79.7	226	15.2	74.6	5.1
Put Option with Maturity Jan-04 and Strike Price 4475										
8				146	10.5	78.4	221	12.6	71.5	6.9
9				158	20.6	77.3	175	26.3	73.5	3.8
10	1	5.0	70.0	93	16.0	72.8	96	32.2	69.4	3.3
11	3	11.7	65.7	90	19.8	66.9	91	52.5	60.4	6.5
12	2	7.0	56.5	178	33.3	60.8	162	35.3	56.7	4.2
13	1	2.0	55.0	139	36.2	57.3	104	24.3	53.0	4.3
14	1	1.0	61.0	210	30.1	64.9	131	43.5	62.5	2.4
15	6	3.7	68.2	197	35.8	70.7	283	16.6	66.3	4.4
16	4	2.3	61.3	133	30.5	63.1	154	26.6	58.4	4.6
Call Option with Maturity Jun-04 and Strike Price 4525										
8				3	5.0	368.7	3	5.0	237.0	131.7
9				15	41.0	328.6	13	39.6	287.7	40.9
10				8	50.0	320.0	8	50.0	304.0	16.0
11				2	50.0	330.0	4	50.0	312.0	18.0
12				13	46.5	342.8	13	46.5	317.6	25.2
13	8	25.6	331.5	21	27.6	334.6	8	40.6	329.8	4.8
14	1	20.0	326.0	14	30.0	328.6	11	40.5	314.4	14.2
15	8	18.1	317.0	32	31.3	322.2	22	32.5	310.2	12.0
16	9	20.0	326.4	23	30.2	332.8	40	35.4	324.1	8.7

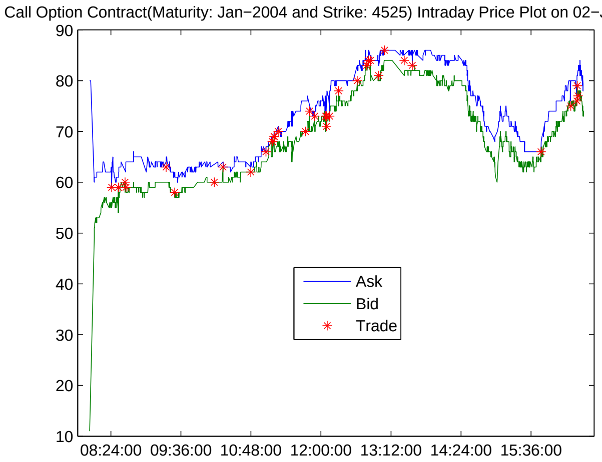
Table 3.5: Hourly Trading and Quotations for Selected Options

This table gives hourly quotation and trading information for selected option contracts on 02-Jan-2004. *H* is the trading hour. The hour 8 covers the time range of 8:00-9:00. For the hour of 16, the time range is 16:00-16:30. *No.* is the number of transactions in each trading hour; and it is the number of quotation updates for *ask quotes* and *bid quotes*. *Vol.* is the average trading volume. *Q-D* is the average quoted depth for bid and ask quotations. *P* is the average trading/ask/bid price per trading hour. *Ind. SP* is the indicated bid-ask spread calculated by subtracting the average bid quotation from the average ask quotation.

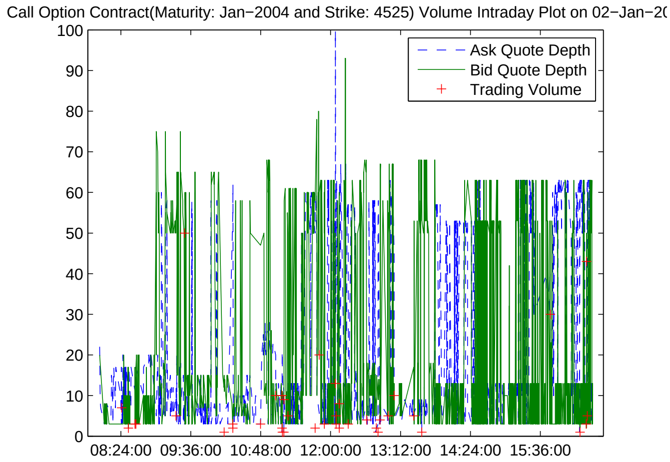
Time	Most Active Call			Most Active Put			Call T. Vol. 550		
H	T-B	A-T	Diff	T-B	A-T	Diff	T-B	A-T	Diff
8	2.92	4.07	1.15						
9	1.47	2.27	0.79						
10	0.63	2.16	1.53	0.56	2.78	2.22			
11	1.28	2.51	1.22	5.27	1.19	-4.08			
12	0.81	3.04	2.23	-0.17	4.35	4.18			
13	2.36	1.21	-1.15	2.03	2.27	0.24	1.75	3.07	1.32
14		-		-1.54	3.93	2.39	11.64	2.57	-9.07
15	0.17	5.21	5.05	1.86	2.54	0.68	6.77	5.22	-1.55
16	2.12	2.97	0.85	2.83	1.81	-1.02	2.37	6.34	3.97

Table 3.6: Hourly Traded-Bid-Ask Price Comparison

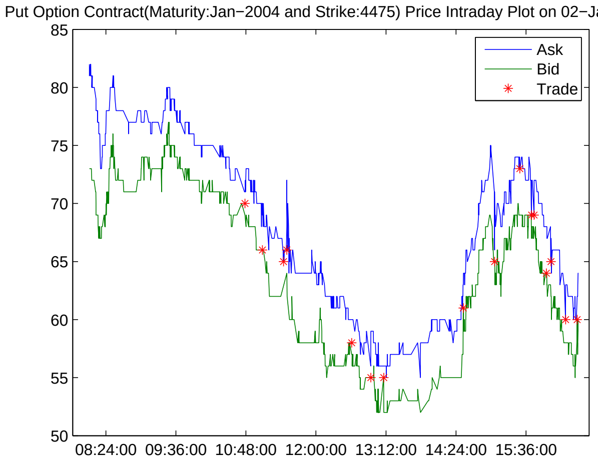
This table gives hourly average traded, bid and ask price differences for three selected options based on the information in Table 3.5. *T-B* is the average traded price minus the average bid price. *A-T* is the average ask price minus the average trade price. *Diff* is the absolute value of *T-B* minus the absolute value of *A-T*.



(a) Intraday Price of Selected Call Option

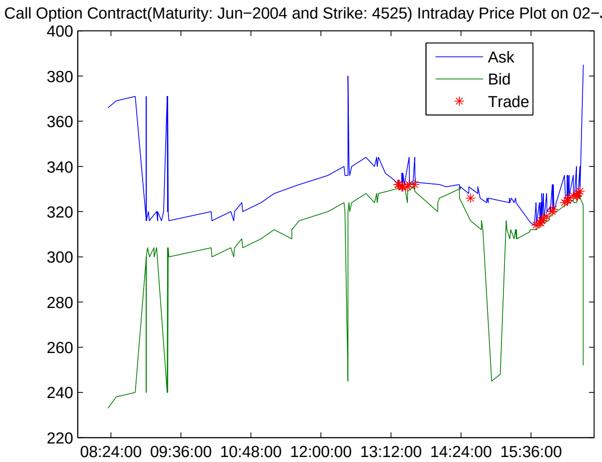


(b) Intraday Volume/Quoted Depth of Selected Call Option

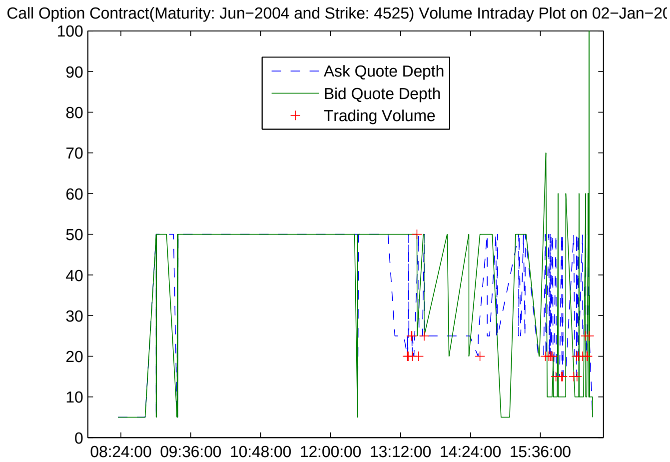


(c) Intraday Price of Selected Put Option

(d) Intraday Volume/Quoted Depth of Selected Put Option

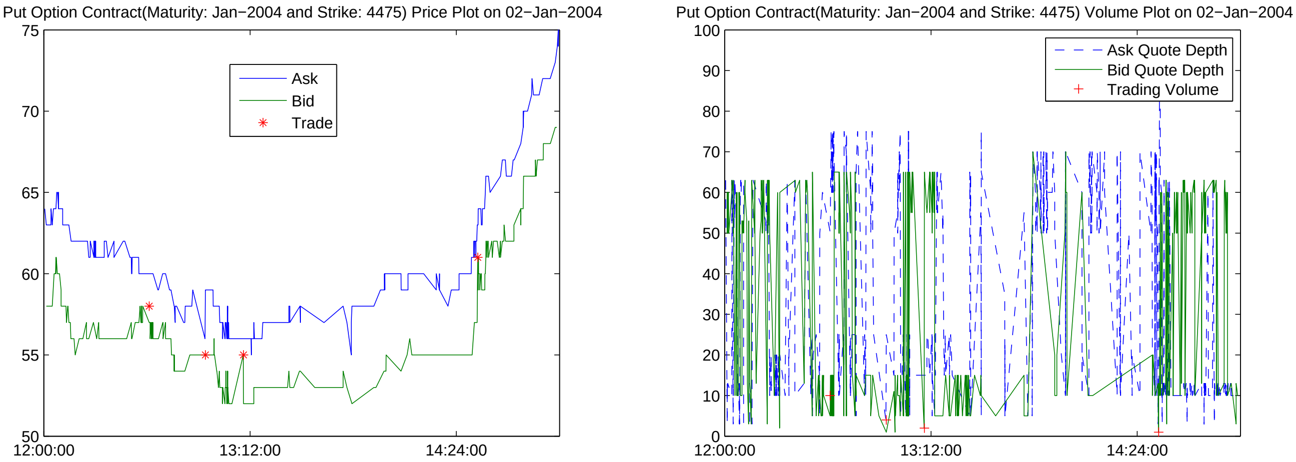


(e) Intraday Price of Selected Call Option



(f) Intraday Volume/Quoted Depth of Selected Call Option

Figure 3.2: Intraday Information of Selected Option Contracts



(a) Intraday Price

(b) Intraday Volume/Quoted Depth

Figure 3.3: Trading/Quotation 12:00-15:00 of Selected Option

3.4 FTSE 100 Futures Data Description

3.4.1 Futures Dataset Overview

A futures contract is an agreement between two parties to buy or sell an asset at a certain time in the future (the delivery month) for a certain price. In this one year intraday futures dataset, there are in total 26,271,084 observations listed in Table 3.7. There are different quotation types, ask quotation, bid quotation, trade and other wholesale trade type “J”, “K”, “S” and “V”, which have a higher trading size, as shown in Table 3.7. The average traded price is 45,133.9, which is less than both the average ask quotation and the average bid quotation. This is because only a portion of quoted contracts are traded, as indicated by the large number of quotation updates. There are around 23 million bid and ask quotation updates. However, only around 3 million trading transactions. The average trading size is 4.1, which is much smaller than the average bid and ask quotation depth, 14.3. The trading time for the FTSE 100 index option is from 8:00 to 17:30. From 26,271,084 records, 15,288 of them are outside of this trading time period and are therefore deleted. Also, 29,236 records from wholesale quotation “J”, “K”, “S” and “V” are also excluded in the following data analysis.

Compared with the option dataset summarised in Table 3.1, where the total number of ask quotation updates is 37.8 million, the total number of bid quotations is 37.7 million and the total number of trades is 0.148 million, the futures dataset has less quotation updates, 11.8 million for ask and 11.5 million for bid. However, it has a larger trading transaction number, 3.0 million.

Quote	No.	Mean Q-D/T-V	Std. Q-D/T-V	Mean Price	Std Price
Ask	11,811,582	14.3	17.9	45,346.9	1,383.0
Bid	11,473,181	14.3	24.9	45,336.7	1,395.1
Trade	2,957,085	4.1	21.7	45,133.9	1,325.7
J	620	565.5	1,126.2	45,005.7	1,201.7
K	1,672	1,867.8	1,960.4	45,607.2	1,189.9
S	12,224	240.4	809.6	45,519.3	1,157.9
V	14,720	142.2	296.5	45,075.7	1,300.5
Total	26,271,084				

Table 3.7: Futures Raw Dataset Quotation and Trading Information Summary

This table gives the number of quotation updates/trading transactions, the mean and standard deviation (*Std.*) of *Q-D/T-V*, the mean and standard deviation (*Std.*) of *Price* for different quotation types. *Q-D/T-V* is Quoted depth for *ask* and *bid* quotations and trading size for *Trade*. *No.* counts the quotation updates for *bid* and *ask* and transaction number for *Trade* and wholesale trading type.

3.4.2 Futures Contract Characteristics

A futures contract is referenced by its delivery month. Table 3.8 lists all available futures contracts in this one year futures dataset, which have been actively quoted or traded. There are 8 contracts with different delivery months. For each of them the number of ask quotation updates, bid quotation updates and trading transactions are listed. This dataset includes observations from 2-Jan-2004 to 31-Dec-2004. The contracts with a delivery month after Jun-2005 are less frequently quoted and barely traded. Contracts with a delivery month Dec-04 are the ones with the largest trading transaction number.

The features of futures market are very different from the options market. In the one year option dataset, 1,667 different contracts have been traded or quoted. There are only 8 different futures contracts in the full year dataset. As shown in Table 3.8, the first four futures contracts are traded intensely. As reviewed in Section 3.3.3, there are 87 different option contracts traded on the first day 02-Jan-04 listed in Table 3.3 and Table 3.4. For the futures market, normally there are only one or two different contracts that are quoted and traded. Maximally there have been four contracts traded or quoted a day. Table 3.9 counts the number of days where only one contract is listed and the number of days with more than one contract listed. For example, there are 3 out of 254 days when four different contracts are traded on the same day. There are 106 out of 254 days when 3 different contracts have updated ask quotation information on the same day.

Maturity	Ask	Bid	Trade
Mar-04	1,168,372	1,183,013	691,363
Jun-04	2,455,513	2,468,081	770,558
Sep-04	2,487,220	2,368,761	687,733
Dec-04	2,864,644	2,831,449	745,679
Mar-05	1,800,359	1,760,493	61,720
Jun-05	920,618	797,331	29
Sep-05	110,334	59,783	3
Dec-05	4,522	4,270	

Table 3.8: Futures Contracts in Raw Dataset

This table lists all available futures contracts in this one year FTSE 100 index futures dataset. The number of ask and bid quotation updates and trading transactions are provided for each contract.

No. of Different Contracts	Ask	Bid	Trade
1	13	13	52
2	58	74	179
3	106	101	20
4	77	66	3
Total	254	254	254

Table 3.9: Futures Contracts Trading/Quoting

This table gives the number of days when there are only one/two/three/four futures contracts traded/quoted per day. The first column lists four different scenarios possible from the dataset; these are the number of different futures contracts listed on the same day. *Ask*/*Bid*/*Trade* gives the number of days for each of these 4 scenarios when only ask quotations/bid quotations/trading transactions are considered.

3.4.3 Daily Information

There are three days in this one year dataset with four different futures traded on the same day, 4-Feb-2004, 12-Feb-2004 and 16-Dec-2004. The one in the middle, 12-Feb-2004 is selected as a sample to investigate futures trading. The summary of trading and quotations on the selected day is provided in Table 3.10.

Sample Day Futures Characteristics

There are four contracts (Mar-04, Jun-04, Sep-04 and Dec04) traded and actively quoted on 12-Feb-2004. The contract with a delivery month Mar-04 is the nearby-month contract. The nearby-month or front-month is used in futures trading to refer to the contract month with an expiration date closest to the current date. The front-month contracts are generally the most liquid futures contracts. It is shown in Table 3.10 that the Mar-04 futures contracts have a large transaction number of 11,353, which is more than half of the ask (bid) quotation updates 19,832 (20,310) on 12-Feb-2004. This means that there is one trade for every two ask (bid) quotation updates. In Section 3.3.3, there are 40 trading transactions from 1,374 ask quotations and 1,914 bid quotations for the most active call option on 02-Jan-2004, which means there is one trade for every 34 ask quotation updates or 48 bid quotation updates. The futures market is a much more liquid market compared with the options market.

The next delivery contract, Jun-04 has been actively quoted. However, there are only 14 trading transactions for it on the selected day. The indicated bid-ask spread for the Jun-04 futures contracts is 15.1, which is twice that of the front-month contract. The indicated bid-ask spreads for the Sep-04 futures contracts and Dec-04 futures contracts are values that are not realistic. This is because they are not actively quoted and the bid quotation for the Sep-04 futures contracts and Dec-04 futures contracts are not regularly updated. There are only 4 bid updates for the Sep-2004 futures contracts on 12-Feb-2004.

The average price for contracts with a longer time-to-expiry is higher than that of the shorter contracts. This is expected because contracts with a longer expiration time have more time value. There is not too much difference between the average trading volume for different contracts on the selected day.

Delivery	Ask Quotes			Bid Quotes			Trades			Ind.
Month	No.	Q-D	Avg.P	No.	Q-D	Avg.P	No.	Volume	Avg.P	spread
Mar-04	19,832	15.2	43,599.3	20,310	13.9	43,591.4	11,353	4.4	43,600.1	7.9
Jun-04	19,747	15.1	43,681.3	20,281	13.8	43,666.1	14	3.9	43,807.5	15.1
Sep-04	3,187	11.8	43,814.7	4	7.0	44,095.0	3	6.0	44,086.7	-280.3
Dec-04	15,202	11.9	43,991.1	8,012	13.5	43,884.7	2	5.0	44,227.5	106.4

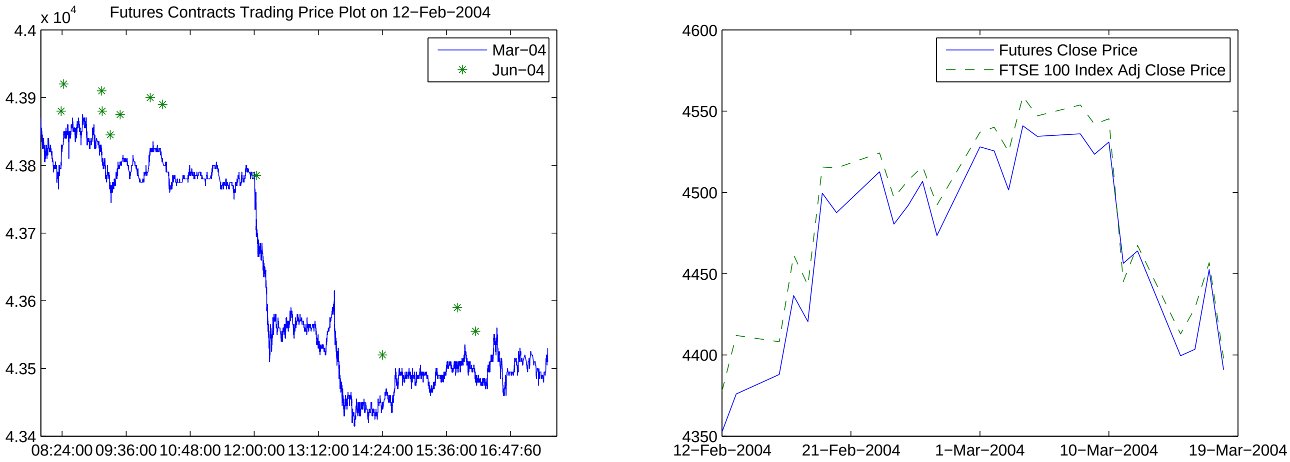
Table 3.10: Futures Contracts Traded on 12-Feb-2004

This table gives aggregated trading and quotation information for all futures contracts traded on 12-Feb-2004. *No* is the number of transactions under the *trades* section; and it is the number of quotation updates under the section of *ask quotes* and *bid quotes* for each contract. *Volume* is the average trading volume for each contract. *Q-D* is the average quoted depth for bid and ask quotations for each contract. *Avg. P* is the average trading/ask/bid price for each contract. *Ind. SP* is the indicated bid-ask spread which is the difference between the average ask quotation and bid quotation. For example, futures contracts with delivery time Mar-04 have 11,353 trading transaction records, 19,832 asking quotation updates and 20,310 bid quotation updates. The average trading volume is 4.4, the average ask quoted depth is 15.2 and the average bid quoted depth is 13.9. Its average traded price is 43600.1, the average ask price is 43599.3 and the average bid price is 43591.4 on this day. The indicated bid-ask spread is 7.9.

Futures Price and Spot Price Comparison

Due to the arbitrage principle and the law of supply and demand, when the delivery date of a futures contract approaches, the futures price will generally converge or even come to equal the spot price as time progresses. This in theory is called spot-futures-parity. The front-month contracts generally have the smallest spread between the futures price and the spot price on the underlying index. In this one year futures dataset, this is not always the case. YAHOO UK and Ireland Finance has an adjusted closing value of 4377.7 for the FTSE 100 index on 12-Feb-2004. The FTSE 100 index futures are quoted in index points and are set at a £10 per index point. The futures price in our dataset is in £. Therefore the average price in index points for these four contracts are 4360.0, 4380.8, 4408.7 and 4422.8. The spreads of futures contracts (Mar-04, Jun-04, Sep-04, Dec-04) and the spot price are 17.7, 3.1, 31 and 45.1 in index points. The Jun-2004 contract actually is the one that has the smallest spread with the spot price. This may be due to the close price of the FTSE 100 index being used instead of the average price. However, the intraday spot price is not available. Therefore, closing prices for Mar-04 and Jun-04 futures contracts have been used to calculate the spot-futures spreads.

It is shown in Figure 3.4a that the futures price actually closes at a lower price than at the open. As a result, the closing price is lower than the average price. The spread between the futures contract and the spot price is actually bigger when the futures closing price is used. The closing traded price of the Mar-04 futures contract is 4353.0 at 17:29:53. The last trade for the Jun-04 futures contract is 4355.5 at 16:08:42. The mid-price of the closing bid price (4359) and ask price (4361), which are taken at 17:29:57 and 17:29:55, for the Jun-04 futures contract is 4360. As a result, although using the closing price for the Mar-2004 and Jun-2004 futures contracts, the spreads with the spot price are 24.7 and 11.7. The difference between the two spreads is a little bit smaller when using the closing price than when using the average price. However, the Jun-04 futures contract is still the one that has the smallest spread with the spot price on 12-Feb-2004.



(a) Intraday Traded Price of Different Contracts

(b) Daily Futures Traded Price Vs Spot Price

Figure 3.4: FTSE 100 Futures Price Series-1

Front-month Futures and Spot Price Comparison

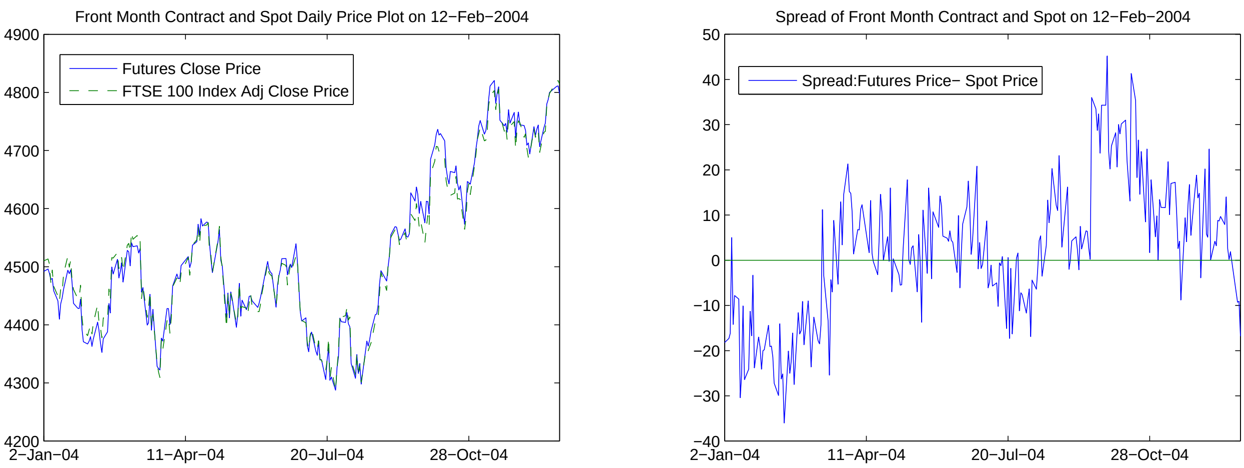
The daily closing traded price for the Mar-04 futures contract is plotted together with the spot daily adjusted closing price in Figure 3.4b. The delivery date for FTSE 100 futures is the third Friday in the delivery month. The Mar-04 futures contract stops trading after the delivery date (19-Mar-2004). It shows in the plot that during the time period of 12-Feb-2004 to 18-Mar-2004, for the majority of the time, the spot price is above the price of the front-month futures contract. There is only one day (11-Mar-2004), when the futures price is slightly higher than the spot price. After 10-Mar-2004, the two prices converge and on 18-Mar-2004, they are nearly the same. In theory, the futures price should always be higher than the spot price except for where there are dividends, as shown in Equation 3.1, where F is the futures price, S is the spot price, r is the risk-free interest rate and q is the dividend yield.

$$F = S * e^{(r-q)T} \tag{3.1}$$

Futures Price Series Construction

The one year daily closing price of front-month futures contracts and the spot adjusted closing price are plotted in Figure 3.5a. The spread of these two time series is in Figure 3.5b. The one year FTSE 100 index daily adjusted closing price is downloaded from YAHOO UK and Ireland

Finance. It is shown in Figure 3.5b that there are a few points where the spread is zero. Most zero spread points happen around the futures delivery date, which are 19-Mar-04, 18-Jun-04, 17-Sep-04 and 17-Dec-04 in the year of 2004. It is noticed that the spread is positive after the 10th of August, i.e. the price of front-month futures contracts is higher than the spot price. The ex-dividend dates of the majority of FTSE 100 constituents are in August. Before the ex-dividend date, the underlying security, the spot index, is more expensive because of the expected dividend yield. After that date, the underlying security holders do not have that advantage. The potential reason for a negative spread of the front-month contracts and the spot price is due to dividend payments.



(a) Intraday Traded Price of Different Contracts

(b) Daily Futures Traded Price Vs Spot Price

Figure 3.5: FTSE 100 Futures Price Series-2

It is found from this one year dataset, that the front-month contract always has the largest trading volume among all traded futures contracts until the delivery date, when the next near to delivery contract is traded more often than the expiring one. Therefore, in the following analysis, the front-month contract is used except on the quarterly delivery date when the next near to delivery one is used. In this way, the most active contract is always used in the analysis. The contract used in each time period analysis is listed in Table 3.11. For example, the first delivery date in 2004 is 19-Mar-2004. The analysis before the date of 19-Mar-2004 is from the Mar-2004 futures contracts.

Time Period	Futures Contract
02-Jan. to 18-Mar.	Mar-2004
19-Mar. to 17-Jun.	Jun-2004
18-Jun. to 16-Sep.	Sep-2004
17-Sep. to 16-Dec.	Dec-2004
17-Dec. to 31-Dec.	Mar-2005

Table 3.11: Futures Contracts Used in Analysis

This tables specifies which futures contracts are used for different time periods.

3.4.4 Intraday Information

For comparison purposes the same day of 02-Jan-2004 used in the option intraday analysis is chosen to do the futures intraday analysis, where the information is from the most active contract Mar-2004. The hourly bid, ask and trade information is provided in Table 3.12a. All trades, bid and ask quotations on 02-Jan-2004 are in Figure 3.7. The price is plotted in Figure 3.6a and trading volume, quoted depth are in Figure 3.6b. The transaction and quotation information in Figure 3.6a and 3.6b is very dense. It is difficult to observe the difference between trade and quotation. Therefore fifteen minutes trading and quotation information from 9:00 to 9:15 is provided in Figure 3.6c for price and Figure 3.6d for volume in order to magnify the market microstructure of the futures trading.

Transactions and Quotations

The trading time for futures is from 8:00 to 17:30. It is observed in Table 3.12a that the number of trading transactions is more than half of ask/bid quotations. The average trading size is less than half of the average ask/bid quoted depth. The average traded price is between the average bid price and average ask price. The bid-ask spread in the opening hour is the biggest in the selected day. The second biggest bid-ask spread is just before lunch time. When the market closes the bid-ask spread is only half of that in the opening hour.

It is observed in Figure 3.6c that the traded price of futures is either very close to the bid quotation or very close to the ask quotation. It is seldom in the middle of the bid and ask quotation. The trading size is generally smaller than the quoted depth in Figure 3.6d.

Intraday Trading Patterns

The full day pattern of the front-month futures contracts in Figure 3.6c is similar to the quotations for the most active call options in Figure 3.2a. Starting from 8:00, the market is very volatile until 08:30, when the price goes very flat and then steadily increases. The first trade is 44800 at 8:00:07. At around 9:20 the price sharply decreases. Ten minutes later the price continues to increase until 13:15, when the traded price reaches the maximum point for the day of 45085. For the next hour, the price decreases slightly and there are some small increasing corrections in the middle. From 14:30 it decreases sharply. The traded price reaches 44825, which is the local minimum in the afternoon. It is very volatile for the next 40 minutes. From 15:40 it starts to sharply increase again until 16:10, when it becomes volatile again. Around 16:35 the price increases to 45010 and then it continues to decrease until the market closes. The last trade has a price of 44920.

The trading after lunch time is the most active time period for futures trading. The number of transactions doubles in the hourly interval from 14:00 to 15:00 compared with the hourly interval from 13:00-14:00; the bid and ask quotation updates also double in the same hourly interval compared with the previous hour. In the next hourly interval from 15:00 to 16:00, the number of transactions is nearly 20 percent of all of the day's transactions, as shown in Table 3.12a. It is the same for quotation numbers. However, the average trading size in this hourly interval is the smallest for the day. In the hourly interval from 16:00 to 17:00, transactions and the ask quotations are less than the hourly interval 15:00-16:00, but more than the hourly interval of 14:00-15:00 except for bid quotation updates.

Futures and Options Comparison

Comparing this with options trading, the bid and ask quotations for the most active option on 02-Jan-2004 are also more active in the afternoon than in the morning, which is shown in Table 3.5. In the last two and a half hours trading, the number of ask quotations is 49.1 percent of all of the day's ask quotations and the number of bid quotations is 44.5 percent of all of the day's bid quotations. In the last 30 minutes of options trading, the quotation updates are more frequent than all the hourly time intervals before 14:00. The options market closes at 16:30 and the futures market closes one hour later. The futures trades in the last 30 minutes from 17:00-17:30 in Table 3.12a is only 14.4 percent of trades in the hourly interval of 16:00-17:00.

Table 3.12b provides the 30-minute breakdown information. It turns out that futures trading from 16:00 to 17:00 occurs mostly in the first 30 minutes. The trades in the first half an hour is 74.6 percent of total hourly trading. This indicates that futures trading is much less active after the options market closes. These two markets are related to each other closely.

Time	Trades			Ask Quotes			Bid Quotes			Ind.
H	No.	Vol.	P	No.	Q-D	P	No.	Q-D	P	SP
8	738	5.8	44762.2	869	17.7	44771.9	973	11.9	44757.3	14.6
9	430	4.6	44789.9	585	19.4	44792.0	674	19.8	44786.2	5.8
10	292	5.9	44830.0	432	21.2	44832.8	406	24.1	44825.9	6.9
11	468	5.2	44924.6	767	14.3	44931.5	610	16.0	44917.0	14.5
12	489	5.4	45005.3	677	19.7	45009.7	751	14.6	45001.9	7.8
13	314	8.4	45060.7	613	15.9	45063.2	622	23.4	45055.8	7.5
14	770	4.1	44952.1	1290	13.2	44959.8	1344	14.6	44951.3	8.5
15	1075	3.5	44881.2	1901	10.8	44884.9	1762	15.3	44878.2	6.7
16	786	5.0	44994.4	1441	19.0	44995.5	1236	14.5	44987.5	8.0
17	113	4.2	44983.7	237	10.8	44988.4	166	5.8	44981.4	7.0

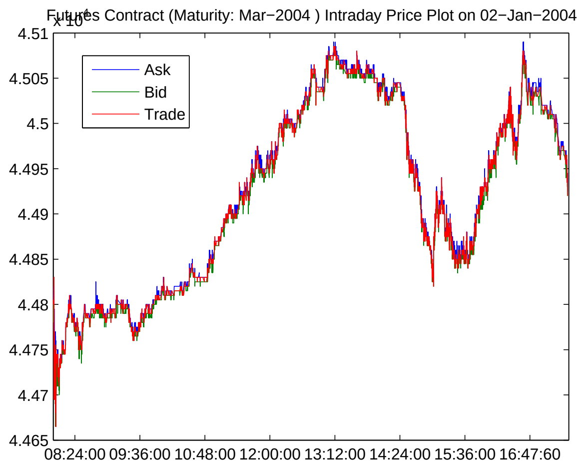
(a) Hourly Trading and Quotations for Selected Contracts on 02-Jan-2004

Time	Trades			Ask Quotes			Bid Quotes		
Interval	No.	Vol.	P	No.	Q-D	P	No.	Q-D	P
First 30-Min	586	44987.8	5.0	1075	44986.6	22.0	891	44976.9	17.1
Last 30-Min	200	45013.7	5.1	366	45021.5	10.0	345	45014.9	7.8

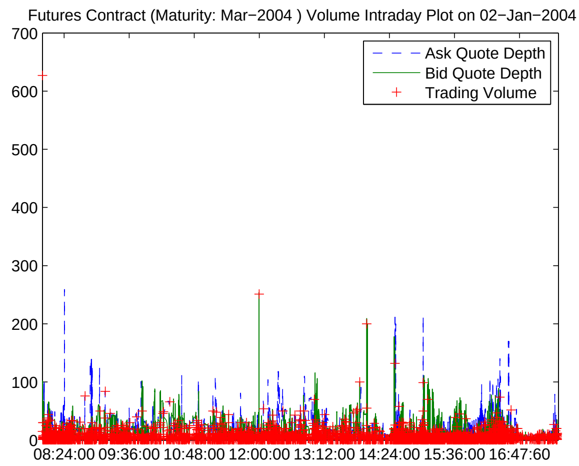
(b) Half an Hour Breakdown for the Hourly Interval 16:00-17:00

Table 3.12: Hourly Trading and Quotations for Selected Contracts on 02-Jan-2004

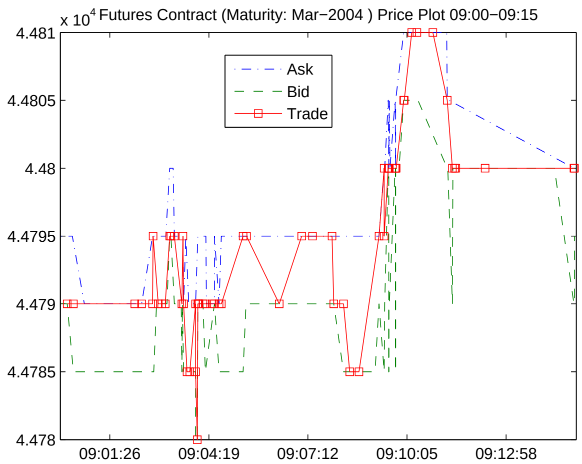
Table 3.12a provides hourly quotation and trading information for selected futures contracts on 02-Jan-2004. Table 3.12b gives 30 minutes breakdown information for the hourly interval 16:00-17:00. *H* is the trading hour. The hour 8 covers the time range of 8:00-9:00. For the hour of 17, the time range is 17:00-17:30. *No.* is the number of transactions(quotation updates) for *trades*(*ask quotes* and *bid quotes*) in each trading hour. *Vol.* is the average trading volume. *Q-D* is the average quoted depth. *P* is the average trading/ask/bid price per trading hour. *Ind. SP* is the indicated bid-ask spread calculated by subtracting the average bid quotation from the average ask quotation.



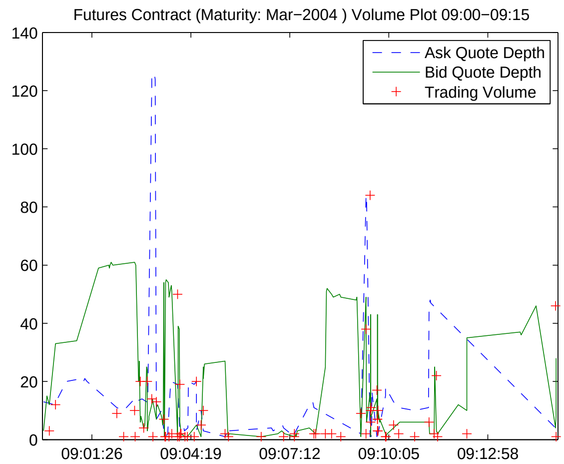
(a) Futures Price



(b) Volume/Quoted Depth



(c) Futures Price



(d) Volume/Quoted Depth

Figure 3.6: FTSE 100 index Futures Trading Information on 02-Jan-2004

3.5 Co-movements of Index Options and Futures

3.5.1 Theoretical Relationship of the Futures and Options

FTSE 100 index options and futures have the same underlying security, the FTSE 100 index. According to one-dimensional diffusion process option pricing models, such as the Black-Scholes model, there are some basic properties between the option price and the underlying price summarised in [89]. Call prices are monotonically increasing and put prices are monotonically decreasing in the underlying asset. The underlying price is the sole source of uncertainty for all of its options. Prices of different options must be perfectly correlated with each other and with the underlying price. The empirical analysis from [89] showed evidence that the S&P 500 options prices and the underlying price does not obey these properties on an intraday basis. The empirical analysis in [4] found similar evidence that the co-movements of index options and index futures quotes, are far from perfectly correlated in periods with option trades, based on intraday data and that market microstructure effects, including stale quotes and aggressive quotes explain the majority of deviations from the benchmark.

3.5.2 Co-movements of Futures and Options Examined by a Sample

In this section the bid, ask quotation and trades for the most active call option(with a maturity Jan-04 and a strike 4525) on 02-Jan-2004 are plotted together with the traded price of front-month futures in Figure 3.7. Due to the very small bid-ask spread and the frequent futures trading discussed above, only the futures traded price is given. The option bid, ask quotation and traded price are at the left vertical axis and the futures traded price is at the right vertical axis in Figures 3.7. Sub-Figure 3.7a and Sub-Figure 3.7b give the full trading day plots. It is noted in Figure 3.7a that there are large differences between the selected option prices and the futures prices. However, they are only due to the first bid quotation on the day, which is 11, as at 08:02:13. Therefore the first bid quotation is excluded in Figure 3.7b, where the bid quotation starts from the second quotation in the day which is 49 at 08:06:54. Sub-Figure 3.7c and Sub-Figure 3.7d only give the price plot for one hour.

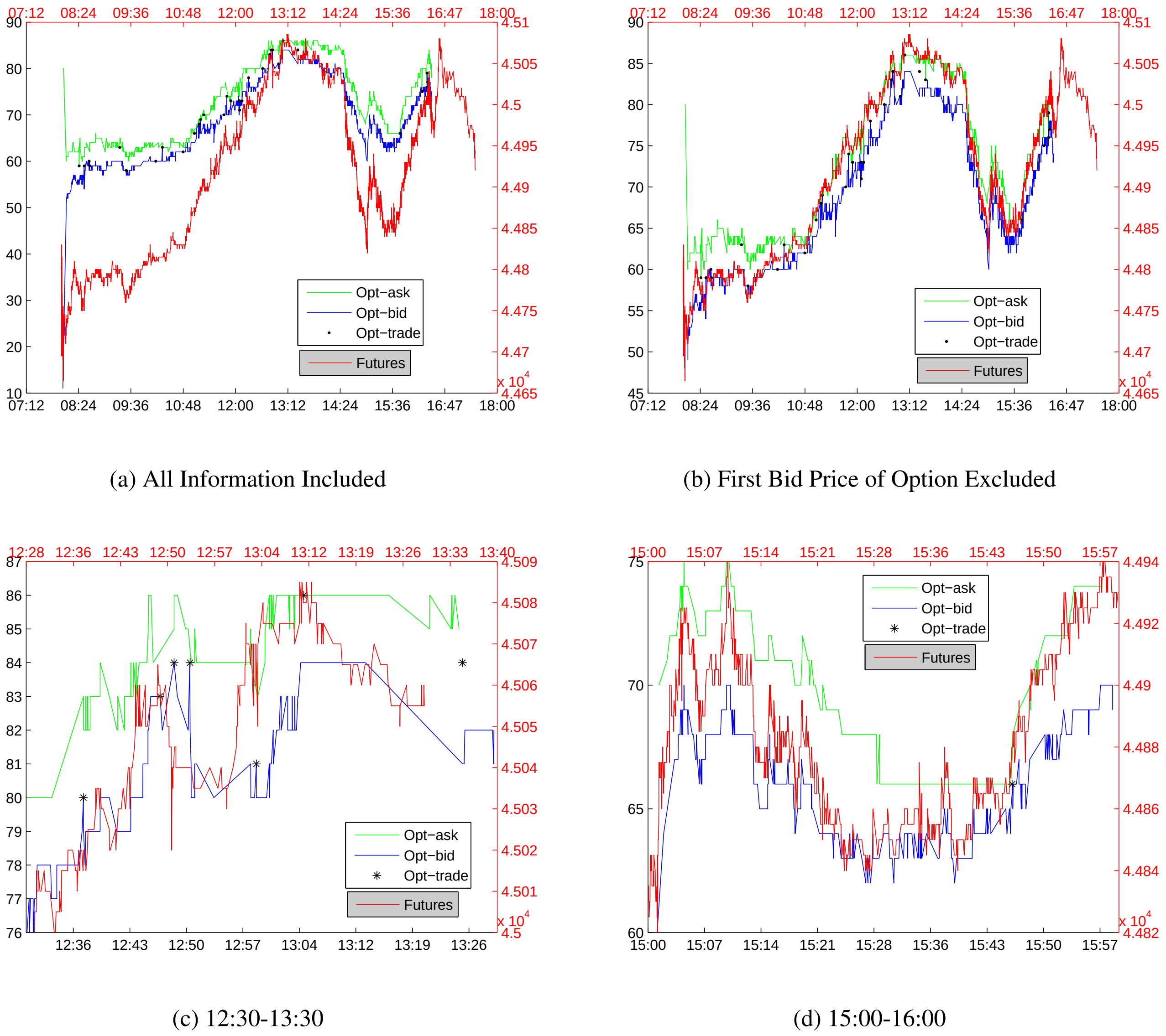
Futures Move Quicker than the Options

It is observed in Figure 3.7b that the futures price has increased at a higher speed than the options price. When the market opens, the futures price is nearly overlapped with the bid price of the option and the option bid-ask spread is quite wide. After 11:00, it is nearly overlapped with the ask price of the option. Therefore the futures price increases at a faster pace before 11:00. After 13:00 the futures price is slightly higher than the option ask price. After 14:20, the option bid-ask spread gets narrower and both the futures and options market falls. The futures traded price falls at a quicker speed than the option prices and it nearly overlaps with both the option bid and ask prices. After 15:40, both markets increase again and as before the futures price increases at a higher speed. It nearly overlaps with the ask price. It is observed in Figure

3.7d in the time periods 15:00-15:05 and 15:28-15:38 that though moving in the same direction the futures price moves more than the option price. The changes in the futures prices are more volatile than the changes in the option prices.

Futures Traded More Frequently than Options

The futures traded price and the option three prices series are magnified in Figure 3.7c for the one hour interval from 12:30 to 13:30 and Figure 3.7d for another hour from 15:00 to 16:00. In the first one hour time period there are a few option transactions. In the second time period both futures and options markets are the most active. The futures have been actively quoted and traded. The option is actively quoted though there is only one option transaction.



(a) All Information Included

(b) First Bid Price of Option Excluded

(c) 12:30-13:30

(d) 15:00-16:00

Figure 3.7: Futures and Most Active Call Option on 02-Jan-2004

Futures and Options Are Not Perfectly Correlated in Refined Time Intervals

It appears from Figure 3.7c and Figure 3.7d that the movements of futures prices and option prices are far from perfectly correlated. Ask and bid prices from the option do not move in the same direction as the futures prices change. For example from 12:30 to 12:33, the option ask price remains flat while the futures price and the option bid price change directions a couple of times. For the next five minutes, the option ask price increases, i.e. only has one direction change while the futures traded price and the option bid price change the sign a couple of times. Another example is as follows; from the time period 13:15 to 13:25, the futures price and the option ask price move direction a couple of times while the bid price only decreases. For the third example, just before 13:30 the futures price increases while the option bid decreases and there are no updates for the option ask. In Figure 3.7d, during the time period 15:30 to 15:45, the option ask prices are flat while the futures prices and option bid prices are very volatile.

Futures and Options Move in the Same Direction in Longer Time Intervals

For longer time intervals, the futures prices change and the option prices move in the same direction though not in the refined time intervals. For the time period in Figure 3.7c from 13:04 to 13:14 and ignoring the futures price changes in the middle, it is noted that both futures prices and option prices are flat. However, in the refined time period in these 10 minutes, the futures price moves a couple of times while there is no change for the option bid and ask prices.

After 16:30 the option market closes. The futures price sharply increases and then decreases. When the futures market closes the futures traded price is close to the price level as at one hour previously when the option market closed.

3.6 Chapter Summary

In this chapter, the two datasets one year FTSE 100 options and FTSE 100 futures are described. On each day, there are over 200 option contracts actively quoted while only around half of them are traded. Within traded option contracts, the most frequently traded contracts are ATM and near to expiry options. Most of the option contracts are only traded one or a few times a day. Compared with this, the FTSE 100 futures have been very actively traded although there are only a few contracts available a day.

There are a few indications drawn from this data analysis for the empirical analysis conducted in the following chapters.

- It is clear from the above analysis that different option segments, ITM, ATM and OTM options have different trading patterns. Therefore they should be modelled separately.
- It is observed from both options and futures datasets that the bid-ask spread changes over time and the traded prices are close to either the bid or the ask. Therefore it is not realistic to use a fixed percentage of the bid-ask spread as a transaction cost charge. In the delta hedging modelling the bid-ask transaction costs have been considered by using the bid price when futures position need to reduce (to sell) and using ask price when futures position needs to increase (to buy).
- The futures quotations are updated more frequently than the options quotations and the bid-ask spreads are much narrower than that of the options. Therefore the bid-ask spread is considered to be a more relevant market condition variable on delta hedging strategy modelling.
- The FTSE 100 futures market is a liquid market. It has potential utility in options hedging.
- The futures intraday trading analysis shows that the price changes are very volatile during the day. Therefore the price ranges including open-close and high-low could be potential predictive factors for the futures daily RV forecast.
- The co-movements of FTSE 100 options and futures are not perfectly correlated in the refined time intervals. The Black-Scholes delta hedging theory may not work under the microstructure effects and the relevant market conditions should be considered to correct the observed bias, such as the futures and options move in different directions or they move in different speeds when the directions are the same.

This chapter has made two contributions, which are listed below.

- This chapter analyses the intraday trading environment of financial derivative markets including the stock index futures market and stock index option market. This analysis gives the statistical summary of the traded price, volume, quotation price and quoted depth for both futures and options.

- This chapter examines a few derivative market theories empirically in a sample of the intraday dataset. These include the put-call-parity, spot-futures-parity and futures and options co-movements. The analysis shows that under the microstructure effects, the examined theories do not work perfectly in the refined time intervals though they do hold in longer time periods.

Chapter 4

Realised Volatility Estimation and Forecasting

4.1 Introduction

In this thesis, two detailed financial modelling studies are conducted on the ultra-high frequency dataset of FTSE 100 index futures and options by a novel approach GP. In Chapter 2, the methodology, GP is introduced. In chapter 3 the one year high frequency dataset is described and analysed. In this chapter, the realised volatility (RV) of FTSE 100 index futures is estimated and then forecasted by using market condition variables. The out-of-sample results are compared with benchmark models. Relations between market information variables and RV are examined through the best individual from GP.

The rest of the chapter is divided into seven sections, Section 4.2 gives the introduction to RV modelling. Section 4.3 provides a literature review in this field including financial return volatility, volatility models and the relation between market conditions and volatility. Section 4.4 gives motivations to apply GP in RV modelling and the empirical test design. Section 4.5 describes data used, explains RV estimation methodology and how GP is applied in this RV forecasting. The benchmark models are also listed in this section. Section 4.6 presents competing model one-day ahead RV forecasting performances and statistical tests. The relations of market conditions and RV are analysed in Section 4.7. The conclusion is then given in Section 4.8.

4.2 RV Modelling

Volatility is a concept that captures the fluctuation from the expectation. It is an important concept in finance and has different indications for different users. From an investment perspective, it shows the degree to which returns tend to fluctuate. Traders would like to capture the volatility caused by positive returns. Risk management is more concerned about the volatility caused by negative returns. Derivatives traders use it as a key pricing element. As the financial market global securitisation and electronic trading become available, more and more people are doing security investment instead of real investment. The trillions in financial trading volume show the large size of this market's participants. The financial time series price changes are caused by trading behaviours, natural events, national/international economic policies, special events, terror attacks, or anything that has the potential to impact on the interests of the participant. The changes/fluctuations reflected by volatility give the monetary policy maker a sign of consumer confidence. The financial market's size and therefore its impacts on the global economy require strict and accurate quantitative risk regulation. Volatility is also a key input to the regulatory capital requirements from The Second Basel Accord. The volatility modelling interests have increased as its role increases in importance.

In a conventional volatility model, volatility is a latent variable. The term of realised volatility has started to be used in [62] among others, which is the sum of intraday squared returns at short intervals. Such a volatility estimator has been shown to provide an accurate estimate of the latent process that defines volatility [98]. A more accurate realised volatility estimate would result in improved forecasting performance, all things being equal. Through this realised volatility estimation the latent volatility process is theoretically observable from past returns. Traditional volatility models developed for daily data, such as GARCH and ARCH, may not be suitable for this realised volatility where volatility is treated as latent. Volatility modelling is a function fitting problem.

Genetic Programming's (GP) model induction has been applied in volatility modelling and has achieved good results in the studies of [44], [40], [43], [48], [51], which are reviewed in Section 4.3.4. However, there are still some questions which have not been addressed. Market conditions have been documented as important volatility indicators or have been shown to have a high correlation with volatility in a number of studies. A sample of these studies include [58] which examined the relation of trading volume and volatility, [144] examined the relation of the number of transactions and volatility, [152] examined the relation of price range and volatility, [157] examined the relation of interest rates and volatility, [137] examined the relation of implied volatility and volatility and [146] examined the relation of bid-ask spread and volatility. A better prediction is expected when these market conditions are taken into the model as inputs.

In this chapter, the realised volatility (RV) is calculated from the one year FTSE 100 index futures five-minute returns. It is well known that the five-minute frequency is a trade-off between accuracy which is theoretically optimised using the highest possible frequency and microstructure noise that can arise through the bid-ask bounce, asynchronous trading, infrequent trading and price discreteness, among other factors [59].

The calculated realised volatility is modelled directly by GP. One-day-ahead RV is forecasted. Modelling results are compared with a number of benchmark models. The relations of RV and market information conditions are also analysed.

4.3 RV Literature Review

The background of volatility modelling is given in Section 4.3.1. RV concept and its estimation methods are reviewed in Section 4.3.2. The conventional parametric RV modelling approaches are covered in Section 4.3.3. The EC based volatility models are summarised in Section 4.3.4. The studies of the relations between volatility and market condition variables are then reviewed in Section 4.3.5.

4.3.1 Background of Volatility Modelling

Volatility [1] is a measure of uncertainty of returns. It is a conditional concept based on returns assumptions. Depending on the return time series assumptions, volatility may take different forms. Such as, the integrated variance over the time period of T, is defined as $\int_0^T \sigma^2(t)dt$ when the instantaneous return is assumed to be generated by the continuous time martingale as in

[1] Variance is the square of the volatility. Volatility is used in the paper as a general concept to cover both variance and volatility. It is defined specifically when necessary.

Equation 4.1, where $W(t)$ denotes the standard Brownian motion/standard wiener process and *p(t)* is the log price process.

$$dp(t) = \sigma(t)dW(t) \tag{4.1}$$

In most modelling works, the target variable is observable such as the asset price prediction where the price is observed from the market directly. Compared with this, volatility modelling is more complicated. In general there are two ways to model it. In the traditional volatility modelling framework, ARCH (Autoregressive Conditional Heteroscedasticity) [41]/ GARCH (Generalised Autoregressive Conditional Heteroscedasticity) [42] and SV (Stochastic Volatility Model) it is modelled as a latent variable. The analyses in [124], [125] and [123] have shown that most latent volatility models cannot fully capture financial time series stylised facts. The second way is to measure/estimate the volatility first, therefore it becomes ex post observable. After that it can be modelled directly, such as [65] and [66]. This application takes the second form. The work of [98] gives an excellent volatility measurement and modelling review, where all traditional volatility models are covered. In this application the realised volatility is estimated and then forecasted.

4.3.2 Realised Volatility (RV)

Under RV concept, the returns are assumed to be a process generated by the stochastic differential equation (Equation 4.1), which is a continuous time stochastic process over a given time period. This time period is divided into i equally small adjacent intervals. The quadratic variation is defined as the limit of the sum of squared returns over these intervals in Equation 4.2 as the length of the sampling intervals goes to zero, where t_i and t_{i-1} are adjacent intervals. This limit is well-defined in the case of the logarithm price process $p(t)$, which is a semi-martingale. In the general semi-martingale case, assuming some (mild) restrictions on the types of leverage, the quadratic variation is an unbiased estimator of the integrated variance, $\int_0^T \sigma^2(t)dt$ and the square root of the quadratic variation is called realised volatility.

$$\lim_{i \longrightarrow \infty} (\sum_i (p(t_i) - p(t_{i-1}))^2) \tag{4.2}$$

Realised volatility has started to be used in [61], among others, to measure the interdaily volatility by summing up the intraday squared returns at short intervals, such as five-minute or fifteen-minute. This concept is very important to volatility modelling. It has been pointed out in [65] that the standard volatility models used for forecasting at the daily level can not readily accommodate the information in intraday data. The models specified directly for the intraday data generally fail to capture the longer interdaily volatility movements sufficiently well. This volatility concept allows us to model volatility by high frequency data, which has a time stamp of up to seconds. This concept also captures the interday volatility stylised facts. The information in high frequency data has been proved to be useful in forecasting at longer horizons [61], [65].

In an ideal world, the quadratic variation from shorter intervals in Equation 4.2 is always closer to the integrated volatility than the one from longer intervals. However, returns measured at intervals shorter than five minutes are plagued by spurious serial correlation caused by various market microstructure effects including nonsynchronous trading, discrete price observations, intraday periodic volatility pattern and the bid-ask bounce, which is concluded in [98]. In reality, prices are observed at discrete and irregularly spaced intervals. There are different sampling schemes to estimate the realised volatility as reviewed in [59]. There is also an increasing interest in controlling microstructure effects when modelling volatility by ultra high frequency data [126], [127]. In this application, the RV estimation approach in [60] is followed. This is not only because we are using the same futures index data, FTSE 100 prices, but also the RV estimated in [60] which successfully captured the volatility stylised long-memory effect. The detailed calculation of this method is discussed in Section 4.5.2.

4.3.3 Conventional RV Forecasting Models

It is well documented that realised volatility is a highly persistent and long memory process. Conventional models have been used to model it including ARFIMA (Augtoregressive Fractionally Integrated Moving Average) initiated by [65] and also extended in [130], HAR (Heterogeneous AutoRegressive) proposed by [66], the simple AR (Autoregressive) type model used in [128], [129] and SV with volatility treated as observable [129]. Recently there have also been HAR type extended models including HAR-GARCH model proposed by [131] to model RV while at the same time capturing its volatility and HAR with Jump proposed by [133].

Empirical works [131], [132], [129] have been done by comparing RV forecasting models. In [131], ARFIMA, HAR and HAR-GARCH are compared based on tick-by-tick transaction prices of S&P 500 index futures data (1985-2004) and HAR-GARCH gives the best forecasting performance in terms of R^2, RMSE (Root of Mean Squared Error), MAE (Mean Absolute Error) and RMSPE (Root of Mean Squared Percentage Error). In [132], AR, ARFIMA and HAR are compared and HAR gives the best result in terms of RMSE, MAE and R^2. This conclusion is drawn on a dataset consisting of tick-by-tick series for USDCHF (1989 to 2003), S&P500 Futures (1990-2007) and 30-year US Treasury Bond Futures (1990-2003). In [129], simple AR, SV and HAR are compared and HAR gives the best forecasting performance in terms of RMSE, MAE, Theil-U and QLIKE on a dataset of equity market indices of SPX and DJIA(1997-2011) and two exchange rates CADUSD and USDGBP(1998-2011). The Theil-U provides a relatively accurate measure by comparing the forecasts with the random walk model. QLIKE is a loss function, which is robust to noise in the volatility proxy. The ARFIMA has been reported in [131] and [132] to give a similar performance as HAR while its estimation procedure is more complex.

4.3.4 EC Based Volatility Models

In volatility modelling besides applications from GP, the applications from other EC are also reviewed as volatility modelling is a key area that this thesis has been devoted to. The applications reviewed are summarised in Table 4.1, Table 4.2 and Table 4.3. In these applications, EC has been used for two purposes, parameter estimation and function form (model) exploration. In the first one, the model function form is fixed and BIO is used to estimate the parameters. In the second one BIO is used to fit a nonlinear function form for the problem given. Unlike the gradient based optimisation method, EC does not require the objective function form differentiable. More importantly, the model induction utility from GP carries out the volatility modelling without unrealistic assumptions. The output model is selected by the actual data feed into GP. Compared with traditional volatility models GARCH and ARCH, the results are improved in most cases.

Research	EC	Data	Definition	Estimation Measure	Benchmark	Out-of-sample Test
[51]	GP	Daily S&P500 index call options 2002-2003	Implied Volatility	MSE	-	included
[48] [49] [50]	GA GP	Intraday 15-minute high-low S&P500(1998-2003)	Integrated Volatility	Accuracy Rate	GRACH(1,1)	included
[52]	EA	European Call/Put Options on DAX index on June 13, 2001	Implied/Local Volatility	Weighted Absolute Option Pricing Error	-	-
[40]	GP	USD/CHF and USD/JPY hourly return (1987-1999)	Realised Mean Daily Volatility	RMSE	ARCH Type Model	included
[43]	GP	USD/DEM and USD/JPY 5 Obs per day 1975-1999	Integrated Volatility	MSE, MAE, R^2	GARCH(1,1) and RiskMetrics Vol. Models	included
[44]	GP	S&P500 (Jan.-Oct.1992) small and Nikkei 225 (Jun. 1989-Apr. 1990)	Conditional Volatility	Mean, Std and Max	-	-

Table 4.1: EC Applications in Volatility Modelling

Research	Fitness function	Population	Operators	Selection	Generation	Replacement
[51]	MSE	100	Crossover(60%) Mutation(40%)	Tournament Selection(4)	400	200 Offspring, Comma Replacement
[48] [49] [50]	Absolute Error	100	Crossover; Mutation(5%)	-	25/50/100	50%
[52]	Weighted Absolute Error	50	Crossover; Mutation	-	120	Increased Selection Pressure
[40]	Logarithmic Residual Error	100	Crossover; 7 Mutation Operators	Fitness Proportional Algorithm	200	Elitism 50%
[43]	MSE	500	Crossover	Performance Based Selection	50	Performance(in selection period) Based Replacement
[44]	SSE	500	Crossover (50%), Mutation (0.33%) and Reproduction	-	100	-

Table 4.2: EC Applications in Volatility Modelling GP Setting

This table gives GP parameters setting of EC applications in volatility modelling. Fitness Function, Population Size, Operators, Selection Method, Generation Number and Replacement Method are key parameters in EC Algorithm.

Research	Function Set	Terminal Set
[51]	+, -,% , ×, Exp, Sqrt, ln, Ncdf	c/k, s/k, t
[48][49][50]	+, -,% , ×, Sin, Exp, Sqrt, Nature Logarithm	x(t-1),......, x(t-4)
[40]	+, *, EMA(Exponential Moving Average), Absolute Value, Square	Constants, Hourly Returns, and Aggregated Return at Selected Time Intervals
[43]	+,-,*,/, Norm, log, Exponential, Square Root, CDF of Stand. Norm. Distri., Data Function(Data, Average, Max, Min and Lag, GEO, MEM and ARCH5)	Daily Fx Returns, Integrated Vol, the Sum of Absolute Value of Intraday Returns and the Number of Days until the Next Business Day
[44]	-	-

Table 4.3: EC Applications in Volatility Modelling

This table gives GP parameter settings of EC applications in volatility modelling. Function Set and Terminal Set are key parameters in GP. *CDF of Stand. Norm. Distri.* is the cumulative distribution function of standard normal distribution.

Parameter Estimation

The EA approach is used in [52] to estimate the volatility surface, which is then used to calculate option prices. A family of pricing models compatible with market prices are returned. This family is then used to quantify model uncertainty and its impact on derivative prices. It provides an example of a coherent risk measure compatible with a set of observed option prices. In this application, the parameters in the predefined volatility surface are optimised by matching the market observed option price and the model option price. This application has highlighted the fact that EA yields a way of analysing model uncertainty besides its numerical advantages, such as avoiding computation of gradients. Compared with the calibration method based on deterministic optimisation, which yields a point estimation for model parameter, EA yields an entire population of solutions to the inverse problem, many of these solutions price the benchmark options with equivalent precision. The heterogeneity of this population reflects the uncertainty in model parameters, which are left undetermined by the benchmark options.

Volatility Model Exploration

Volatility forecast is a function fitting problem. Traditional parametric volatility estimation methods including GARCH and ARCH all pre-specify the volatility structure, which may be too rigid to get a good result. ARCH family models use daily data originally therefore it may not be suitable for high frequency data. To better utilise the intraday data/tick data a more flexible method is needed to capture its complex structure. GP's model induction ability has been applied in this area [40], [43], [44] and [49]–[51]. These applications are reviewed below. A recursive GP (RGP) was used in conditional volatility estimation [44]. This RGP dynamically utilise the training data by way of a rolling window composed by a major sample and marginal sample. In RGP, a size of representative GP trees have been used as modelling results instead of the best tree in standard GP. The proposed method estimates volatility by simultaneously detecting and adapting to structural changes. When RGP discovers structural changes, it will quickly suggest a new class of models so that overestimation of volatility due to ignorance of structural changes can be avoided. Testing data sets include S&P 500 and Nikkei 225. This application showed that RGP is an adaptive cognitive system.

A GP with syntactic restriction to impose a specific symmetry property and local search techniques to optimise the coefficients is used in [40] to explore the volatility forecasting models for foreign exchange rates (USD/CHF and USD/JPY). In the experiment, the control of tree complexity effectively reduced the over fitting problem and saved optimisation time. The solutions from GP consistently outperform ARCH-type models in out-of-sample forecasting. The function from GP indicates that cross products of returns at different time horizons improve the forecasting performance. That is, the realised volatility has dependency on past price evolution.

GP has been compared with GARCH(1,1) and the Risk Metrics volatility forecasting model in the forecasting of USD/DEM and USD/JPY integrated volatility (by five irregularly spaced intra day observations) at 1 day, 5 days and 20 days time horizons in the study of [43]. The full data sample is divided into a training period, selection period and out-of-sample period. Six measures of forecasting performance are reported: mean squared error, mean absolute error, R^2, mean forecast bias and kernel estimates of the error densities and generalised mean-squared forecast error matrix tests. Two methods are used to utilise ten trials of GP for each forecasting task. One is the equally weighted forecast using the arithmetic average of the forecasts from each of the ten GP trials. The other way is the median weighted forecast, which takes the median forecast from the set of ten GP forecasts. Within six measures, mean absolute error and error bias criterions clearly prefer GP method at all horizons. With regard to the rest measures, none of the tested forecasting methods is clearly superior. Two suggestions have been proposed from the study results; using the mean absolute error as fitness function and increasing the intra day observations frequency.

The works of [49] and [50] forecasted one day integrated volatility (IV) by GA and GP together by feeding wavelet transformed data. In this application, GA and GP are used to find a IV forecasting rule. The inputs for the potential rule are four lagged data points, i.e. IV at time point t is forecasted by the rule with previous 4 data points at time *t-1*, *t-2*, *t-3* and *t-4*.

GA is used first, the inputs and the forecasted IV are in a form of prespecified ranges, i.e. they belong to one of 4 ranges ($(-\infty, a], (a, b], (b, c], (d, \infty]$), where a-d are specified based on the observation of the time series plots as well as the analyst's risk requirement. The rule in the GA case is a If-then clause; the if part is a combination of inputs and logic operators including AND and OR; the then part is the forecasted IV class. The forecasted result is measured in forecasting accuracy, i.e. the rate of correct guess. This is also the fitness function in GA training. Results are compared with GRACH(1,1) in a way that the results from GRACH(1,1) are converted to the same 4 ranges. The GA method is superior to the GARCH approach.

GP is then used to forecast IV values based on the rules from the GA part. The mean absolute error is used as the fitness function. GP's parameter settings are in Table 4.3. The forecasted results measured in accuracy rate are improved further.

In [51]'s application, GP is used to forecast Black-Scholes implied volatility. Experiments are based on real data from S&P500 index options. GP's forecasting ability is compared in different training data groupings. The implied volatility patterns including volatility smile, volatility term structure and volatility surface are produced for the best models returned from GP. The analysis shows GP models are able to produce some well-documented volatility features. The author suggests that GP can be used to forecast implied volatility of stochastic volatility models and models with jump.

4.3.5 Volatility and Market Conditions

All the models discussed above rely on RV lagged information and return information. The volatility forecasting models in the studies of [158]–[161] also include other economic and market information as explaining factors, which are summarised in Table 4.4, which only includes research studies that have the volatility forecasting performance measured. In the study of [158], there are 5 category predictive variables considered including equity market variables and risk factors, interest rates, spreads and bond market factors, FX variables and risk factors, liquidity and credit risk variables and macroeconomic variables. Monthly volatility is the modelling target in this study. This chapter considers the short time horizon (daily) volatility forecasting, therefore the long-term factors, such as, macroeconomic factors are not considered.

In fact the relation between volatility and other market information has received increasing attention from academic researchers. There are concentrated studies that focus on the relation of trading volume and volatility. A sample of these studies include [56], [57], [142] and [164]. The relation of implied volatility and volatility is also well researched in [53], [134], [136] and [163] to name but a few of these studies. The studies of [144], [145] are devoted to the relation of transaction number and volatility. The relation of interest rate and volatility is investigated in the studies of [54], [55], [156] and [157]. The relation of volatility and price information including intraday price range ([152]–[155]) and bid-ask spread ([146]–[149]) have also been researched.

Volatility Forecasting Studies	Predictive Variables Considered
[159]	Daily High-low Range
[160]	realised range, realised power variation, realised bipower variant and volume
[161]	trading volume and implied volatility
[158]	5 Categories (38 variables) tested
[136] [163] [53] [134] [137] [135]	Implied volatility
[142]	volume

Table 4.4: Market/Economic Condition Variables Used in Volatility Forecasting

This table gives market/economic information variables used in volatility forecasting in different studies besides the lagged volatility and return information.

Forecasting Volatility with Interest Rates

In the empirical finance literature the effects of interest rates on market volatility is inconclusive. The work of [157] shows there is a negative relation between interest rates and volatility. The lagged nominal interest rates have predictability for the conditional volatility. The empirical analysis of [156] provides convincing evidence that the combination of the Baa-Aaa spread, the commercial paper-Treasury spread, the one year Treasury yield and the dividend yield has predictable power in return volatility. However, the evidence from [54] and [55] only show weak predictive power of interests rate in stock volatility. In both [54] and [55], contemporaneous interest rates or their changes have been included in the conditional variance equation in a GARCH model framework. The study of [54] also includes interest rate in the mean equation.

Forecasting Volatility with Implied Volatility

Volatility is a key input for an option pricing model. The volatility backed out from BSM through matching the model option price and market observed option price is called implied volatility from the option pricing model. It is often interpreted as a market's expectation of volatility over the options maturity. In most empirical works implied volatility has been found to have predictive power for return volatility.

The studies of [163] and [53] show that implied volatility provides accurate realised volatility forecasts. The analysis from [134] also demonstrates that implied volatility contains information in forecasting realised volatility and the relationship is stable under different volatility measurements. The evidence from [135] further confirms this conclusion. The study of [137] also draws the same conclusion though it was pointed out that the implied volatility may overestimate or underestimate the realised volatility for stocks with different characteristics. However, the evidence from [136] shows implied volatility has poor forecasts for subsequent realised volatility.

Forecasting Volatility with Volume

- **Hypotheses of the Relation Between Volatility and Volume**

There have been intensive studies about the relation between trading volume and volatility. There are two prevailing theories about this, the sequential information arrival hypothesis (SIAH) from [139] and the mixture of distribution hypothesis (MDH) from [138]. According to the MDH, returns and trading volume are driven by the same underlying latent news arrival or information flow where the contemporaneous relation between volatility and volume is concerned. However, the SIAH suggests that lagged value of trading volumes may have the ability to predict current return volatility. From a forecasting point of view, the causal relations between return volatility and trading volume are more relevant.

- **Summary of Reviewed Studies**

Most empirical analyses including [140]–[143], [145] and [164] have adopted the GARCH framework to assess the relation of volatility and volume, where the trading volume is added in the conditional variance equation as an explaining variable. Evidence supporting a positive contemporaneous volume and volatility relation may be found in studies [57] and [141]. The empirical analysis in [56] supports a long run MDH. Evidence supporting the fact that lagged volume has predictive ability to return volatility includes [140], [142], [143] and [164]. However, the problem of how to efficiently utilise volume in volatility forecasting is still a question that must be answered. As discussed below, [142] found a significant Granger Causality relationship from volume and return volatility while the study did not find a strong explaining power from volume in the volatility forecasting. The analysis has attributed this unexpected result to not being able to find a suitable function form.

- **Detailed Reviewed of Key Studies**

Using daily NYSE aggregate volume and Dow Jones composite, the study of [142] first examined linear and nonlinear Granger Causality of return volatility and volume. A bidirectional feedback relationship was found under both tests. Also, the results indicate that incremental nonlinear causality exists once linear cross- and auto-dependence have been removed from the series. Further, this study examines the effects of using lagged volume as a predictor for volatility. Comprehensive 10 types of model are compared including a naive model, which formulates tomorrow's forecast of volatility as being equal to the realised value today, the long-term mean model, moving average models, exponential smoothing models, exponentially weighted moving average models, autoregressive volatility models, symmetric GARCH models, asymmetric GARCH models, a neural network and augmentation of forecasting models with lagged volume. The model performance results measured by the mean squared error and mean absolute error show all models consistently over-predict the true realised volatility. The lagged volume measures have little contribution to the out-of-sample volatility forecasting. Although the best model for the pre-crash data according to the MSE criterion includes volume, occasionally it can lead to very erratic forecasts. The neural network models in this study give a reasonable performance. However, the author reported that the complexity and loss of any diagnostic information associated with the use of this technique renders them hardly worth the additional effort in this case. The author analysed that the evidence of nonlinear Granger causality gives the researcher no clue as to the appropriate functional form for the nonlinear forecasting model, and this may explain the reason for the disappointing forecasting performance. That is, the variables are correct, but the functional form may be wrong. Another explanation for the lack of additional explanatory power of the volume series could be that the measure of volume used has been transformed.

Using a GARCH framework, intraday returns from NASDAQ stocks are analysed in [140].

Contemporaneous and lagged volume and bid-ask spreads are used as explaining variables in the conditional variance equation separately. Statistically significant coefficients for all of them are obtained, however the numerical values are all very small.

Using intraday transaction data the results of [164] support SIAH theory. The trading volume and return volatility are found to follow a clear lead-lag pattern in a large number of DJIA stocks. The empirical evidence in [143] from Short Sterling movements shows that the trading volume has a significant coefficient by either using a contemporaneous or lagged value of it when estimating conditional volatility in the GARCH method.

Relation between Volatility and Number of Transactions

The study of [145] shows the positive volatility and volume relation actually reflects the positive relation between volatility and the number of transactions. Using the vector autoregressive analysis (VAR) model [144], it draws the conclusion that trading volume measured by the number of transactions has predictive power on realised volatility.

Relation between Volatility and Bid-Ask Spread

A positive relationship between the bid-ask spread and price volatility is documented empirically in [146], [147] and [149]. The price volatility is a dependent variable in the tested model in [146] while the bid-ask spread is a dependent variable in [147] and [149]. In the study of [148] a positive relation is found between bid-ask spread and intensity of information flow. Although this is not a direct analysis of bid-ask spread and volatility, we can expect a relation from bid-ask spread and volatility as the relation of volatility and intensity of information flow is well documented.

Relation between Volatility and Price Range

The daily range of the price series contains extra information about the course of volatility over the day. This has been evidenced by a series of studies. The study of [150] examined the problem of estimating capital asset price volatility parameters from the high, low, opening and closing prices and the transaction volume. The study of [152] used present and past high, low and close prices to forecast the currency volatility, which is defined as the standard deviation of daily price changes in the logarithm of the exchange rate. The study of [151] used the high and low prices to estimate the variance of the rate of returns. The studies of [153], [154] and [155] and used the price range in the estimation of stochastic volatility models.

Market Conditions Used in Volatility Forecasting

The study of [160] used GARCH framework and realised volatility from five-minute returns as the target to test the predictability of lagged realised volatility, realised range, realised power variation and realised bipower variant and volume, which were added in the conditional variance equation as explaining variable separately. One-day-ahead forecasts from these models for NYSE stocks were compared. GARCH with the realised power variation provided the most accurate results. The author also reported that the forecast combination of all four intraday measures excepting volume produced the smallest forecast errors in about half of the sampled stocks. Finally, the authors did a market conditions analysis, which revealed that the additional use of intraday data on day t-1, to forecast volatility on day t, was the most advantageous when day t was a low volume or an up-market day. The study of [159] jointly considered absolute daily returns, daily high-low range and daily realised volatility, to build a forecasting model based on their conditional dynamics. The one month ahead forecasts of the S&P 500 index from the developed multiplicative error model matched the VIX index well.

4.4 Motivations of Applying GP in RV Modelling

RV transfers intraday return information to an observable volatility (Section 4.3.2), and it allows volatility to be modelled directly. RV lagged value is the only input the conventional RV modelling methods (Section 4.3.3) can take to forecast the future value. At the same time, it has been documented that trading volume, transaction number, price range (including the range of open and close, high and low), bid-ask spread and implied volatility have predicative information or explaining power for volatility (Section 4.3.5).

It has been pointed out from [160] that different market information is likely to capture distinct subtle aspects of the volatility process, the relative prominence of which may vary over time. Also different market information may suffer to different extents from the market microstructure biases. The out-of-sample result in [160] confirmed that the combined result from GARCH models, with different volatility predictors, gives the smallest forecast errors in about half of the tested stocks.

In the reviewed cases, most of the studies ([54], [55], [140]–[143], [145], [157], [160] and [164]) used market information to explain/forecast conditional volatility in a GARCH type framework. The market information was added in the conditional variance equation as an explaining factor linearly.

The nonlinear Granger causality test conducted in [142] shows there is extensive evidence of bidirectional feedback between volume and volatility. However, in the forecasting race the model, including lagged volume, has not improved the result as expected. The author has concluded the potential reason might be that the function form used is not correct, although the variables are correct. How to use available market information to improve RV forecast is challenging to the conventional volatility parametric modelling methods.

GP is a model induction optimisation methodology. It evolves the function form for the problem given, driven by training data. In this application, GP is used to pick up the suitable explaining variables and link them to RV by the suitable function forms automatically. The function form returned from GP is then used to forecast a one-day-ahead RV by using explaining variables it has picked up during training. RV lagged value and other market information including price range, bid-ask spread, trading volume, number of transactions, corresponding implied volatility and interest rates are used as the potential explaining factors. GP is re-trained each day on a rolling window approach. We are assuming that the relative importance of each predictive variable changes dynamically overtime.

Since GP has been applied in the volatility modelling area it has achieved good results. This application aims to address questions that have not been covered by the previous applications. GP's model induction utility is used to explore a flexible function form, for a realised volatility, calculated from FTSE 100 futures price five-minute returns. This application tries to explore the relationship between realised volatility and exogenous variables, which have been documented to be highly correlated with volatility. To avoid the bias from the log transformation, RV is modelled directly instead of the logarithm of RV. It is reported in [131] that the prediction results are similar in direct RV modelling and logarithm of RV modelling.

Model performance is compared with four benchmark models, including traditional models (ARMA model, HAR and GARCH), which only use RV lagged information and a linear regression model, which uses the same explaining factors that are used to train GP. Lastly, the dynamic market conditions picked up by GP in RV forecasting are analysed, which will enhance the current understanding of the relations between market conditions and volatility.

4.5 Data Description and Methodologies

4.5.1 Data

FTSE 100 index futures tick-by-tick transactions and bid-ask quotation data were collected for the full year of 2004. The traded prices are used in the RV estimation. Intraday information including trading volume, bid-ask information, price range and the number of transactions are used as the explaining factors. FTSE 100 index options for the full year of 2004 were also collected, from which the implied volatility was calculated. Bank of England LIBOR rates (overnight, one-week and six-month) in 2004 were collected from Datastream. The detailed original data process is in Section 3.4. The data post process and used in RV modelling is described in the following sections. The first six month data is used for in-sample training and the other six month data is used for out-of-sample testing. There are 129 trading days RV forecasted in out-of-sample testing. Training data has been used as a rolling window approach. Each day's forecast is from the information up to the day before. Subject to only one year data being available we try to use as much data as possible to do the out-of-sample forecasting. For the first day's forecast (the first day of July), data from January 9th to June 30th is used. For the last day's forecast (the last day of December), data from January 9th to December 30th is used. The first five days in January are excluded as lagged information required.

Traded Prices

There are in total, 2,957,085 transactions recorded for all maturity contracts. The traded prices are used instead of the bid and ask prices in the RV estimation. As the most active traded contract is the one nearest to delivery, the prices from the nearest matured contract are used apart from on the expiry date, when the next contract is used. Therefore returns are always calculated from the prices of the contract that has the highest traded volume. Trading starts at 8am and finishes at 5:30pm. However some abnormal days have been discovered in Table 4.5. The out-of-hours trading records are excluded.

Date	Missing Time
22-Jan-2004	16:36-16:55
28-Jan-2004	13:45-13:55, 15:30-16:05
09-Feb-2004	16:08-17:30
20-Feb-2004	14:40-16:00
11-Mar-2004	12:35-12:40
31-Aug-2004	13:40-13:45
10-Nov-2004	16:55-17:30
24-Dec-2004	12:30-17:30
31-Dec-2004	12:31-17:30
02-Apr-2004	start at 7am end at normal

Table 4.5: Missing Transaction Data Time Period

The above table outlines the time periods where no transaction data was available. The last time period, 02-Apr-2004, includes trading outside the trading time.

4.5.2 Realised Volatility Calculation

The latent volatility can be accurately estimated by realised volatility, which is the sum of intraday squared returns, as long as the sampling process is frequent enough. However, it is concluded in [98] that returns measured at an interval shorter than five minutes are plagued by various market microstructure effects. Following this route the intraday return frequency is set to five minutes. The realised volatility in [60] shows that the distribution of volatility measured daily is similar to lognormal, whilst the volatility time series has persistent positive autocorrelation that displays long-memory effects. The distribution of daily returns, standardised using the measures of realised volatility, is shown to be close to normal. These features have also been observed by other empirical studies [63], [64]. The same estimation method in [60] is used to construct the realised volatility in this application. There are three steps in this RV calculation, intraday return calculation, intraday variance multipliers estimation and realised volatility calculation, which are explained in the following sections.

Intraday Returns

The trading time of FTSE 100 index futures was from 8:00 am to 5:30 pm in the year of 2004 therefore there are 114 five-minute intraday returns each day in our dataset, which are calculated from the latest prices before each five-minute mark in Equation 4.3, where $ln(p_{t,j})$ is the logarithm price for the *j-th* five-minute interval on day t and $r_{t,j}$ is the *j-th* intraday return on day t with j=*1,2 . . . , 114*.

$$r_{t,j} = ln(p_{t,j}) - ln(p_{t,j-1}) \tag{4.3}$$

The statistics of five-minute returns are in Table 4.6. Besides returns from the latest close price of each five-minute time mark, returns from the simple average price and volume weighted price in each five-minute interval are also calculated. The returns from the latest close prices are the same as in [60] and are used for RV calculation. The average returns are nearly zero for all return series. The standard deviation of returns from the simple average and weighted average price are lower than that of returns from latest prices. Negative skewness indicates the longer left tail. The positive kurtosis indicates these returns are from a distribution that has a sharper peak and a longer tail than a normal distribution. The maximum return happened in the first 5 minutes on 19 March 2004 (Friday). The minimum return happened in the first 5 minutes on 10 May 2004 (Monday). There are 3,264 zero-returns for the latest closing price series, which is 11.34 percent of the total number of returns (28,780 returns). The autocorrelations of the returns are close to zero. In [60], it is found that the autocorrelation of five-minute returns from the interpolated prices are higher than returns from the latest prices. The same results are found here that the returns from the simple average prices, and volume weighted prices, have a higher autocorrelation than returns from the latest close prices. Therefore the returns from the latest prices are used in the realised volatility calculation. The daily returns from the latest close prices by summing up the five-minute intraday returns are in Figure 4.1.

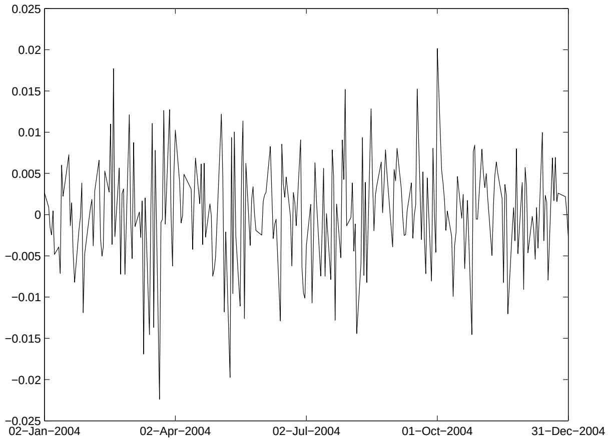


Figure 4.1: Daily Returns of FTSE 100 Futures

The plot gives daily returns of FTSE 100 futures by summing up the five-minute returns

	R-Closing	R-Simple Avg.	R-Volume Weigh.
Mean	2.42E-06	2.43E-06	2.40E-06
Median	0	5.02E-06	4.62E-06
Standard Deviation	6.26E-04	5.48E-04	5.55E-04
Skewness	-8.96E-01	-1.27E+00	-1.22E+00
Kurtosis	4.52E+01	6.66E+01	6.21E+01
Maximum	9.07E-03	9.61E-03	9.39E-03
	19 March 2004	19 March 2004	19 March 2004
	08:00-08:05	08:00-08:05	08:00-08:05
Minimum	-1.36E-02	-1.39E-02	-1.41E-02
	10 May 2004	10 May 2004	10 May 2004
	08:00-08:05	08:00-08:05	08:00-08:05
Number of Zeros	3264	18	6
% of Total Obs.(28,780)	11.34%	0.06%	0.02%
Autocorrelations			
Returns			
Lag 1	-2.76E-02	1.48E-01	1.35E-01
Lag 2	-1.32E-02	-3.46E-02	-3.46E-02
Lag 3	2.74E-03	-2.08E-03	-2.67E-03
Lag 4	4.65E-03	6.64E-04	-2.99E-04
Lag 5	1.66E-02	1.02E-02	1.02E-02
Absolute Returns			
Lag 1	1.51E-01	1.58E-01	1.72E-01
Lag 2	1.08E-01	1.08E-01	1.09E-01
Lag 3	9.55E-02	9.31E-02	9.50E-02
Lag 4	9.79E-02	1.01E-01	9.87E-02
Lag 5	8.80E-02	8.45E-02	8.18E-02

Table 4.6: Summary Statistics for Five-minute FTSE 100 Index Futures Returns

This table gives statistics for three five-minute intraday return series. *R-Closing* are returns from the latest closing price. *R-Simple Avg.* are returns from the simple average prices. *R-Volume Weigh.* are returns from the volume weighted prices. The returns from the latest closing prices are used in RV estimation.

Intraday Variance Multiplier Estimation

Following [60], the intraday volatility is modelled by assuming that there is a fixed multiplicative effect that may vary across the days of the week. The volatility for one day's return is from each five-minute time interval. The variance contribution from the same fixed time interval is assumed to be the same across different days.

It is found that the proportion of the time period from the previous day's closing to the first five-minute is substantial. For example, both the maximum and minimum returns are from this period in Table 4.6. This is because of the closing time period, which is from the close price on day *t-1* until the open price on day t. As such, the closing time periods are considered in this section.

Let $r_{t,j}$, $0 \leq j \leq n$, represent a set of $n+1$ intraday returns for day t, so that $j=0$ represents the closed-market period from the close on day $t-1$ until the open on day t, $j=1$ represents the first five-minute on day t,... concluding with $j=n$ representing the final five-minute on day t. Daily return r_t and the latent level of volatility σ_t for a day t are defined below.

$$\begin{aligned} r_t &= \sum_{j=0}^{n} r_{t,j} \\ var(r_t|\sigma_t) &= \sigma_t^2 \end{aligned} \tag{4.4}$$

Multiplicative volatility terms are defined by assuming that

$$\begin{aligned} var(r_{t,j}|\sigma_t) &= \lambda_j \sigma_t^2 \\ with \sum_{j=0}^{n} \lambda_j &= 1 \end{aligned} \tag{4.5}$$

The multiplier, λ_j is the proportion of a trading day's total return variance that is attributed to period j. It is the same for all days t. Here the intraday returns are assumed as uncorrelated. The proportion of open-market variance is defined by k_j in Equation 4.6.

$$\begin{aligned} k_j &= \frac{\lambda_j}{1-\lambda_0} \\ with \sum_{j=1}^{n} k_j &= 1 \end{aligned} \tag{4.6}$$

The variance proportions can be estimated as below in Equation 4.7. The summary over days t can be from all days or all Mondays, etc.

$$\begin{aligned} \hat{\lambda}_j &= \frac{\sum_t r_{t,j}^2}{\sum_t \sum_{i=0}^{n} r_{t,i}^2} \\ \hat{k}_j &= \frac{\sum_t r_{t,j}^2}{\sum_t \sum_{i=1}^{n} r_{t,i}^2} \end{aligned} \tag{4.7}$$

The variance proportion of the closed-market λ_0, from day *t-1* close to day *t* open, is substantial, particularly when days t are restricted to all available Mondays as shown in Table 4.7, which provides the estimated close-market proportion, $\hat{\lambda}_0$ and the first five-minute proportion on day t, $\hat{k_1}$, from different datasets. For example, 31.17 percent and 4.96 percent are estimated from all Mondays. The last column gives estimations from all available days.

The estimation of $\hat{\lambda}_0$ is 24.96 percent from all available days, increasing to 31.17 percent when only Mondays are used in the estimation, where the closed-market period is from Friday's close to Monday's open. Compared with the variance proportion of the closed-market, the first five-minute proportion from the open-market variance is much smaller. The estimation from all available Tuesdays gives the highest first five-minute volatility proportion (7.50 percent). This is reasonable as the close-market volatility proportion on Tuesday is the lowest one, at 17.51 percent.

The general pattern of the open-market variance proportions is given in Figure 4.2, in which the proportions are estimated from all available days. The estimated $\hat{k}_j$, starts from the high level at the first five-minute interval, then it declines until the interval of 08:20-08:25. This is followed by a general decline until a sharp increase in the interval from 13:30-13:35. It then gets very volatile. Starting from 14:30 until 16:30 the variance proportion remains at a high level while it is less volatile. After a short comparably low and smooth time period it closes at a high level.

Day Set	Monday	Tuesday	Wednesday	Thursday	Friday	All Days
$\lambda_0(\%)$	31.17	17.51	28.77	25.11	21.06	24.96
$k_1(\%)$	4.96	7.50	4.86	4.92	4.63	5.30

Table 4.7: Variance Proportion for Close-market and First Five-minute Interval

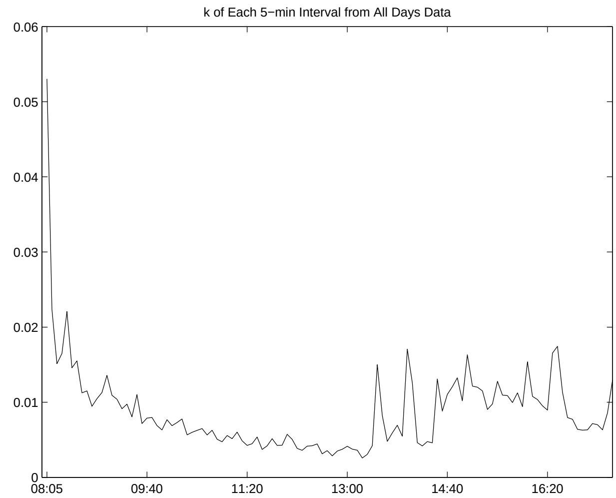


Figure 4.2: Open-Market Variance Proportions

The plot gives open-market variance proportions estimated for FTSE 100 Futures 2004

Realised Volatility

The realised volatility is used to measure the intraday return volatility. The realised variance for trading day *t*, from the close on day *t-1* to the close on day *t*, is estimated by weighting the intraday squared returns as in Equation 4.8.

$$\hat{\sigma}_t^2 = \sum_{j=0}^{n} \omega_j r_{t,j}^2 \tag{4.8}$$

To ensure conditionally unbiased estimates when intraday returns are uncorrelated, so that $E[\hat{\sigma}_t^2|\sigma_t^2] = \sigma_t^2$, it is necessary to apply the constraint $\sum_{j=0}^{n} \lambda_j \omega_j = 1$ [60]. Two sets of weights have been chosen for this purpose. The first method, referred to as equal weights is in Equation 4.9. The second method, referred to as optimal weights is in Equation 4.10, where $\hat{\lambda_0}$ and $\hat{k_j}$ are from Equation 4.7.

$$\omega_j = 1, 0 \leq j \leq n \tag{4.9}$$

$$\omega_j = \begin{cases} \frac{1}{(1-\hat{\lambda_0})n\hat{k_j}}, 1 \leq j \leq n \\ 0, j = 0 \end{cases} \tag{4.10}$$

Table 4.8 gives the summary statistics of the annualised realised volatility from equal weights and optimal weights. The average annualised volatility is 10.17 percent for equal weights and

10.37 percent for optimal weights. This is very close to the annualised daily return standard deviation, 10.26 percent, therefore, any bias caused by autocorrelation among intraday returns is small.

Table 4.9 gives the summary statistics of the logarithm of the daily realised volatility. The distribution of $ln(\sigma)$ is almost symmetric and approximately Gaussian when the optimal weights are used, the skewness and kurtosis then being 0.3459 and 4.2741 respectively. These moments of a Gaussian distribution are zero and 3. The skewness and kurtosis from the equal weights are much higher at 0.6736 and 4.4617. Therefore, the realised volatility from the optimal weights is preferred and used in the following modelling section. The augmented Dickey-Fuller test provides highly significant evidence that the realised volatility process does not contain a unit root. The low decline in the autocorrelations of the realised volatility series suggests a long memory process. The calculated RV is in Figure 4.3.

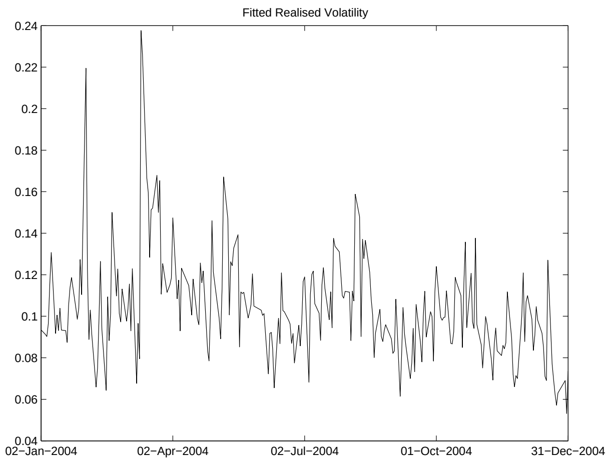


Figure 4.3: Annualised Daily Realised Volatility

RV is calculated from the optimal weights by five-minute returns, which is from the latest close prices.

$\sigma\sqrt{254}$	Optimal Weight	Equal Weight
Mean	0.1037	0.1017
Median	0.1001	0.0948
Standard Deviation	0.0253	0.0321
Skewness	1.6050	2.0571
Kurtosis	8.6006	9.5797
Maximum	0.2367	0.2751
	11 March 2004	10 May 2004
Minimum	0.0529	0.0499
	30 December 2004	06 September 2004
Autocorrelation		
Lag 1	0.5080	0.3385
Lag 2	0.3302	0.2931
Lag 3	0.2539	0.2318
Lag 4	0.2677	0.1983
Lag 5	0.3240	0.2678
Lag 6	0.2900	0.2832
Lag 7	0.1941	0.1762
Lag 8	0.1729	0.1448
Lag 9	0.1590	0.1086
Lag 10	0.1256	0.0075
Augmented Dickey-Fuller Test		
	Rejected	Rejected
Stat.	-8.9833	-11.0822

Table 4.8: Statistics of Intraday Volatility

$\sigma\sqrt{254}$ is the annualised realised volatility. There are 254 trading days in 2004.

$ln(\sigma)$	Optimal Weight	Equal Weight
Mean	-5.0579	-5.0905
Median	-5.0664	-5.1204
Standard Deviation	0.2271	0.2756
Skewness	0.3459	0.6736
Kurtosis	4.2741	4.4617
Maximum	-4.2057	-4.0554
	11 March 2004	10 May 2004
Minimum	-5.7039	-5.7622
	30 December 2004	06 September 2004
Autocorrelation		
Lag 1	0.5122	0.3485
Lag 2	0.3494	0.3094
Lag 3	0.2868	0.2548
Lag 4	0.3000	0.2189
Lag 5	0.3429	0.2921
Lag 6	0.2989	0.3052
Lag 7	0.1836	0.1902
Lag 8	0.1749	0.1630
Lag 9	0.1655	0.1120
Lag 10	0.1571	0.0455
Augmented Dickey-Fuller Test		
	Rejected	Rejected
Stat.	-8.9039	-10.9064

Table 4.9: Summary Statistics for the Logarithm of the Intraday Volatility

$ln(\sigma)$ is the logarithm of daily realised volatility.

4.5.3 Predictive Market Information

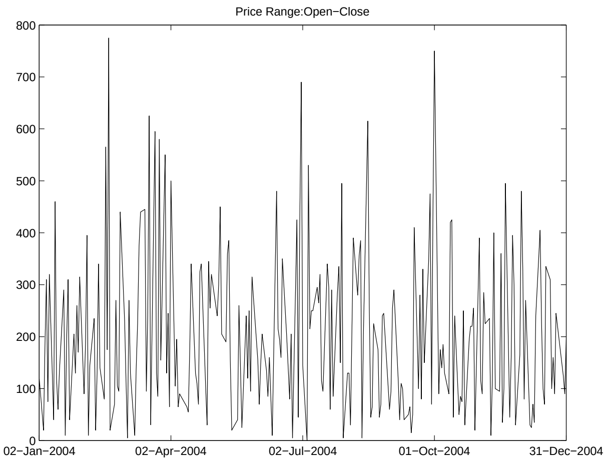
The potential explaining factors in the proposed RV modelling are RV lagged values and other market conditions including price range, bid-ask spread, trading volume, number of transactions, corresponding implied volatility and interest rates. The lagged information in Table 4.10 is used in the RV modelling directly to achieve better forecasting results. An attempt has been made to model RV by the contemporaneous predictive information first and then use the lagged information to forecast. The out-of-sample results from this deteriorated significantly compared with the very good in-sample fitting result. As such the lagged information is used in RV modelling.

There are two daily information variables created for the price range category including the absolute difference of open price and close price and the absolute difference of day highest price and day lowest price. The squared daily return is also used as the explaining variable. It is a proxy of the absolute price range from the close of adjacent days. The price range information is shown in Figure 4.4. There are two variables created for trading volume, total trading volume per day and average five-minute trading volume. The daily total transaction number and average trading duration are used directly. These variables are shown in Figure 4.6. For bid-ask spread category there are two variables created, daily average bid-ask spread, the maximum bid-ask spread is shown in Figure 4.7. The implied volatility of the at-the-money option with one month time to expire is calculated from a volatility surface using all option contracts traded in the previous day. It is shown in Figure 4.8, which has similar patterns as RV including long memory and persistence. Therefore one to five lagged information of implied volatility is included in the modelling. The squared RV is included based on the thought that it is an indicator of RV's volatility. Overnight Libor, one week and six-month Libor are used as the nominal interest rate proxies shown in Figure 4.5.

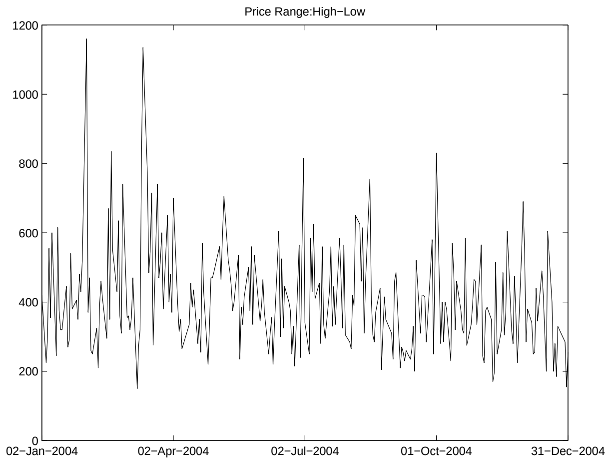
Label	Variable Name	Definition
s1	RV lag1	RV with one day lag
s2	RV lag2	RV with two days lag
s3	RV lag3	RV with three days lag
s4	RV lag4	RV with four days lag
s5	RV lag5	RV with five days lag
s6	Price Range(open-close)	The absolute difference of day open and close price
s7	Price Range(high-low)	The absolute difference of day highest and lowest price
s8	Total trading volume	The daily total trading volume
s9	average five-minute trading volume	The average five-minute total trading volume
s10	Transaction number	The daily total transaction number
s11	average Bid-ask Spread	The average daily absolute difference of bid and ask price
s12	Maximum Bid-ask Spread	The maximum daily absolute difference of bid and ask price
s13	IV	The implied volatility of at money option with one month to expire
s14	average Duration	The average daily trading duration in second
s15	Squared Daily Return	The squared daily return
s16	Squared RV	Squared RV
s17	IV lag2	IV with two days lag
s18	IV lag3	IV with three days lag
s19	IV lag4	IV with four days lag
s20	IV lag5	IV with five days lag
s21	Libor daily	Daily Libor
s22	Libor weekly	Weekly Libor
s23	Libor six months	Six month Libor

Table 4.10: Explaining Factors Used in RV Modelling

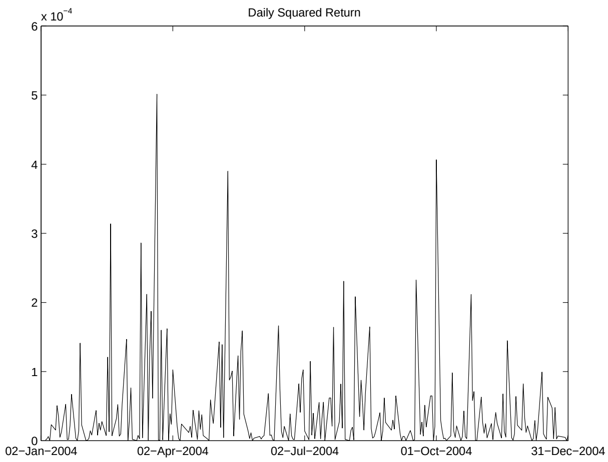
This table lists all explaining variables used in RV modelling. Variables *s6*-*s16* and *s21*-*s23* are one day lagged information.



(a) Absolute Difference of Open to Close

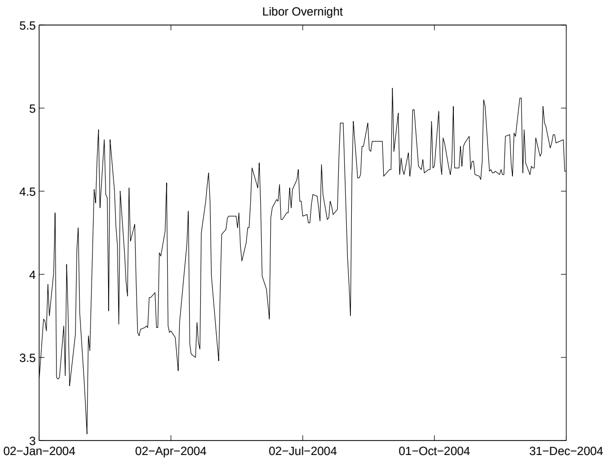


(b) Absolute Difference of High to Low

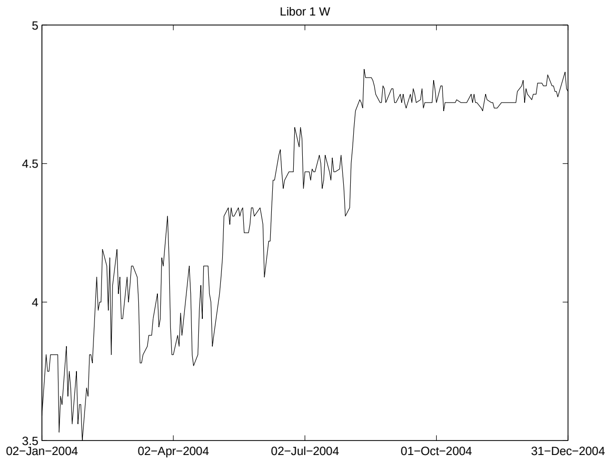


(c) Daily Squared Return

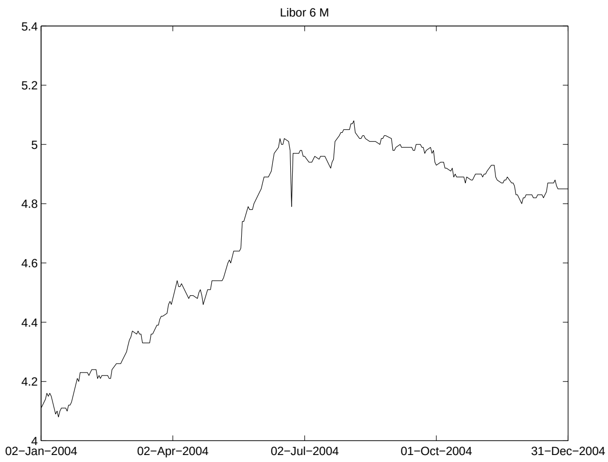
Figure 4.4: Daily Price Range Information in RV Forecasting



(a) Daily Overnight LIBOR

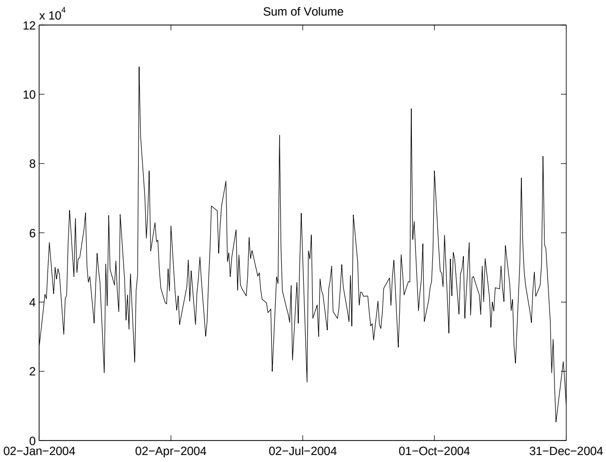


(b) Daily 1-week Libor

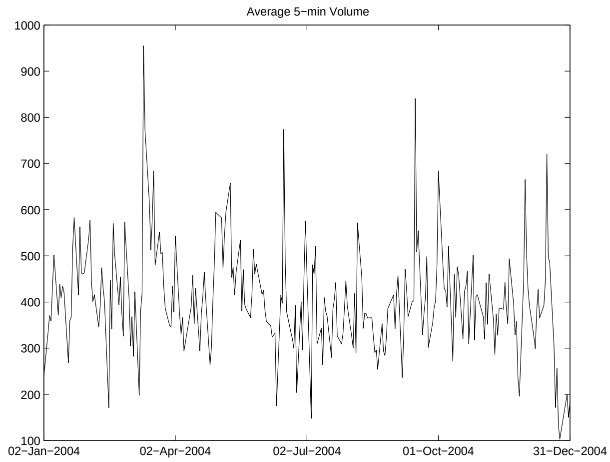


(c) Daily Six-month Libor

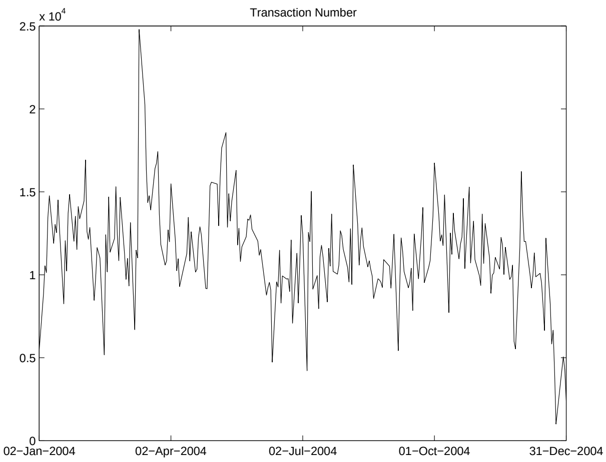
Figure 4.5: Libors in RV Forecasting



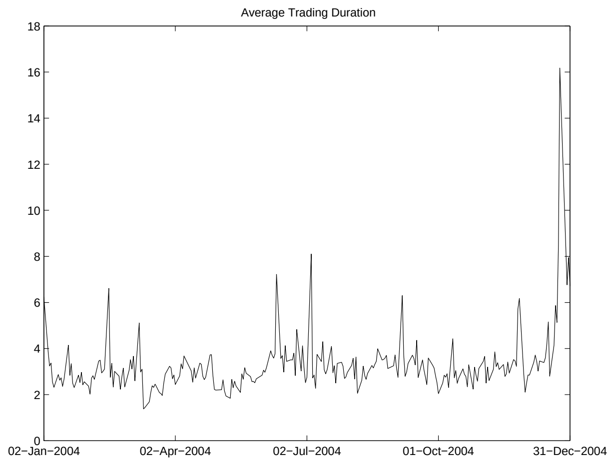
(a) Total of Trading Volume



(b) Average 5-minute Trading Volume

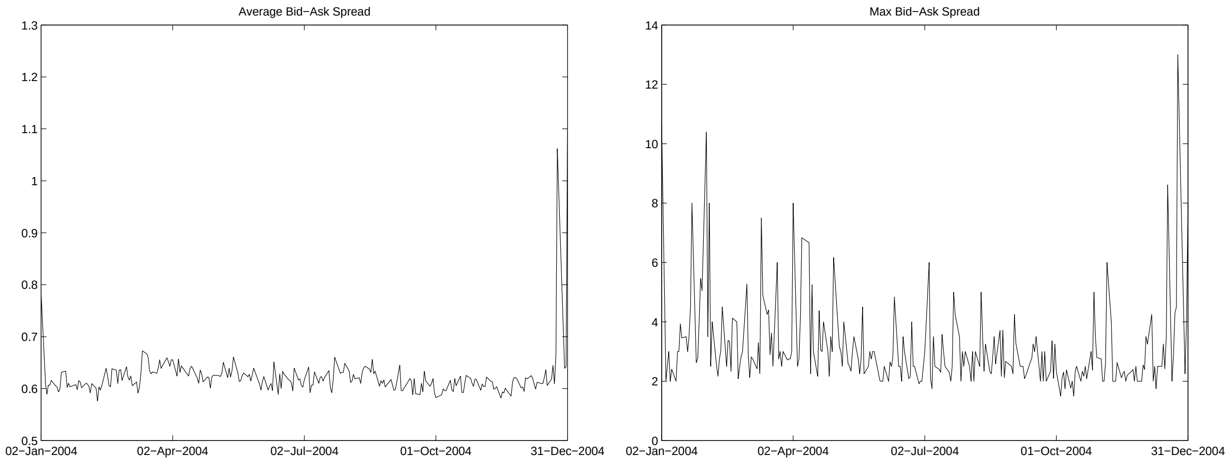


(c) Transaction Number



(d) Average Trading Duration

Figure 4.6: Daily Trading Action Measurements in RV Forecasting



(a) Average Bid-Ask Spread

(b) Max Bid-Ask Spread

Figure 4.7: Daily Bid-Ask Spread Variables in RV Forecasting

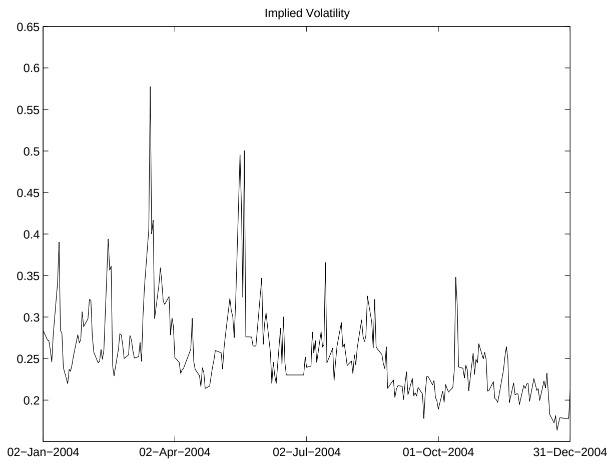


Figure 4.8: Implied Volatility of ATM Option Expiry in One Month

The implied volatility of the at-the-money option with one month to expire is calculated from a volatility surface, which is fitted by all available trading information of FTSE 100 Index Options on the previous day.

4.5.4 GP In RV Modelling

GP is a symbolic regression method. In this RV modelling application, the target variable is the realised volatility. GP in this application creates individuals, which are forecasting equations composed by explaining variables and mathematical functions. The evaluation of an individual is a predicted RV. All elements in a GP individual are from the GP's terminal set in Table 4.10 and function set in Table 4.11. Terminal set and function set are the basic GP parameter setting. The terminal set includes all explaining variables. The function set includes all prespecified functions.

Each individual in GP has a fitness value, which indicates how good they are when tested in the training dataset. The fitness function, in this application, is the mean squared error defined in Equation 4.11, where the RV_{target} is the target RV value, RV_{ind} is the evaluation of the individual and $Number_{Days}$ is the number of data points in the training dataset.

GP is trained with a rolling window approach. For each day's forecast in the out-of-sample dataset, the training data is from the start, the earliest available data, until the previous day of the day to be forecasted and there are 10 runs of GP performed. This process is demonstrated in Figure 4.9. In each run of GP there are 50 generations with the population size 50,000. The new population will be filled by three methods from the old generation. 55 percent of the population will be filled by a crossover method, 40 percent from a mutation method, and 5 percent from reproduction. To avoid an over-fitting problem the maximum depth size is set to six. The fitness function leads GP to return a best fit individual for a run. Therefore there are 10 out-of-sample predicted results for each day in the time period of 1 July 2004 to 31 December 2004. In theory, a bigger population usually gives a better result. However, the population size is limited by the computer power. Here a population size of 50,000 and 10 runs are used, which is subject to the limited computation resource to do the experiment. This forecasting method is time-consuming and has a large computation requirement. Therefore it is suitable for a short horizon forecast where the training dataset is comparably small. In this application, the average result from ten runs of GP is used as the predicted value. The method of using average GP result as the final solution has been used in [43], [44] and [52].

$$Fitness = \sqrt{\frac{\sum(RV_{target} - RV_{ind})^2}{Number_{Days}}} \tag{4.11}$$

Function Label	Function Name
+	Addition
-	Subtraction
$*$	Multiplication
/	Division
NormC	Cumulative Distribution Function of Normal Distribution
Exp	Exponential Function
rlog	Nature Logarithm Function
Sqrt	Square Root
$Sqrt_3$	Cube Root
Sine	Sine Function
Cosine	Cosine Function

Table 4.11: Function Set in GP RV Modelling

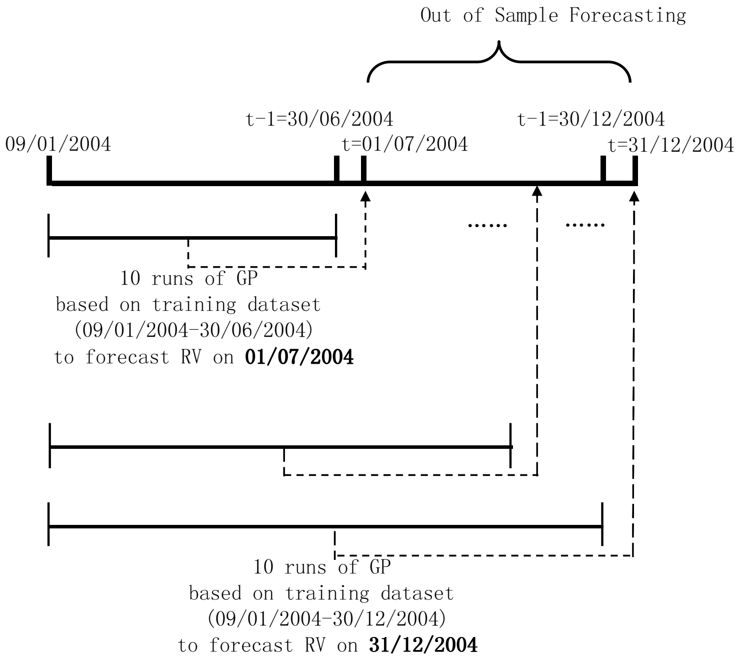


Figure 4.9: GP in RV Forecasting

This diagram demonstrates the process, in which GP forecasts RV. For each day's RV forecasting, there are 10 runs of GP. The training data is from the start until the previous day of the day, on which the RV is to be forecasted.

4.5.5 Benchmark Models

There are a number of benchmark models used in this RV forecast. The first model chosen is the ARMA, which is the most common statistic model and has been used in [128] and [129]. As reviewed in Section 4.3.3 the HAR model gives the best forecasting result in empirical studies [129] and [132] therefore it is also included here. Inspired by [131] and [159], a GARCH model is also fitted for RV, where RV and its volatility are modelled jointly. In all three benchmark models the only input factor is RV lagged information. The fourth benchmark model is the stepwise selected linear regression model, which can potentially take all market information variables as inputs.

ARMA Model

In the Autoregressive and Moving Average (ARMA) model, RV is assumed to be a combination of its own lags and a series of white noise error terms. The *ARMA(p,q)* model in Equation 4.12 has p autoregressive terms, $RV_{t-1}, \cdots, RV_{t-p}$ and q moving average terms, $\varepsilon_{t-1} \cdots, \varepsilon_{t-q}$, where the error terms, ε_{t-i} are assumed to be independent identically distributed random variables, sampled from a standard normal distribution and c is a constant.

$$RV_t = c + \sum_{i=1}^{p} \alpha_i RV_{t-i} + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i} \tag{4.12}$$

HAR Model

In the Heterogeneous Autoregressive(HAR) model [132], RV is modelled by its own lagged information, including RV one day before, average RV in the last week and average RV in the last month. This model is in Equation 4.13, where c, α, β and γ are constants.

$$\begin{aligned} RV_t &= c + \alpha RV_{t-1} + \beta RV_w + \gamma RV_m \\ RV_w &= \tfrac{1}{5} \textstyle\sum_{i=1}^{5} RV_{t-i}, \\ RV_m &= \tfrac{1}{21} \textstyle\sum_{i=1}^{21} RV_{t-i} \end{aligned} \tag{4.13}$$

GARCH Model

In the generalised autoregressive conditional heteroscedastic (GARCH) model, RV and its volatility, υ are modelled jointly in Equations 4.14, in which Equation 4.14a is the mean equation and Equation 4.16c is the conditional variance equation. z_t in Equation 4.14b is a white noise process. c, α_i, β_i, ϕ_i and η_i are constant coefficients. This is the same in Equation 4.12, ε_{t-i} are assumed to be independent identically distributed random variables sampled from a standard normal distribution.

$$RV_t = c + \sum_{i=1}^{p} \alpha_i RV_{t-i} + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i} + \nu_t \tag{4.14a}$$

$$\nu_t = \upsilon_t z_t \tag{4.14b}$$

$$\upsilon_t^2 = k + \sum_{i=1}^{p} \phi_i \upsilon_{t-i}^2 + \sum_{i=1}^{q} \eta_i \varepsilon_{t-i}^2 \tag{4.14c}$$

Stepwise Selected Linear Regression Model

Stepwise regression is a systematic method for adding and removing explaining factors from a multilinear model based on their statistical significance in a regression. The method begins with an initial model and then compares the explanatory power of incrementally larger and smaller models. At each step, the p value of an F-statistic is computed to test models with and without a potential factor. If a factor is not currently in the model, the null hypothesis is that the factor would have a zero coefficient if added to the model. If there is sufficient evidence to reject the null hypothesis, the factor is added to the model. Conversely, if a factor is currently in the model, the null hypothesis is that the factor has a zero coefficient. If there is insufficient evidence to reject the null hypothesis, the factor is removed from the model.

In this stepwise regression model, all market information variables in Table 4.10 are potential explaining factors. The stepwise regression model is fitted for each day's RV forecast based on the in-sample training dataset, which is from the starting day until one day before the day to be forecasted and this is the same as GP's training dataset explained in Section 4.5.4. As shown in Equation 4.15 the factors and factor number in each day's forecasting model may vary in this stepwise selected linear regression model. By default there is always a constant intercept c.

$$RV_t = c + \alpha_1 Var_1 + \alpha_2 Var_2 \cdots + \alpha_i Var_i \tag{4.15}$$

4.6 Forecast Results

4.6.1 Forecast Evaluation

Competing model performance is evaluated by three layers of tests. The forecast errors are measured firstly. Model prediction results are tested by a hypothesis if different models give equal accuracy. At the end, regression-based forecast efficiency tests are performed to answer the question as to whether predicted content from one model encompasses another.

Forecast Error Measures

The forecast errors are firstly measured by MAE (Mean Absolute Error), MAPE (Mean Absolute Percent Error), RMSE (Root Mean Square Error) in Equations 4.16, T is the data points number in the test, $Predicted_i$ is the predicted RV from competing models.

$$MAE = \frac{1}{T}\sum_{i=1}^{T} \mid RV_i - Predicted_i \mid \tag{4.16a}$$

$$MAPE = \frac{1}{T}\sum_{i=1}^{T} \frac{\mid RV_i - Predicted_i \mid}{RV_i} \tag{4.16b}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}(RV_i - Predicted_i)^2} \tag{4.16c}$$

Comparing Forecast Errors of Different Models

There are three Diebold-Mariano tests proposed by [162] to test the equality of forecast accuracy between two models. The tests relate prediction error to some very general loss function and analyse loss differential derived from errors produced by two competing models. The three tests include an asymptotic test that corrects for series correlation and two exact finite sample tests based on the sign test and the Wilcoxons signed-rank test. The last two sign-based tests in particular work well for small samples.

In this application, the loss differential is defined as the difference of the squared forecast errors from two competing models. The loss differential series, d_i are calculated in Equation 4.17, where $Predicted_i^a$ is the predicted value by model a and $Predicted_i^b$ is the one from model b for RV observation at i and $i=1,\cdots,T$. Under the null hypothesis, the two competing models give equally accurate results. The alternative hypothesis is that two prediction models do not give equally accurate results. Diebold-Mariano tests do not tell which one gives the better result. Three test statistics are shown in [162] to follow a standard normal distribution. The null hypothesis will be rejected at the 5 percent significant level if $|S| > 1.96$.

$$d_i = (RV_i - Predicted_i^a)^2 - (RV_i - Predicted_i^b)^2 \tag{4.17}$$

- The Asymptotic Test: According to the central limit theory, when the sample size is large, the sample mean of the loss differential approximately follows a normal distribution with constant mean and variance. $\overline{d}$ is the sample mean and $\widehat{avar}_{\overline{d}}$ is the estimate of the asymptotic long-run variance of $\sqrt{T}\ \bar{d}$. In this application the forecast is only one step ahead therefore no correlation adjustment is needed and $\widehat{avar}_{\overline{d}}$ is calculated as the variance of loss differential series. The test statistic is in Equation 4.18.

$$S_1 = \frac{\overline{d}}{\sqrt{\frac{\widehat{avar}_{\overline{d}}}{T}}} \sim_A N(0,1) \tag{4.18}$$

- The Sign Test: When the sample size is small, a finite sample test such as the sign test can be conducted. The sign test statistic is constructed in Equation 4.19, where $I(d_i)$ is one for $d_i > 0$ and otherwise zero.

$$S_2 = \frac{\sum_{i=1}^{T}[I(d_i)] - 0.5T}{\sqrt{0.25T}} \sim_A N(0,1) \tag{4.19}$$

- Wilcoxon's Signed Test: The test statistic is in Equation 4.20, where $rank(|d_i|)$ is the rank of the absolute values of loss differential series.

$$S_3 = \frac{\sum_{i=1}^{T}[I(d_i) * rank(|d_i|)] - 0.5T}{\sqrt{0.25T}} \sim_A N(0,1) \tag{4.20}$$

Forecast Information Content Tests

The regression-based method for examining the informational content of forecasts entails regressing the RV on the predicted value as shown in Equation 4.21, where α and β are constant coefficients. Depending on the forecast, the prediction is unbiased only if $\alpha = 0$ and $\beta = 1$. The R^2 tells the predictive power [98].

$$RV = \alpha + \beta Predicted \tag{4.21}$$

Additional forecasts can be added to the right-hand side of Equation 4.21 to check for incremental explanatory power. The first forecast is said to subsume information contained in other forecasts if these additional forecasts do not significantly increase the adjusted regression R^2. In this application there are five models used to forecast RV, among which ARMA, HAR and GARCH only use RV lagged information while GP and stepwise regression use market information variables as inputs besides RV lagged information. It is interesting to see what are the relations among forecasts from different models. This test answers the question whether forecasts from different models contain the same information.

4.6.2 Empirical Results

The out-of-sample results are reported in this section. The forecast errors are measured in Tables 4.12a for the full out-of-sample dataset, 129 days from July to December in 2004. It is obvious from the error measures that the result from the GP average prediction gives the minimum MAE, MAPE and RMSE and also the highest predict power in terms of R^2. The results are mixed for the rest of the models in this table. The SR model provides for the second best estimator in terms of MAE and MAPE. The HAR model provides for the second best estimator in terms of RMSE. The second best estimator in terms of the R^2 is the HAR model.

	MAE	MAPE	RMSE	R^2
ARMA	0.014340	0.158315	0.017398	29.17%
HAR	0.014306	0.158370	0.017351	30.42%
GARCH	0.014411	0.160022	0.017442	29.71%
SR	0.014189	0.154038	0.017488	25.39%
GP-avg	0.013060	0.142854	0.016081	39.32%

(a) From 1-Jul-2004 to 31-Dec-2004 (129 Days)

	MAE	MAPE	RMSE	R^2
ARMA	0.014120	0.147907	0.017739	20.90%
HAR	0.014018	0.147675	0.017642	22.32%
GARCH	0.014077	0.148209	0.017644	22.01%
SR	0.014377	0.146706	0.018103	17.56%
GP-avg	0.012951	0.132870	0.016552	30.25%

(b) From 1-Jul-2004 to 30-Sep-2004 (65 days)

	MAE	MAPE	RMSE	R^2
ARMA	0.014564	0.168885	0.017044	26.84%
HAR	0.014599	0.169232	0.017051	26.99%
GARCH	0.014751	0.172020	0.017235	25.25%
SR	0.013998	0.161485	0.016842	23.57%
GP-avg	0.013172	0.152993	0.015587	38.28%

(c) From 1-Oct-2004 to 31-Dec-2004 (64 days)

Table 4.12: Out-of-sample Forecast Error Measure

For easy comparison reasons the R^2 from the linear regression is also listed for each model, which regresses RV on the predicted value. The first table gives results for the full out-of-sample dataset. The following sub-tables give the results in the first half and the second half of the full time period.

To further examine the model performance, the full out-of-sample dataset is split in two. The forecast errors are measured in Table 4.12b for the first half time period from July 2004 to September 2004. Table 4.12c gives results for the second half time period from October 2004 to December 2004. The results are similar to those found in the full dataset. GP is the consistent best estimator in these two time periods in terms of MAE, MAPE, RMSE and R^2. The second best estimator in terms of R^2 is still the HAR model in both datasets. The second best estimator is either the HAR or SR models in terms of MAE, MAPE and RMSE. The HAR model performs better in the first half time period and SR is better in the second half time period. All five models show better predictive ability in the second half time period, this is indicated by the higher R^2

and lower RMSE. This may due to the longer training time period available compared with the first half time period. However, all five models show bigger MAE and MAPE in the second time period.

		ARMA	HAR	GARCH	SR	GP-avg
X1	ARMA		0.2160	-0.1733	-0.2316	3.9600
X2			-0.2641	-1.1446	0.9685	3.7859
X3			0.2409	-0.3867	0.6029	4.8858
X1	HAR	0.2160		-0.4731	-0.3303	3.3858
X2		-0.2641		-0.0880	0.7924	2.5533
X3		0.2409		-0.5559	0.2809	3.4519
X1	GARCH	-0.1733	-0.4731		-0.1054	3.4055
X2		-1.1446	-0.0880		0.7924	3.6098
X3		-0.3867	-0.5559		0.8545	3.6776
X1	SR	-0.2316	-0.3303	-0.1054		4.1235
X2		0.9685	0.7924	0.7924		2.7294
X3		0.6029	0.2809	0.8545		3.3744
X1	GP-avg	3.9600	3.3858	3.4055	4.1235	
X2		3.7859	2.5533	3.6098	2.7294	
X3		4.8858	3.4519	3.6776	3.3744	

Table 4.13: Diebold-Mariano Tests

This table gives D-M tests including the asymptotic test(X1), sign test(X2) and Wilcoxon's signed test(X3) on the full out-of-sample period. The null hypothesis that two models give equal accuracy results will be rejected at 5 percent significant level if $|X| > 1.96$.

Diebold-Mariano tests including the asymptotic test, sign test and Wilcoxon's signed test are done pair-wisely for five competing models on the full out-of-sample time period. The statistics are given in Table 4.13, which indicate whether the model performance measured above is statistically significant. All three tests give similar results that the prediction from GP is significantly different than the other four models. From the forecast error measures and R^2 we know that GP gives significantly more accurate results. The hypothesis that two competing model predictions are equally accurate is not rejected for the other four models when they are compared to each other.

	ARMA	HAR	GARCH	SR	GP-avg
α	-0.0010	-0.0048	-0.0132	0.0028	-0.0302
p-Value	0.9393	0.7302	0.3863	0.8471	0.0346
β	0.9718	1.0043	1.0876	0.9545	1.2813
p-Value	0.0000	0.0000	0.0000	0.0000	0.0000
R^2	29.17%	30.42%	29.71%	25.39%	39.32%
Adj-R^2	28.61%	29.87%	29.16%	24.80%	38.84%

(a) Regression Analysis with One Regressor and Intercept

Adj-R^2	ARMA	HAR	GARCH	SR	GP-avg
ARMA		29.33%	28.93%	28.05%	45.38%
HAR	29.33%		29.35%	29.33%	40.20%
GARCH	28.93%	29.35%		28.69%	39.24%
SR	28.05%	29.33%	28.69%		44.38%
GP-avg	45.38%	40.20%	39.24%	44.38%	

(b) Regression Analysis with Two Regressors and Intercept

Table 4.14: Out-of-sample Forecast Information Encompassing Test

The above two tables give forecast information encompassing test results on the out-of-sample time period. The first table gives a regression fitting result when regressing the RV on the predicted value from each of five models as shown in Equation 4.21. The second table gives the adjusted R^2 when extra prediction results from another model is added in to the right-hand side of the Equation 4.21 as a second regressor. An increased adjusted R^2 indicates that the first model can not subsume the second model and the second model does give extra prediction power.

The forecast information encompassing tests are performed by regression analysis in the full out-of-sample time period and the results are displayed in Table 4.14. Firstly, single factor analysis is performed for each model, where RV is the dependent variable and the prediction from each model is the explaining variable as in Equation 4.21. The coefficients fitted in the single factor regression analysis in Table 4.14a show that forecast results from ARMA, HAR, GARCH and SR are closer to an unbiased prediction than GP. The intercept α are very close to zero and the coefficient for the model prediction, β are closer to one in these 4 models. In the GP case, α is significant as it is not zero at the 5 percent level and the β, 1.2813 is significantly higher than one. However, indicated by R^2 the prediction power from GP is much higher than the other models. It is important to distinguish between bias and predictive power. An unbiased forecast is useless if forecast errors are always big. For a good forecast, the residual error should be small and the R^2 should be big [98].

The prediction result from another model is added to the right-hand side of Equation 4.21 as an extra explaining factor to test if it can further improve the prediction result. The adjusted R^2 from such a test is reported in Table 4.14b. An increased adjusted R^2 indicates that the second prediction has extra predicted information, which is not included in the first model. For example, the adjusted R^2 of the regression that ARMA is the single explaining factor for RV is 28.61 percent. When the prediction from HAR is added as the second explaining factor the adjusted R^2 is 29.33 percent, which is slightly higher than 28.61 percent. As such the prediction of ARMA does not subsume the prediction of HAR. While the adjusted R^2 from the regression RV against prediction from HAR is 29.87 percent, which is higher than 29.33 percent therefore the prediction result from HAR subsumes the prediction from ARMA. The result from the analysis finds that ARMA subsumes SR; HAR subsumes ARMA, GARCH and SR; and GARCH subsumes ARMA and SR. Even more interesting is that the GP does not subsume any of the other four models though it has the highest prediction power. When the other four model predictions are added separately, the adjusted R^2 are 45.38 percent, 40.20 percent, 39.24 percent and 44.38 percent accordingly, which are all higher than the adjusted R^2 of 38.84 percent, from GP itself. The highest combination 45.38 percent is from ARMA and GP. This may suggest that a joint forecast from the GP and other models could potentially increase the predictive power.

4.7 Market Conditions Analysis in RV Modelling

The average predicted value from 10 runs of the GP gives the best forecast performance among the competing models, which is discussed in Section 4.6.2. Within 10-run results, the best one gives performance above the average value when compared against the target value to be forecasted. In reality this best prediction cannot be used for forecasting because its in-sample training result is not guaranteed to be the best one and there is not a way to select it until the target is known. However, this does not stop us from using its functional form to investigate the relationship between market variables and RV. In this section the relationship between market information variables and RV is analysed through the best solutions from GP.

The R^2 from the best individuals nearly doubled from the average results in Tables 4.15b and

4.15c. The superior performance in Table 4.15 is from 129 individual trees, within which seven best individuals are selected to conduct the detailed analysis. They are functional forms evolved by GP for RV forecast on the first day of each month and the last day in the out-of-sample time period.

	MAE	MAPE	RMSE	R^2
GP-avg	0.013060	0.142854	0.016081	39.32%
GP-best	0.008845	0.097760	0.012304	67.94%

(a) From 1-Jul-2004 to 31-Dec-2004 (129 days)

	MAE	MAPE	RMSE	R^2
GP-avg	0.012951	0.132870	0.016552	30.25%
GP-best	0.008440	0.087120	0.012766	60.24 %

(b) From 1-Jul-2004 to 30-Sep-2004(65 days)

	MAE	MAPE	RMSE	R^2
GP-avg	0.013172	0.152993	0.015587	38.28%
GP-best	0.009258	0.108566	0.011815	71.83%

(c) From 1-Oct-2004 to 31-Dec-2004(64 days)

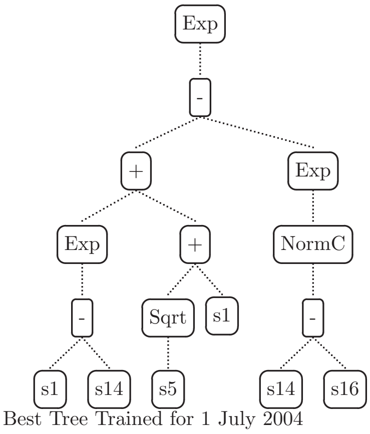
Table 4.15: Out-of-Sample Forecast: Best-Run and Average Comparison

This table compares the out-of-sample performance between best GP within 10-run and GP average prediction.

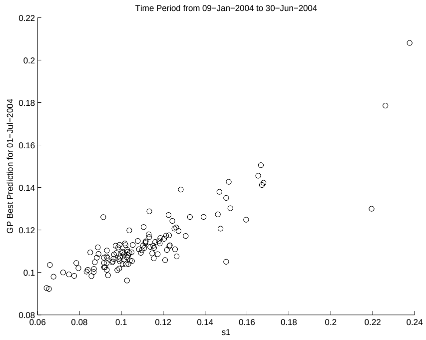
Target Date	Factors Selected
01-Jul	*s1*, *s5*, *s14*, *s16*
02-Aug	*s1*, *s5*, *s14*, *s16*, *s17*
01-Sep	*s1*, *s5*, *s8*, *s10*, *s11*, *s14*, *s15*
01-Oct	*s1*, *s5*, *s14*
01-Nov	*s1*, *s5*, *s7*, *s11*, *s12*, *s13*, *s14*, *s22*
01-Dec	*s1*, *s5*, *s6*, *s16*, *s22*
31-Dec	*s1*, *s5*, *s14*, *s16*, *s17*

Table 4.16: Factors in Selected GP Best Individuals

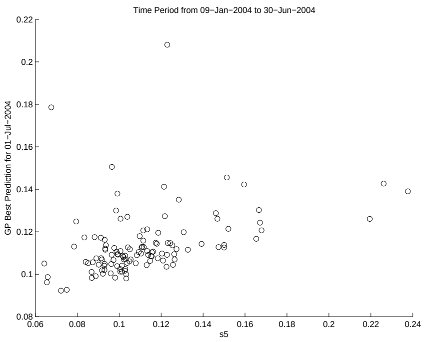
This table lists selected factors in the best GP individuals. *s1*:RV lag1; *s5*: RV lag5; *s6*: the previous day's price range, the absolute difference between open price and close price; *s7*:the previous day's price range, the absolute difference between day highest and day lowest price; *s8*: the previous day's total trading volume, *s10*:the previous day's transaction number; *s11*:the previous day's average bid-ask spread; *s12*:the previous day's maximum bid-ask spread; *s13*: the previous day's IV of at money option with one month to expire; *s14*: the previous day's average trading duration; *s15*: the previous day's squared daily return; *s16*: Squared RV with 1 day lag; *s17*: IV with 2 days lag; *s22*: the previous day's weekly Libor. More factor details refer Table 4.10.



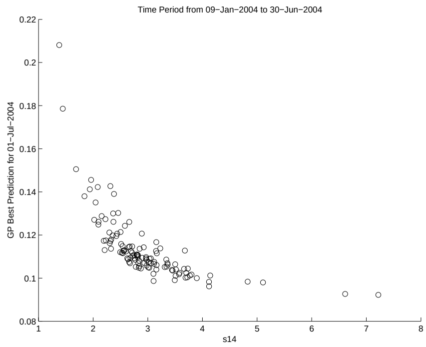
(a) Best Individual from GP



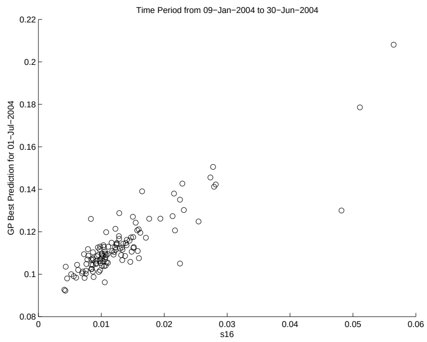
(b) Factor s1 vs Predicted Value



(c) Factor s5 vs Predicted Value



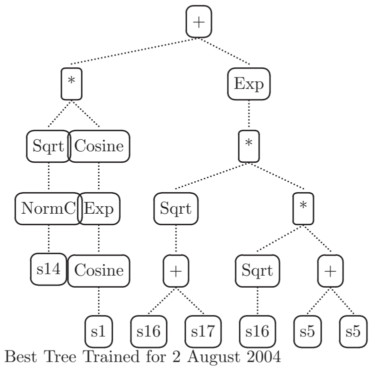
(d) Factor s14 vs Predicted Value



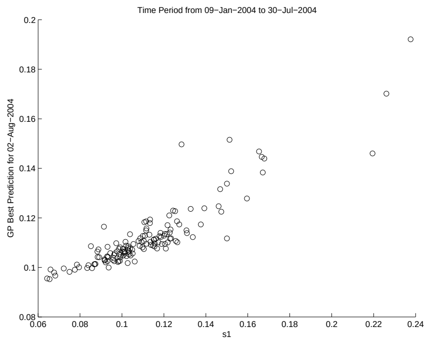
(e) Factor s16 vs Predicted Value

Figure 4.10: RV and Market Conditions for 1-Jul-2004

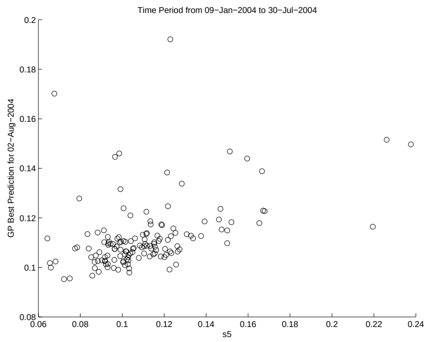
The first sub-figure gives the GP best individual, which is the closest to the target RV value among GP 10 runs trained for 01-Jul-2004. This GP tree is used to do the market information and RV analysis. The following sub-figures are scatter plots, where the y axis is the predicted RV value, x axis is the factor selected in GP tree and data covers the time period from 09-Jan-2004 to 30-Jun-2004.



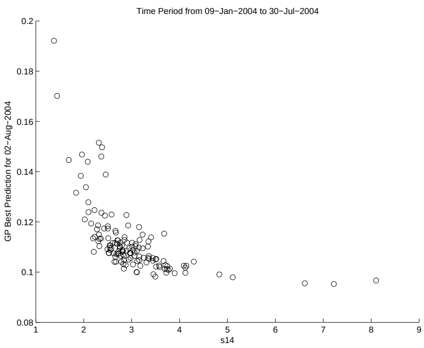
(a) Best Individual from GP



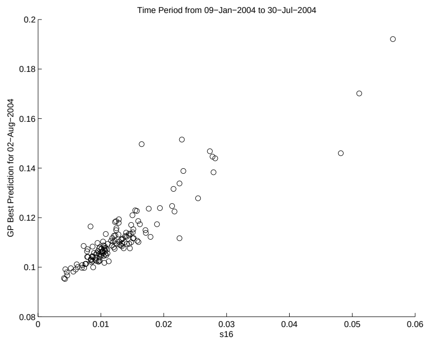
(b) Factor s1 vs Predicted Value



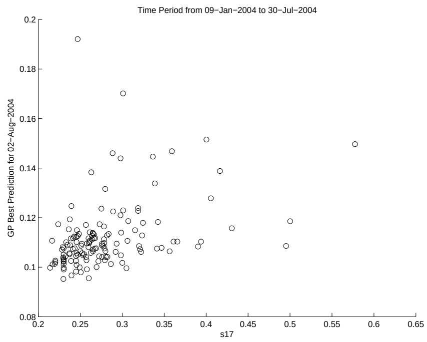
(c) Factor s5 vs Predicted Value



(d) Factor s14 vs Predicted Value



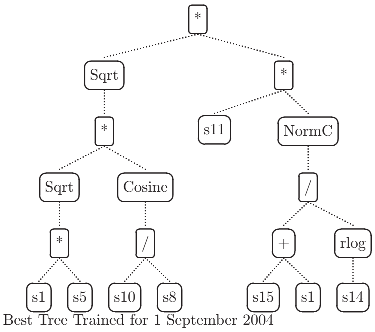
(e) Factor s16 vs Predicted Value



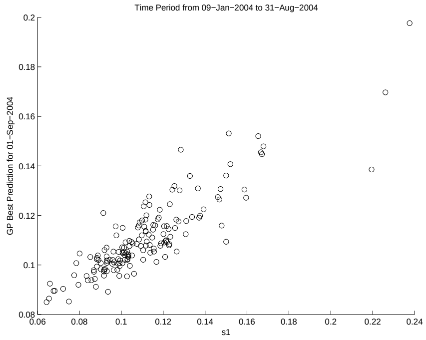
(f) Factor s17 vs Predicted Value

Figure 4.11: RV and Market Conditions for 2-Aug-2004

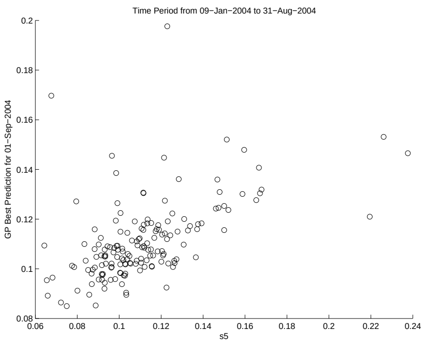
The first sub-figure gives the GP best individual, which is the closest to the target RV value among GP 10 runs trained for 02-Aug-2004. This GP tree is used to do the market information and RV analysis. The following sub-figures are scatter plots, where the *y* axis is the predicted RV value, *x* axis is the factor selected in GP tree and data covers the time period from 09-Jan-2004 to 30-Jul-2004



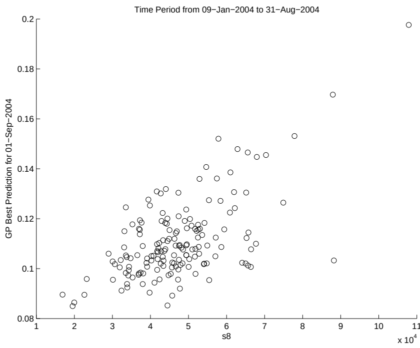
(a) Best Individual from GP



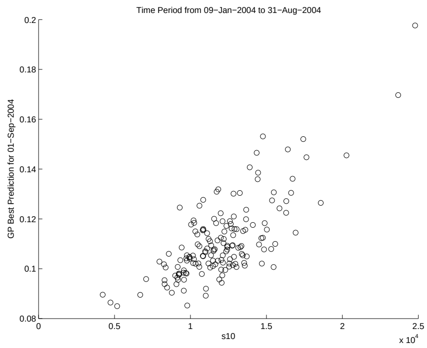
(b) Factor s1 vs Predicted Value



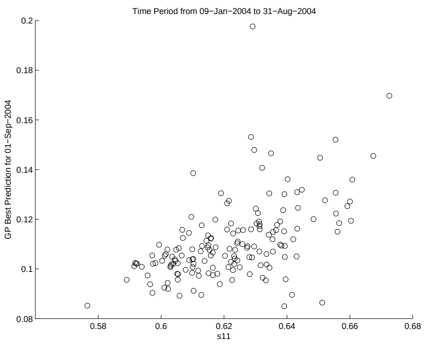
(c) Factor s5 vs Predicted Value



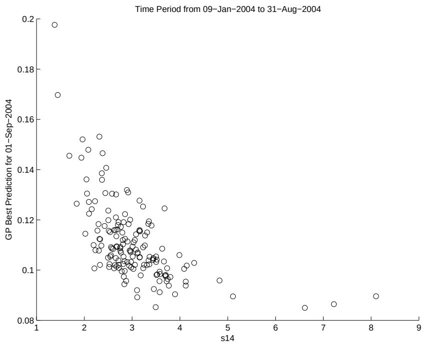
(d) Factor s8 vs Predicted Value



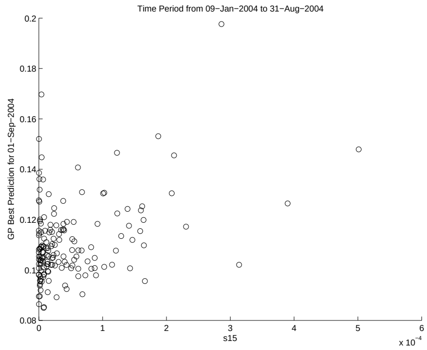
(e) Factor s10 vs Predicted Value



(f) Factor s11 vs Predicted Value



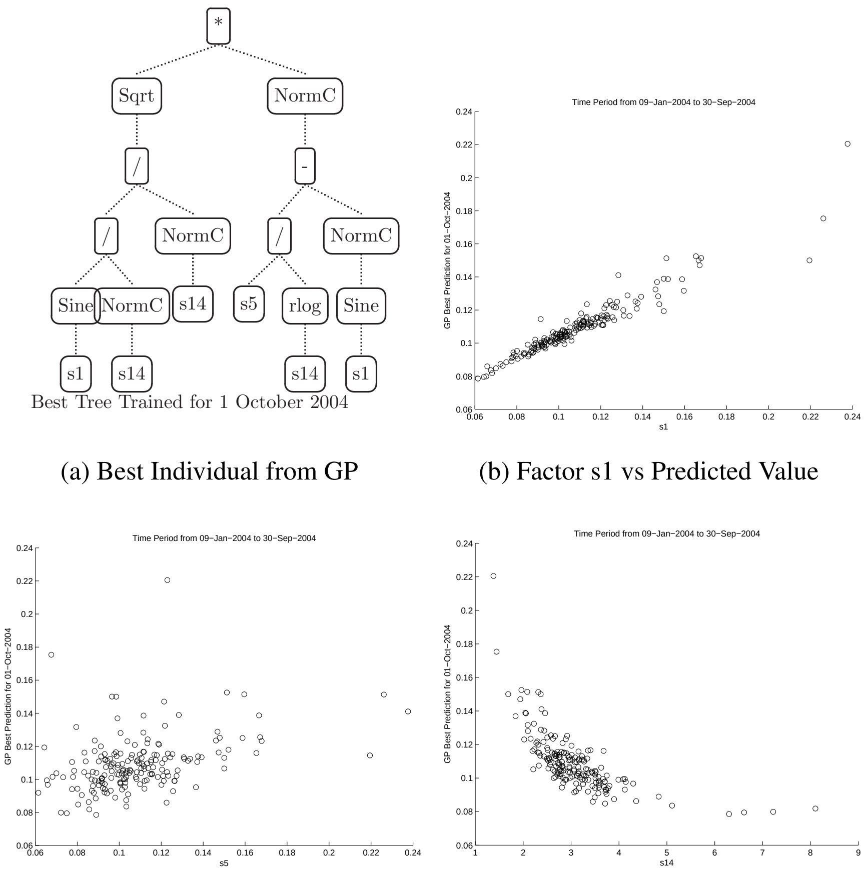
(g) Factor s14 vs Predicted Value



(h) Factor s15 vs Predicted Value

Figure 4.12: RV and Market Conditions for 1-Sep-2004

The first sub-figure gives the GP best individual, which is the closest to the target RV value among GP 10 runs trained for 01-Sep-2004. This GP tree is used to do the market information and RV analysis. The following sub-figures are scatter plots, where the *y* axis is the predicted RV value, *x* axis is the factor selected in GP tree and data covers the time period from 09-Jan-2004 to 31-Aug-2004



(a) Best Individual from GP

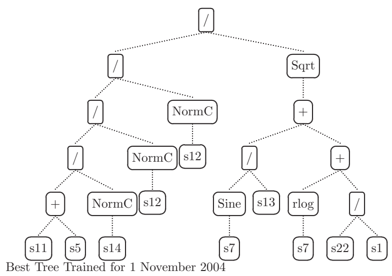
(b) Factor s1 vs Predicted Value

(c) Factor s5 vs Predicted Value

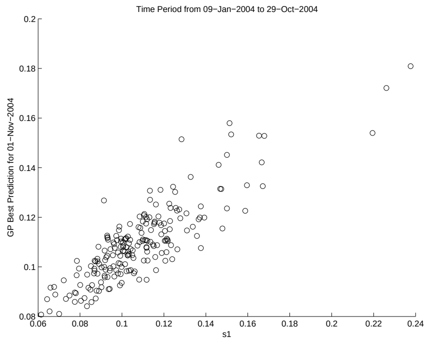
(d) Factor s14 vs Predicted Value

Figure 4.13: RV and Market Conditions for 1-Oct-2004

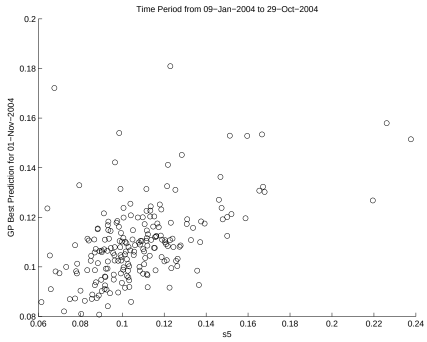
The first sub-figure gives the GP best individual, which is the closest to the target RV value among GP 10 runs trained for 01-Oct-2004. This GP tree is used to do the market information and RV analysis. The following sub-figures are scatter plots, where the y axis is the predicted RV value, x axis is the factor selected in GP tree and data covers the time period from 09-Jan-2004 to 30-Sep-2004



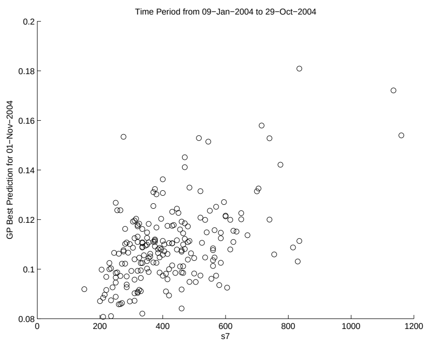
(a) Best Individual from GP



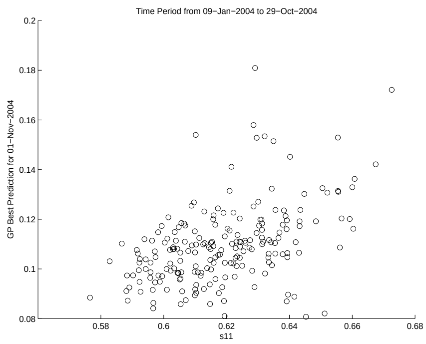
(b) Factor s1 vs Predicted Value



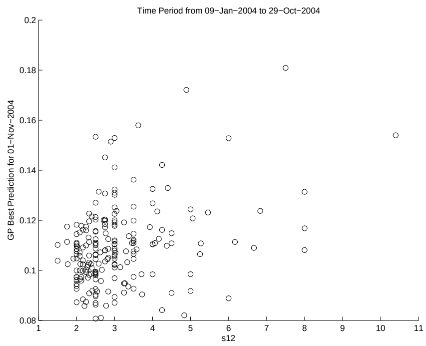
(c) Factor s5 vs Predicted Value



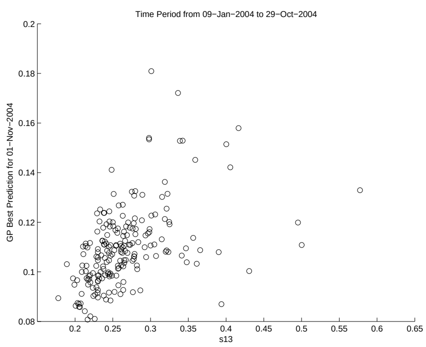
(d) Factor s7 vs Predicted Value



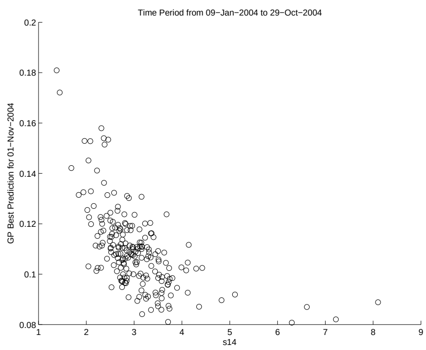
(e) Factor s11 vs Predicted Value



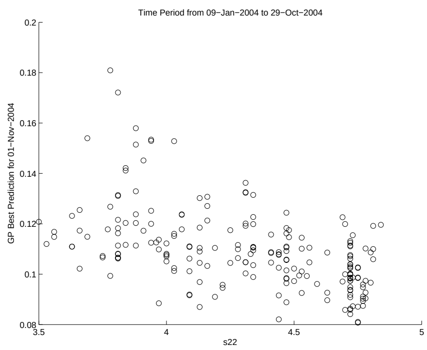
(f) Factor s12 vs Predicted Value



(g) Factor s13 vs Predicted Value



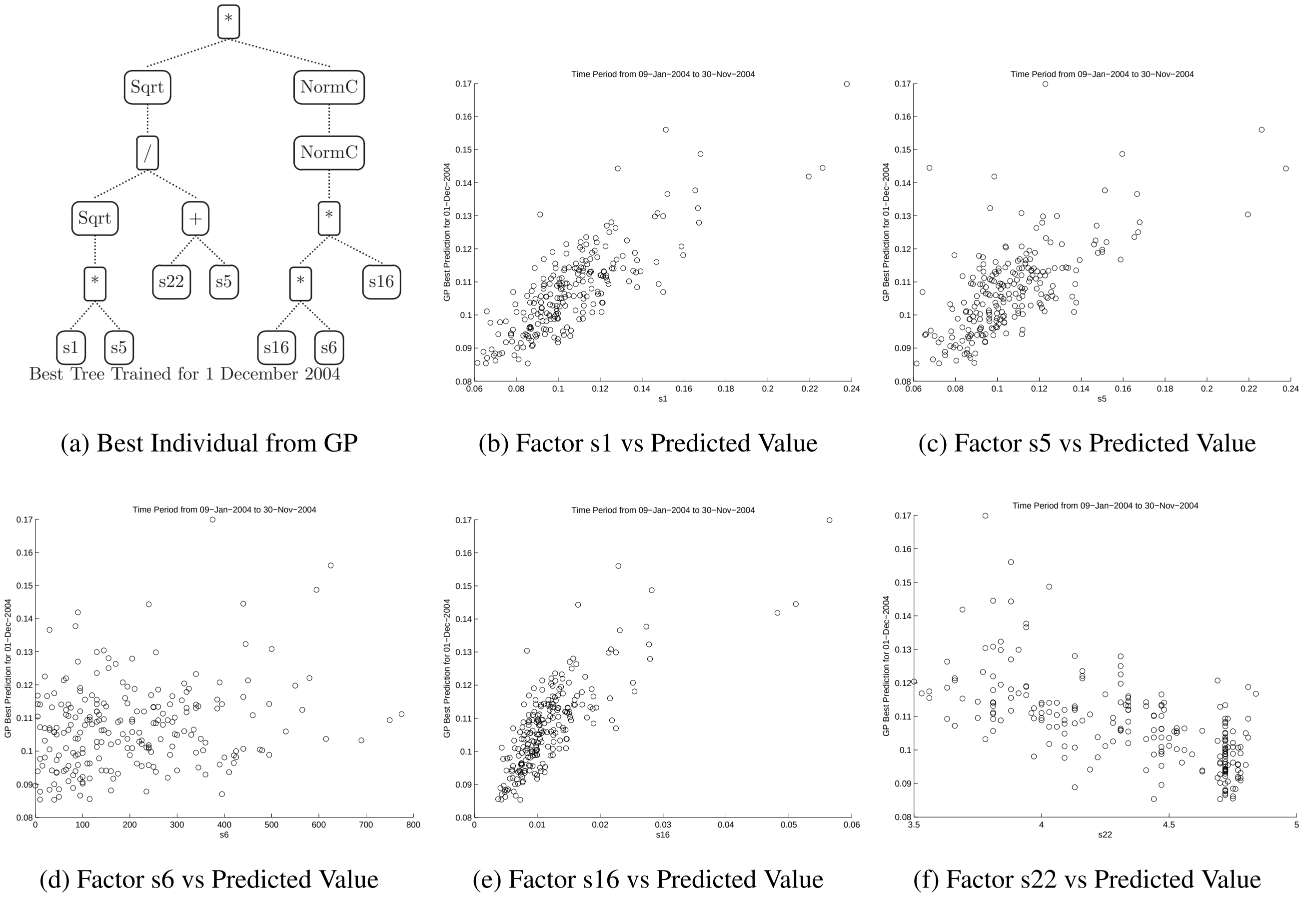
(h) Factor s14 vs Predicted Value



(i) Factor s22 vs Predicted Value

Figure 4.14: RV and Market Conditions for 1-Nov-2004

The first sub-figure gives the GP best individual, which is the closest to the target RV value among GP 10 runs trained for 01-Nov-2004. This GP tree is used to do the market information and RV analysis. The following sub-figures are scatter plots, where the y axis is the predicted RV value, x axis is the factor selected in GP tree and data covers the time period from 09-Jan-2004 to 29-Oct-2004



(a) Best Individual from GP

(b) Factor s1 vs Predicted Value

(c) Factor s5 vs Predicted Value

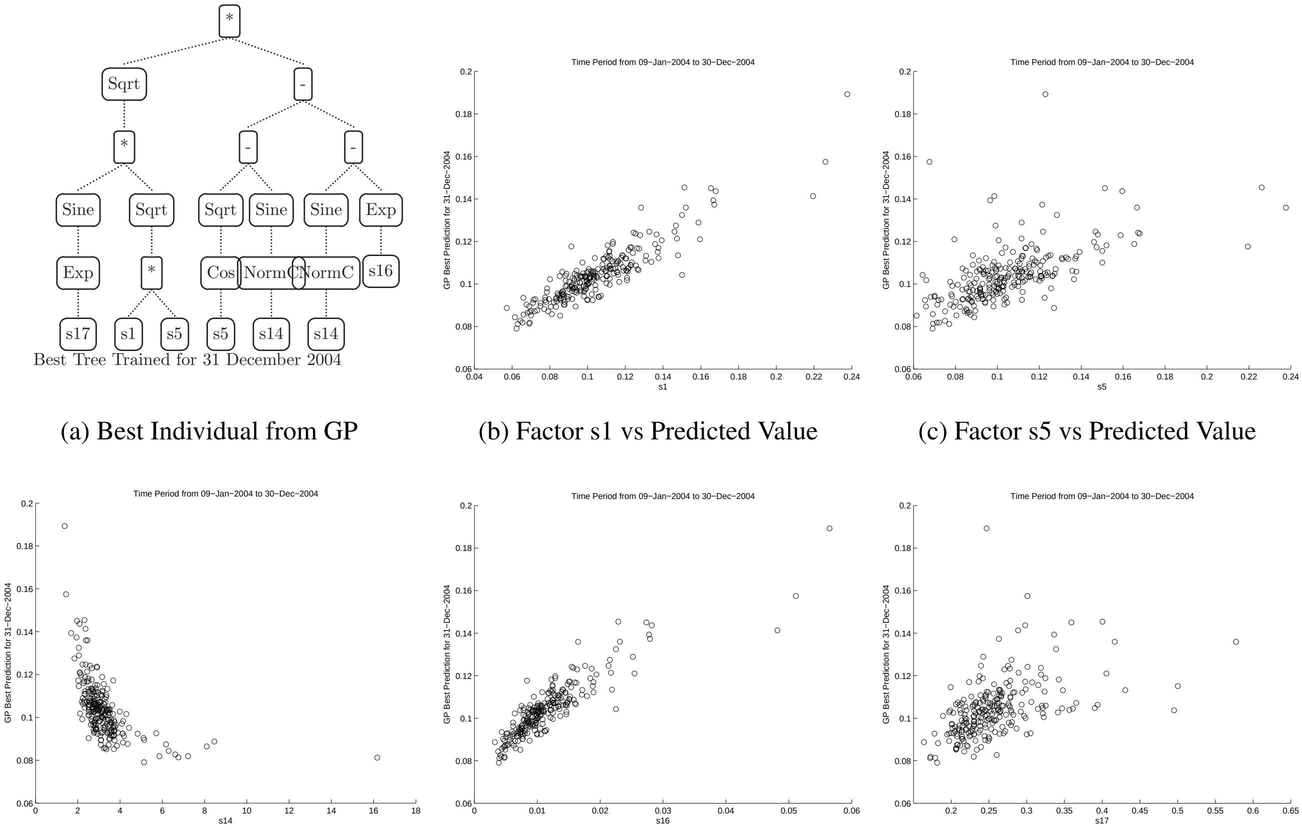
(d) Factor s6 vs Predicted Value

(e) Factor s16 vs Predicted Value

(f) Factor s22 vs Predicted Value

Figure 4.15: RV and Market Conditions for 1-Dec-2004

The first sub-figure gives the GP best individual, which is the closest to the target RV value among GP 10 runs trained for 01-Dec-2004. This GP tree is used to do the market information and RV analysis. The following sub-figures are scatter plots, where the y axis is the predicted RV value, x axis is the factor selected in GP tree and data covers the time period from 09-Jan-2004 to 30-Nov-2004



(a) Best Individual from GP (b) Factor s1 vs Predicted Value (c) Factor s5 vs Predicted Value

(d) Factor s14 vs Predicted Value (e) Factor s16 vs Predicted Value (f) Factor s17 vs Predicted Value

Figure 4.16: RV and Market Conditions for 31-Dec-2004

The first sub-figure gives the GP best individual, which is the closest to the target RV value among GP 10 runs trained for 31-Dec-2004. This GP tree is used to do the market information and RV analysis. The following sub-figures are scatter plots, where the *y* axis is the predicted RV value, *x* axis is the factor selected in GP tree and data covers the time period from 09-Jan-2004 to 30-Dec-2004

All factors in the selected GP best individuals are listed in in Table 4.16. Factor *s1*, RV with one day lag and factor *s5*, RV with five days lag have been in all of these seven best individuals. This indicates that RV is a self autoregressive process consistently across the whole out-of-sample time and lag one and lag 5 are critical to RV one-day-ahead forecast. Another factor *s14*, average trading duration with one day lag is nearly in all selected GP individuals, which indicates its strong relation with RV. Nearly all provided market information variables are in the selected GP best individuals at least once, except the previous day average five-minute trading volume, the previous day Libor daily and Libor monthly, IV lagged value from lag three to five and RV lagged value with lag two to lag four.

The scatter plots of each predictive factor against the predicted RV value are given for all selected best GP individuals in Figures 4.10 to 4.16. From Figures 4.10b 4.11b 4.12b 4.13b 4.14b 4.15b and 4.16b, the factor *s1*, RV with one day lag has a close to positive linear relationship with RV. As noted from Figures 4.10c 4.11c 4.12c 4.13c 4.14c 4.15c and 4.16c, factor *s5*, RV with five days lag has a positive relationship with RV, however the pattern for this relationship does not seem to be linear. An interesting factor is *s14*, as observed from Figures 4.10d, 4.11d, 4.12g, 4.13d, 4.14h and 4.16d it has a clear nonlinear relationship with RV, very close to one of rectangular hyperbola in the first quadrant, therefore it is inversely proportional to RV. From Figures 4.10e, 4.11e, 4.15e and 4.16e factor *s16*, the squared RV with one day lag, has a similar positive relationship as factor *s1*. This indicates that the volatility of RV lagged value is also important when estimating RV. This further confirms the results from [131] and [159] where volatility estimation performance improves when volatility of volatility is considered.

The lagged information of implied volatility of ATM option with one month to expire is in the best GP individual three times. From Figures 4.11f, 4.14g and 4.16f, IV lagged information has a positive nonlinear relationship with RV.

The ratio of lagged volume and transaction number is in the selected GP best individuals once. This ratio is linked to the predicted RV by multiple nonlinear functions including cosine, multiplication and square root, which make the relationship complicated. As observed from scatter plots, Figures 4.12d and 4.12e, both factors have a positive relationship with RV. The pattern of RV and lagged transaction number is closer to linear than lagged total volume.

Three lagged price range factors, the absolute difference of day open and close, the range of day highest and day lowest, the squared daily return are in the selected best individuals once. As in Figures 4.12h, 4.14d and 4.15d. The pattern of the range of highest and lowest price is clearer than the other two, which have a positive relationship with RV.

The lagged average bid-ask spread, factor *s11* is in the selected best individual twice, where different patterns are found in Figures 4.14e and 4.12f. GP has picked up different function forms for this variable in different time periods. However, in both cases it has a positive relationship with RV. The lagged maximum bid-ask spread is also in the GP best individual once.

As observed in Figure 4.14f, its relationship to RV is not very clear. It shows in the tree form in Figure 4.14a that it is linked by a cumulative normal distribution function. The lagged weekly Libor variable, *s22* is in the selected GP best individual twice as in Figures 4.15f and 4.14i. The patterns are similar in both cases, where it is inversely proportional to predicted RV though the exact function forms linked to it are different on two occasions.

The adaptive markets hypothesis proposed in [36] pointed out that investment strategies perform well in certain environments while may perform poorly in other environments. A strategy may decline for a time and then return to profitability when environmental conditions become more conducive to such trades. We get the same conclusion from the market conditions and RV forecasting analysis, that all market information variables have more or less predictive power for RV forecast. A factor may have a strong predictive power for a time and then less at another time. Factors that have nearly been in all of the selected best GP individuals in forecasting RV include RV with one day and five days lag and average trading duration. The relationships between these factors and RV seem robust over time. However, their contribution at different time periods vary, which is indicated by the subtle shape change in scatter plots from these factors against predicted RV at different time periods.

4.8 Chapter Summary

Daily return volatility is a fundamental risk measurement. Forecasting daily return volatility is very importance in finance. Traditionally, volatility is modelled by its own lagged information only. The relationship of market information variables and volatility has been studied for a few decades while there is no theory to show the suitable function form to be used on these market information variables when forecasting volatility. The question of how to utilise market information variables in volatility forecast is a challenging task. This chapter tries to forecast one-day ahead volatility by market conditions through a GP methodology.

Different to other modelling work, volatility is not directly observable from the market. The one year realised volatility of FTSE 100 index futures is first estimated by five-minute log returns. It is then modelled directly. Its lagged information and other intraday market information variables including price range variables, bid-ask spread variables, trading volume, transaction number, trading durations, nominal interest factors and implied volatility from at money options written on the FTSE 100 index with one month to expire are all used as inputs in this GP-RV-forecast modelling. The average value of 10-run results of GP is used as the daily RV prediction.

Model performance is compared with benchmark models including ARMA, GARCH, HAR and a stepwise regression. The error measures and R^2 select GP as the best estimator. The Diebold-Mariano tests confirm that this tournament result is statistically significant. Further, the regression-based forecast informational compassing tests show that the forecast information from GP does not fully overlap with the rest of the competing models, which indicates that a combination forecast from GP and conventional models could potentially improve the forecast performance.

The relationships between market information variables and RV are examined through the best individual from 10-run of GP. Three factors including RV with one-day lag, RV with five-day lag and the previous average trading duration have been in GP's formula consistently at different time periods although the exact function forms, which link them together, may vary. Other category market information variables are in GP's formula occasionally. These findings indicate that the relationships of market information variables and RV changes dynamically, which is consistent with the adaptive markets hypothesis market efficiency proposed in [36].

This chapter has made a few contributions, which are listed below.

- This chapter gives an up-to-date literature review of realised volatility modelling. The volatility concepts, traditional volatility modelling approaches and EC based modelling approaches are reviewed. The relation of volatility and market conditions including trading volume, number of transactions, bid-ask spread and price range are then covered. Empirical volatility modelling works are also summarised and the research gap in RV modelling is identified.

- This chapter forecasts one-day horizon realised volatility by market conditions through a GP approach. The out-of-sample forecasting results are compared with benchmark models including ARMA, GARCH, HAR and a stepwise regression. In the first three benchmark models only RV lagged information is used. All potential market condition factors provided to GP are also used in the stepwise regression. GP's forecasting results are significantly better than the benchmark models.

- The relationship of volatility and market conditions are examined through the best result from GP. GP's outstanding forecasting performance indicates that the relations between RV and market conditions are nonlinear. The relationships of market conditions and realised volatility are further analysed dynamically through the best individual from the GP. The factor of average trading duration has been shown to have a consistent nonlinear relation with RV.

Chapter 5

Empirical Analysis of Option Delta Hedging

5.1 Introduction

In this thesis, two detailed financial modelling studies are conducted on the ultra high frequency dataset of FTSE 100 index futures and options by a novel approach GP. In Chapter 2, the methodology, GP is introduced. In Chapter 3, the one year high frequency dataset is described and analysed. In Chapter 4, GP is used to model realised volatility by market condition variables. In this chapter, the Black-Scholes model(BSM) delta hedging is examined in one year high frequency FTSE 100 index futures and options dataset by three rehedging strategies, which include rehedging at uniform time intervals, rehedging when the underlying asset moves by a fixed number of ticks and rehedging based on a change in the delta of the option. Finally, this chapter develops an optimal delta hedging strategy based on the BSM delta hedging ratio, in which rehedging is triggered by the output from a GP that bases the rebalancing decision on a nonlinear function of a number of liquidity and volatility measures related to market conditions. Transaction costs are also taken into account in all hedging strategies by using the corresponding bid and ask futures prices when trading the underlying security to maintain a hedged portfolio. The relations between market information variables and the optimal delta hedging strategy are examined through the best individual from GP.

The rest of the chapter is divided into seven sections. Section 5.2 gives an introduction to this study. Section 5.3 reviews the theoretical and empirical works in delta hedging. Section 5.4 gives the motivations for this empirical analysis in delta hedging. Section 5.5 explains the tests that have been designed for this empirical analysis. Section 5.6 describes data and the methodologies used in the analysis. Section 5.7 presents the analysis results. The relations of market conditions and the optimal hedging strategy are analysed in Section 5.8. The chapter summary is then given in Section 5.9.

5.2 Delta Hedging

Derivatives market makers will usually take a position in the underlying stock, futures contract or another option, to dynamically hedge the naked position in the derivative contract. The objective of a hedge is to minimise the risk from the position in the derivative security. The position in the hedging security offsets the position in the derivative contract and in a perfect Black-Scholes type market a derivatives market maker who hedges their position will bear no risk.

Delta hedging is an options strategy that aims to hedge the option risk associated with underlying price movements by trading the underlying assets. Compared with other hedging strategies, such as the locally variance optimal strategy, it is simple to understand and easy to implement. The analysis in [118] shows when the realised distribution is not correctly estimated, delta hedging outperforms the locally variance optimal strategy. The study in [104] concludes that for a European call option, delta hedging performs similarly to mean square optimal hedging, and this is further evidenced in the empirical analysis in [119].

The delta of a stock option, Δ, is the ratio of the change in the price of the stock option to the change in the price of the underlying stock. A derivatives market maker will need to sell Δ shares of the underlying stock to hedge a short put option. The gain (loss) from the short put option offsets the loss (gain) from the short stock position. According to the Black-Scholes model (BSM) [67], as long as the hedging portfolio including the underlying stock, is re-balanced continuously with Δ re-calculated continuously, the portfolio will be perfectly hedged with a zero hedging error, i.e. the payoffs from the positions in the option and in the underlying stock offset each other. However, in real world financial markets this is not the case as the assumptions in the BSM do not hold. The payoff from the hedging portfolio will not be the same as the derivative payoff and the difference is called a hedging error.

The BSM assumes that there are no transaction costs and that security trading is continuous. However, recent advances in theory have relaxed these assumptions. A sample of these studies including [68], [69], [72] and [82] have shown that optimal hedging involves a trade-off between rebalancing costs and risk. However, in reality the question of what is the optimal hedging frequency in real world financial markets cannot be easily answered without empirical tests using real data. This is because in reality financial markets are incomplete and many of the assumptions in the more advanced theoretical models mentioned above will not hold. For risk management purposes, option traders are often required to close their book or limit their exposure during periods of no trading of the underlying asset, therefore they need to rebalance the option hedge back to a delta-neutral position at least daily. Empirical tests in the literature so far have assumed a maximum of daily rebalancing.

Financial high frequency data has been available for a number of years now. Intuitively high frequency data can be used to test hedging performance when portfolio rebalancing becomes more and more frequent. The expectation is that when rebalancing frequency increases, for example from weekly to daily to hourly etc., the hedging error will decrease. However, as far as the authors are aware, there has been no research testing derivative hedging performance using an intraday rebalancing frequency. One potential reason is that market microstructure effects exist in intraday data apart from transaction costs. A recent strand of literature has focused on market microstructure effects and volatility modelling. Microstructure effects are present in high frequency data but are particularly prominent in very high frequency data (higher than 5 minutes or more frequent) resulting in the typical choice for modelling frequency to be 5 minutes, or less frequent, see [101]. The rebalancing frequency in current hedging empirical work is only up to a daily frequency.

This application tries to fill this gap in the literature by examining discrete hedging error using high frequency data. Round trip transaction costs are considered in the empirical tests by using either bid or ask prices instead of traded prices as used in [106]. BSM delta hedging is systematically examined using high frequency data and a number of different hedging strategies are tested. The bid-ask transaction cost impact is analysed. According to the adaptive market hypothesis [36], an optimal hedging strategy should be the one that best adapts to the market environment. GP is used to find such a hedging strategy by utilising the available market conditions.

A few contributions result from this study. The first is to provide an up-to-date literature review of theoretical and empirical works in delta hedging. The second is to examine the empirical performance of BSM delta hedging theory in a high frequency intraday setting. The relationship between rebalancing frequency and hedging error is investigated. The third is to compare time based rebalancing strategies, underlying price move triggered rebalancing strategies and delta move based strategies using high frequency data with transaction costs considered. The fourth is to assess the bid-ask transaction cost impact on delta hedging. The designed delta hedging tests have been carried out in two steps. The first one does not consider transaction costs and the second one does. The last is to present an optimal control hedging strategy, using GP, that triggers a rehedging decision which is conditional on intraday market conditions.

5.3 Literature Review

The theoretical and empirical delta hedging studies have been reviewed separately, which are shown in Figure 5.1.

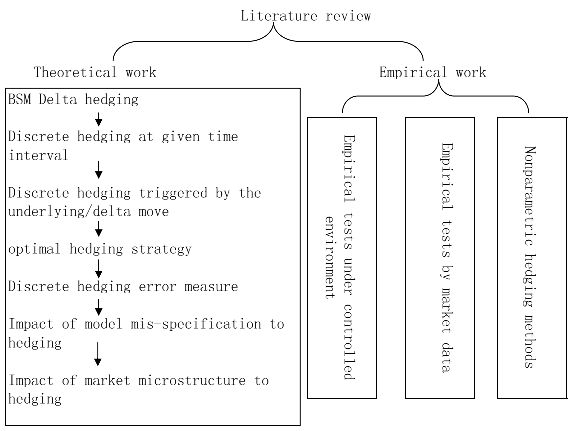


Figure 5.1: Delta Hedging Literature Reviews

In the theoretical work section, the BSM delta hedging theory is firstly reviewed in Section 5.3.1. The works of hedging under discrete trading and transaction costs are reviewed in

Sections 5.3.2, which covers the discrete hedging at given time intervals, the discrete hedging triggered by an underlying price move and delta move and the optimal hedging strategy. The discrete hedging error measure follows in Section 5.3.5. The impact of model mis-specification on hedging is reviewed in Section 5.3.6. The market microstructure impact on hedging is studied in Section 5.3.7.

The empirical studies of delta hedging are reviewed in the following Section 5.3.8; the analysis carried out by simulated environment is reviewed firstly and then the empirical works by market data are reviewed. Section 5.3.9 looks at the nonparametric methods used in delta hedging.

5.3.1 BSM Delta Hedging in BSM

In the Black-Scholes model [67] the underlying stock price S at time t is assumed to follow a geometric Brownian motion as in Equation 5.1 below:

$$\frac{dS}{S} = \mu dt + \sigma dz, \tag{5.1}$$

where dz is a standard Wiener process, μ is the drift and σ is the volatility of the stock and these are assumed to be constant. In the BSM, the principle of no arbitrage opportunities applies. A portfolio composed of an option and Δ units of the underlying stock earns the risk-free rate as long as the portfolio is rebalanced continuously to update the Δ. The riskless portfolio with one short call (put) option needs to be long (short) Δ $(1-\Delta)$ shares of the underlying stock at any given time, where the Δ of a European call option with dividend is given as in Formula 5.2.

$$\Delta = e^{-qT} N(d_1) \tag{5.2}$$

where

$$d_1 = \frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{5.3}$$

where $N(x)$ is the cumulative probability distribution function for a standardised normal distribution, K is the strike price of the option, r is the continuously compounded risk-free rate, q is the rate of dividend yield and T is the time-to-maturity of the option.

The most restrictive assumptions in the BSM from the perspective of derivatives market makers are the assumptions of continuous trading and no transaction costs. Recent theoretical advances have relaxed these assumptions to examine option pricing and hedging in the presence of transaction costs and discrete time trading.

5.3.2 Rebalancing at Given Time Interval

One of the earliest studies to examine discrete hedging was [68] which analysed the main components of returns from a discretely re-balanced hedge portfolio. Leland [69] explicitly proposed a modified option replicating strategy based on the BSM where the hedging strategy itself depends on the percent transaction cost and the revision interval. A number of researches followed this direction including [77], where the hedging strategies proposed were able to cover large transaction costs or small time-intervals between rebalancing, and [78], where the strategy developed includes fixed cost structure and also reduces the modified variance described by Leland in the case of a single option. Parallel with this route, [73] proposed a hedging strategy covering transaction costs from a binomial lattice framework.

Research Works	Revision Frequency
[68]	1 day
[69]	1 week,4 week,8 week
[77]	0.26day, 0.52 day, 2.6 day
[73]	1 day,1 week, 2 months
[82]	1 day,1 week, 1 months and 6 moths

Table 5.1: Rebalancing Frequency in Discrete Hedging Strategies

The rebalancing frequency in the models discussed above happens at a given time interval as in Table 5.1. Theoretically the more often the portfolio is re-balanced, the less risk, however more costs are incurred, therefore the hedging strategy is a trade-off between transaction costs and residual hedging errors when transaction costs are included. The analysis of [80] gives the optimal rebalancing frequency, approximately every week under a very strong assumption that the underlying growth rate is more than the risk-free rate. This assumption seems reasonable in the long run. However, it is hardly met in a short and volatile period.

5.3.3 Rebalancing Triggered by Underlying Price/Delta Move

In BSM delta hedging, the underlying price is the only changing element shown in Equation 5.2. There are hedging strategies with the revision triggered by an underlying price change in the studies of [75] and [82] or by delta change itself in the studies of [76] and [77]. The optimal hedging frequency is still not answered. It is not clear what the best threshold is, in the move of the underlying price or delta, to trigger the hedging portfolio revision.

In the study of [75], the move based hedging strategy and regular time based hedging strategy were compared. The approximated expressions were provided for expected transaction costs and the variance of the total cash flow for both strategies. Toft [82] simplified these expressions and computed general input parameters. The conclusions is that the move based trading strategy is not always better than the time based strategy. It depends on the underlying volatility and transaction costs. When volatility is low and transaction costs are high then the time based one is better. In this study, the rebalancing frequencies for a move based strategy were tested from an underlying price changed by 0.3 percent to 6.3 percent.

5.3.4 Optimal Hedging Strategy

The optimal hedging strategy in the presence of proportional transaction costs was proposed in [72] and [74] through “Utility Maximisation”. The option writing price was obtained in [74] by comparing the maximum utilities available to the writer by trading in the market with and without the obligation to fulfil the terms of an option contract at the exercise time. Optimality in their model is attractive. However, it is slow to compute as it usually results in three or four dimensional free boundary problems. This optimal method rebalances the portfolio whenever a control variable hits the boundary of a no transaction region. This control variable is optimised endogenously. The analyses were extended in [84] under a general cost function with fixed and proportional costs.

A few papers made the effort to improve the computation speed for this optimal hedging strategy including asymptotic analyses proposed in [83] and [87] and the analytic approximation approach in [105].

Compared with the models discussed in Section 5.3.2, these optimal hedging strategies give endogenous re-balancing frequency. The model indicates when to rebalance. The optimal re-balancing frequency is solved theoretically in this optimal hedging strategy. However, the drawback is obvious. Besides computational problems, it also requires the investors' risk preference to be specified and requires constant monitoring of the market.

5.3.5 Discrete Hedging Error Measure

The analysis in [82] gave the closed form solutions for the expected hedging error, transaction cost and variance of the cash flow from a discrete time delta hedging strategy in the special case of geometric Brownian motion. Further, this study characterised the cost and risk of a move based hedging strategy without resorting to Monte Carlo simulations.

Different to the studies above where the focus is the trade-off between the magnitude of tracking errors and the cost of replication, the work of [90] looked at only one of these two issues, discrete trading. This study characterised the asymptotic distribution of the replication errors arising from delta hedging derivatives in discrete time and introduced a notion of "temporal granularity", which is a particular function of a derivative contract's terms and the parameters of the underlying stochastic process. Through this measure, the extent to which the continuous time models are implemented discretely is linked with the efficiency of the replication strategy.

A more general study in [103] gave the asymptotic distribution of the hedging errors in a continuous *Itô* process driven by a multi-dimensional Brownian motion. There was an extended work to process with jumps in [109].

5.3.6 Impact of Model Mis-specification on Hedging

The expected value and variance of the hedging error for a contingent claim when trading in discrete time were computed by a laplace transform approach in [117]. The method applies to a fairly general class of models, including Black-Scholes and Merton's jump-diffusion. Based on the developed method, the effect of a misspecified volatility parameter on the hedging error in the BSM framework was studied. An extended study in [118] investigated the effect of different types of model mis-specifications on the performance of the hedging. The analysis shows that when implementing hedging under the right realised distribution, the performances from different hedging strategies are similar. When the skewness and the excess of kurtosis were incorrectly estimated, overall the BSM delta strategy performs better than the locally variance optimal strategy. That is because in the latter case, one has to adequately fit the realised distribution. The study in [120] also assessed the impact of model mis-specification on hedging. Through the hedging error distribution analysis, the conclusion was drawn that accounting for stochastic volatility is the most important in order to reduce hedging errors arising from model mis-specifications.

A computationally efficient algorithm to determine the moments of the distribution of dynamic hedging error under discrete trading was developed in [104]. By analysing the moments of hedging error, the Merton model based delta hedge [45], the BSM based delta hedge and the mean square optimal hedge were compared. The results indicated that when the underlying asset was modelled as a jump-diffusion process, the mean square optimal strategy for a call option outperformed the delta hedging strategies. An extension of these algorithms to a market with transaction cost was provided in [107]. The Black-Scholes delta hedge and Leland adjusted hedge were compared through the hedging error distribution and moments analysis. As expected, under transaction cost, the Leland adjusted hedge was superior to the Black-Scholes delta hedge. The trade-off curve relating the number of trading opportunities to the cost of a hedge and the suggestive optimal number of discrete trades was given under a predefined transaction cost. However, strategies with path dependence cannot be easily analysed in this approach.

5.3.7 Impact of Microstructure on Hedging

The microstructural hedging error in particular due to discrete trading and microstructural noise on the price was investigated in [116], where the microstructure noise caused by the tick size was studied. Two hedging strategies were compared. In the first strategy, the hedging portfolio is re-balanced every time that the transaction price moves. In the second strategy, the hedging portfolio is only re-balanced when the transaction price is varied by more than a selected value. The asymptotic distributions of the hedging error showed that the second re-balancing strategy reduced the hedging error significantly in the presence of microstructure noise. A numerical study confirmed this theoretical analysis, where every-price-move rebalancing strategy was compared with the every-5-tick rebalancing strategy through a Monte Carlo simulation.

5.3.8 Empirical Tests of Delta Hedging

There is a clear trend for the theoretical work as reviewed in Sections 5.3.1 to 5.3.7. From the start, the BSM was improved to accommodate the discrete trading and transaction costs. Then different methodologies were developed to measure the statistics of the discrete hedging errors. This allows for the impact of the model/parameter mis-specification on hedging error to be analysed. While there are very few studies about the hedging error in the presence of microstructure noise, there are some studies. Compared with this, the empirical work is behind. There are empirical tests for hedging theories comparison done in a controlled environment with simulated data. For most hedging tests in real market data, hedging was used as a tool to examine other theories. To the author's knowledge, there is no real market data assessment of the discrete hedging error.

Empirical Tests under Controlled Environment

The work from Martellini and Priaulet [95] provided a systematic comparison of four popular methods for option hedging in the presence of transaction costs within a unified mean-variance framework and using an extensive dataset of simulated daily asset prices. There were four methods tested, optimal control strategy, delta move based approach, single-scale time based strategy, multi-scale strategies, which essentially consist of rebalancing different fractions of an option portfolio at different time frequencies, and strategies based on moves in the underlying asset. Under the conditions of proportional transaction costs and constant volatility, the optimal control or delta move based strategies dominated other types of strategies. The advantage of move based methods over time based methods increased with a decrease in the drift of the underlying asset, and with an increase in the volatility of the underlying asset. Move based strategies were hurt by the introduction of stochastic volatility and a fixed component in transaction costs. The utility/optimal control based hedge was one of the successful approaches to option hedging with transaction costs. However, its major drawback is the lack of a closed-form solution and the calculation is time-consuming. The study of [105] proposed a simple yet efficient analytic approximation of a utility based approach and compared it with another

4 hedging strategies including two asymptotic strategies for a utility based hedge from [83] and [87], Black-Scholes hedge and Leland's hedge at fixed regular intervals. The proposed hedging strategy outperformed the others in the simulated empirical testing within the unified mean-variance, mean-VaR and mean-ES (expected shortfall) frameworks.

Empirical Tests by Market Data

Hedging was used in the applications below for comparing different option pricing models. Four alternative option pricing models including the BSM, stochastic volatility, stochastic volatility-stochastic interest rates and stochastic volatility and jumps were compared in [85]. The deterministic volatility function option valuation model was examined in [86]. The deterministic volatility models and stochastic models were compared in [93]. The BSM, Gamma and Weibull option pricing models were compared in [102]. The traders' rules and stochastic volatility option pricing model were compared in [110]. For the above applications, the focus is on the model performance and not on the issues raised by discrete time hedging and transaction costs. The statistical properties of delta-hedging in two applications [96] and [112] were used to examine the volatility risk premium. Hedging was used in [70] to test S&P 500 futures option market efficiency. The paper [89] studies the differential information in the pricing and hedging of, short-term versus long-term equity options by comparing alternative option pricing models. The focus of this study is to test if long-term and short-term options contain different information and if long-term options can better differentiate among alternative models.

Most empirical works in option markets focus on the option pricing or volatility modelling. They are useful for hedging but indirectly as option pricing and hedging are related to each other. The applications below worked on hedging directly. Hedging is a reflection of the dynamic relation between underlying security and derivative security. The model to describe this relationship is referred to as "underlying model" here. The hedging ratios are different from different underlying models. The hedging performance was assessed in [99] and [111] to compare different underlying models. The study in [99] examines whether the delta hedging performance of the BSM model can be improved by taking into account the inverse movements between volatility and the underlying stock price. The paper of [111] studied the performance of locally risk-minimising delta hedging strategies for a large class of diffusion-type stochastic

Research Works	Revision Frequency
[99]	daily
[111]	monthly, weekly and daily.
[119]	daily

Table 5.2: Rebalancing Frequency in Empirical Hedging Tests

volatility models. There are parameters in the underlying model. There are different calibration methods to estimate these parameters. The study from [119] assessed the impact of model calibration on different hedging strategies including standard delta, minimum variance delta, delta-gamma and delta-vega hedging. The rebalancing frequencies in these applications are in Table 5.2. Using daily FTSE 100 options data, the study of [106] demonstrates that including transaction costs by using the ask and bid price of an underlying security as inputs, rather than the mid price, leads to superior pricing and hedging performance in the context of the BSM pricing model through a non-parametric neural network model. Various aspects of hedging were assessed in these empirical analyses. However, the discrete hedging error is not on the list.

5.3.9 Nonparametric Hedging Method

Although parametric option hedging formulas are preferred when they are available, nonparametric methods can be useful substitutes when the underlying asset's price dynamics are unknown, or when the pricing equation associated with the no-arbitrage condition cannot be solved analytically. Biologically-inspired algorithms draw metaphorical inspiration from biological systems to create mathematical algorithms [6]. They belong to data mining methods. Within this family, Neural Nets (in [81], [100] and [106]) and Genetic Programming (in [88], [97] and [108]) have been used in the option pricing and hedging area. In those applications the data was allowed to determine both the dynamics of the underlying price and its relationship to the prices of options with minimal assumptions on underlying price and the option pricing model. The underlying asset price, strike price and time to maturity etc. were taken as inputs and option price was defined to be output into which the learning algorithm maps the inputs. When properly trained, the algorithm gave the option pricing formula, which was then used for pricing and delta hedging. The re-balancing frequency in the applications discussed here was exogenous and it happened at a given time interval.

5.4 Motivations of Empirical Delta Hedging Analysis

Following the route used in [95], this application tests different delta hedging strategies within a unified mean variance framework by the same underlying model, the BSM. The difference is that in this application, real live data is used in the analysis, instead of simulated data. Discrete delta hedging strategies including a time based approach, underlying move based approach, delta move based approach and a GP based optimal control approach are compared.

High frequency financial data are time stamped observations on all quotes and transactions denoted ultra high frequency data by Engle [91]. Advances in computer technology, data recording and data storage have made these datasets increasingly accessible to researchers and have driven the data frequency to the ultimate limit for some financial markets including stock markets, option markets and foreign exchange markets. These high frequency financial data have been applied in volatility modelling for a few years. The applications include risk management, option hedging, execution of transactions, portfolio optimisation and forecasting [121]. In this application, high frequency data is used to analyse delta hedging directly.

As reviewed in Section 5.3.1 considerable theoretical work has been devoted to discrete option hedging in the presence of transaction costs based in the BSM world. Theoretical works as reviewed above relaxed the assumptions of continuous trading and transaction costs while still keeping the assumption of underlying stock price following a geometric Brownian motion process. Though the BSM disagrees with reality in a number of ways, it is still widely employed as a useful approximation. A proper application of BSM theory requires an understanding of its limitation in real markets. This application assesses BSM delta hedging in two steps with and without costs using high frequency data under different hedging strategies including a GP based optimal control hedging method where the rebalancing is triggered endogenously by market information variables.

5.4.1 GP Based Optimal Control Hedging

Delta hedging is dynamically trading the underlying to hedge the option position, therefore the gain (loss) from the option position offsets the loss (gain) from the underlying security position to achieve a status so that the return of the overall portfolio composed by the option and underlying security is zero. This may sound easy if there is only one hedging point that must be assessed, as hedging is just minimising the portfolio variation in terms of its monetary value. However, in reality, the lifetime of an option contract normally spans a few months. The final hedging result depends on all rehedging actions during this time window. The hedging error depends not only on the initial and final market condition, but also on the entire sequence of the market changes in between. Through BSM delta hedging, it tells us how much to hedge. However, we do not know when to hedge. It is not possible to rehedge continuously as it is too expensive. In reality the hedger has to decide the rehedging frequency. It is natural to rehedge when time has changed by a certain, or when the underlying price has changed by a certain. Or alternatively rehedge can be triggered by a certain move in delta itself. However, in these approaches, the rehedge is triggered exogenously. The threshold values are prespecified without considering market environment.

In the utility based optimal hedging strategy as discussed in Section 5.3.4, rehedge is triggered endogenously by maximising hedgers' utility. The working mechanism under the utility based optimal hedging strategy in [72] and [74] is actually simple. There are no-transaction regions and transaction regions defined by the control variable called *hedging band* here. If the current hedging ratio lies within this *hedging band* then no action is needed. If the current hedging ratio is outside of the hedging band, rehedging is triggered and the hedging ratio needs to be brought back to the nearest boundary of the *hedging band* by changing the quantity of the underlying security held. As reviewed in Section 5.3.4 there is no close form solution for this utility based optimal hedging strategy. More specifically there is no close solution for this control variable. Asymptotic analysis in [83] and [87] and analytic approximation in [105] have been used to get an approximate solution for it. However, the solution forms from these approaches are all different though they all share some common elements including underlying price, time to maturity, risk-free interest rate, a proportional ratio of transaction cost and a measure of the hedger's risk aversion. Some of them may be closer to the exact strategy (by numerical methods) than others in some range of model parameters under a simulated environment [105]. In practice, it is suspected that a good result can be returned when this optimal hedging strategy through current approaches is applied to real world problems. As in reality the variable could change dramatically from a simulated parameter range and also the hedger's risk aversion is hardly measured with a unified standard.

GP is one type of population based optimisation tool as reviewed in Chapter 2. It belongs to Evolutionary Computation, which draws metaphorical inspiration from the process of natural evolution, such as survival of the fittest. The optimised answer from GP is driven by the data inputs. Therefore people call this "Let data speak on behalf of itself". One of the particularly interesting aspects of GP is that both the solution form and associated parameters are co-evolved. This offers particular utility in the optimal hedging here. With intraday data available this is a data-rich area. While many plausible explanatory variables exist the interrelationship among the relevant variables is uncertain. Also GP's fitness function allows the complex path dependent hedging problem to be coded into GP's optimisation search.

This chapter uses GP's model induction utility to investigate the control variable's function form. It is called GP based optimal control hedging. Using GP to solve the hedging problem is actually a path dependent minimisation problem based on lots of unknown points where the market conditions are different during the option's trading window. The utility maximisation in the utility based optimal hedging strategy is simplified to minimise the hedge error in this GP approach. The *hedging band* in this GP based optimal hedging is a nonlinear function of a number of market variables including recent traded price, trading volume, implied volatility, etc., which are used to detect the market change. When the BSM delta moves out of this band, it gives an instruction for rehedge; when the BSM delta is within the band, it indicates that there is no dramatic market change therefore no action is needed.

5.5 Empirical BSM Delta Hedging Tests Design

This application considers an investor who writes options and then delta hedges their exposure on an intraday basis by trading in the underlying futures contracts. In the proposed empirical test, the hedged position is a short European call position (FTSE 100 index option). The hedging instrument is stock index futures (FTSE 100 index futures). The money to buy the underlying stock is assumed as borrowed at a risk-free rate. The hedging error has three components, return from trading the option position, return from futures trading and the accumulated financing cost. Starting from the call option sold, instantaneously, a delta ratio of futures contracts are bought from financing with interests cost charged. Under different hedging strategies the short option position is rehedged regularly by trading the FTSE futures contract according to BSM delta, which is updated at each rehedging point by the most up-to-date market information. The option position is closed out just before its expiry time. The option traded prices are used in the option trading. At the end of each trading day, the accumulated funding cost is charged at a daily rate. In the case of the option being closed out before the end of day, the accumulated funding cost is still charged as a day. The option bid-ask transaction costs are ignored. There are only two option transactions in the tests, open the position and close out the position, therefore the option transaction cost impact is assumed to be low with regard to the hedging error.

5.5.1 Testing Structure

This application is an empirical analysis of discrete option (BSM) delta hedging in the presence of transaction costs. Therefore, there are two critical points to be assessed, one is discrete hedging, the other is the impact of transaction costs on delta hedging. For simplicity reasons BSM delta hedging is assessed in two steps. In the first step, there is no transaction cost assumed and traded prices are used in the underlying trading calculation. The testing results from the first step are used for discrete trading assessment. In the second step, bid-ask price information is added in the hedging performance analysis. This testing structure allows the transaction cost impact on the hedging performance to be assessed. Under both steps, hedging strategies are the same except GP based optimal control hedging is trained on an enhanced input dataset with bid-ask information included in the second step as in Figure 5.2.

In the reviewed theoretical work the transaction costs were considered as a fixed proportional ratio of the transaction. In reality this does not hold as the bid-ask spread is changing. The work of [106] demonstrated that including transaction costs by using the ask and bid price of an underlying security as inputs, rather than the mid price, leads to superior pricing and hedging performance. The route of [106] taking account of transaction costs is followed in this study. The cash flows of trading underlying positions are calculated by using corresponding bid or ask prices. When the underlying position needs to reduce, i.e. to sell, the bid price is used; When the underlying position needs to increase, i.e. to buy, the ask price is used.

It is well known that options of different option moneyness, which is the current underlying price divided by option contract strike price, have different behaviours, which is also demonstrated in Chapter 3. The hedging tests are therefore segmented by the option moneyness, which is calculated from the traded option price. The same tests have been done separately for in-the-money (ITM) call options, which have the option moneyness >*1.03*, at-the-money (ATM) call options, which have the option moneyness between and equal *0.97* and *1.03* and out-of-the-money (OTM) call options, which have the option moneyness <*0.97*. The frequency settings for time based strategies and tick move based strategies stay the same for all three segments. The frequency settings for delta move based strategies are different for each of these segments. GP is trained separately for these three segments as in Figure 5.2.

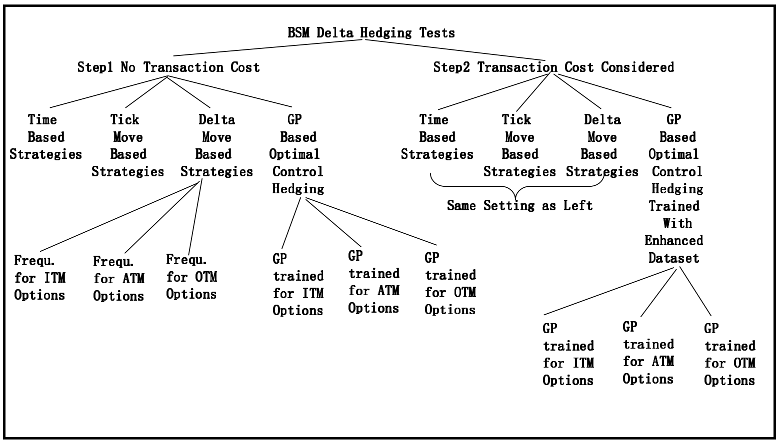


Figure 5.2: BSM Delta Hedging Testing Structure

5.5.2 Testing Strategies

The BSM assumes continuous trading. In reality this is impossible. However, with high frequency data, it is available to assess whether the hedging error will decrease as the rebalancing frequency increases. The testing results will have very useful indications as this is an examination of the dynamic relation of the futures market and the options market. High frequency data hold the promise of new insights about the co-movements of option prices and the prices of the underlying asset [114], in which the evidence was reported that co-movements of index options and index futures quotes differ sharply from perfect correlation during a period with option trades. The testing results here will provide another layer of insight into these two markets movements.

There are three different methods used to define when to rehedge in the exogenously rebalancing approach. In the first method, portfolio revision is triggered by a certain passed time, i.e. rehedge based on uniform time intervals; the second method is when the underlying price has changed a specified amount, i.e. rehedge when the price of the underlying asset has moved by a fixed number of ticks; the last one is when the delta has changed by a percentage amount, i.e. rehedge when the delta of the option being hedged has changed by a defined percentage. These three methods are called *time based strategies*, *tick move based strategies* and *delta move based strategies* in the following. The first method only takes account of the time information to trigger the portfolio revision. The second one only takes account of the underlying price information. The third one is based on delta change. Delta calculation is in Equation 5.2. The delta move based strategies take account of much more information than the first two methods. For each of these methods, different threshold values are used for the frequency setting. To compare like to like, threshold values to define the frequency have been selected based on two criteria. The first is that the number of the underlying trading decided by the rehedging frequency should be around the same for all three hedging strategies at the same frequency level. The second is that the value difference between adjacent threshold values for the same strategy should be roughly even.

Time based strategies

In time based strategies, the rebalancing happens in each uniform prespecified time interval. There are seven rebalancing frequency (level) strategies: rehedging every 5-minute, 10-minute, 20-minute, 30-minute, 1-hour, 5-hour and 1-day. The 5-minute interval is selected as the minimum rebalancing time interval, as the typical choice for modelling frequency is 5-minutes or lower to avoid the microstructure effects [101]. This application carries out an intraday hedging therefore the lowest frequency is daily.

The FTSE 100 index futures market starts at 8:00 and ends at 17:30. For the highest frequency 5-minute, there are 114 trading opportunities each day. There is only one trading opportunity for the lowest frequency 1-day as in Table 5.3.

Frequency Level	Rehedging/Trading Number
5-m	114
10-m	57
20-m	28
30-m	19
1-h	10
5-h	2
1-d	1

Table 5.3: Time Based Rehedging Strategies

Tick move based strategies

For tick move based strategies, rebalancing happens when the underlying price has changed by 3-tick, 5-tick, 10-tick, 15-tick, 20-tick, 30-tick, 40-tick and 50-tick. Tick is the size of the minimum price movement of a trading instrument. For FTSE 100 index futures, it is 0.5 of the index points and £5 in value.

Threshold values (tick numbers) to define frequency settings in the tick move based strategies were derived from an intraday data analysis in Section 5.6.4, in which the average tick change and trading time duration were analysed. This analysis shows that the time stamped price change starts from 0 tick, where the adjacent trades at different time stamps have the same traded price. Based on this, further analyses were developed. Multi-tick size, occurring number and trading duration were dynamically analysed for each trading day. Take 3-tick size for example, starting from one day's open price, the time of the next traded price with more than a 3-tick change than the open price is recorded. The time period between this recorded time and the opening is the duration for the first 3-tick size change that occurred on this day. The current price is then changed from the open price to the last 3-tick change occurred price. The next price with more than a 3-tick size change is then located and so on and so forth until the end of the day. The total occurrences of 3-tick size price changes are recorded for this day. The summary results of this analysis for 254 trading days are in Table 5.4, where the first column gives the tick size used in each frequency level in the tick move based strategies. The second column gives the average daily occurrence number for different tick size. The third column gives the equivalent trading time based on the assumption that x-tick size occurs evenly during a day. There are 570 minutes trading time in each trading day. The duration is therefore calculated by dividing 570 by the occurrence number. The *Average No. Per day* and *Equivalent Duration (in minutes)* show that the current selected threshold values give a roughly same revision number as the revision number used in the time based strategies at the same frequency level.

Table 5.4 gives average occurrence numbers and equivalent time durations for threshold values used in tick move based strategies. In this approach, 50-tick size was selected as the lowest frequency, which occurs on average 0.9 times per day and the equivalent time duration for trades with at least 50 ticks size change is 621 minutes on average. It roughly matches the lowest frequency level, daily, in time based strategies. Based on the second selection criteria that adjacent threshold values should be evenly distributed, 5-tick should be the highest frequency. However, 3-tick was selected as the highest one. This is because the occurring number for 5-tick size is 79.1, which is less than the 5-minute one, 114. Because that 5-tick change is recommended in [116], it is kept in as the second most frequent threshold setting. 3-tick size is chosen as the highest frequency setting in the tick move based strategy.

Tick	Average No. Per Day	Equivalent Duration (in Minutes)
3	186.7	3
5	79.1	7
10	23.2	25
15	10.9	53
20	6.2	92
30	2.8	205
40	1.4	402
50	0.9	621

Table 5.4: FTSE 100 Index Futures Trading Opportunities by Tick Changes

Delta move based strategies

For the delta move based strategies, threshold values are selected separately for different option moneyness segments. This is because ITM options commonly have high delta and OTM options have low delta as in Figure 5.7. The threshold values should be different for different option segments according to the selection criteria. Similar dynamic analysis as in tick move based strategies was also carried out for delta move based strategies based on part of the options used in the hedging analysis.

It should be noted that the options delta changes during the hedging window are unknown before hedging starts in reality. Therefore only 80 percent of options in each segment, which are also used as the training sample in GP optimal control hedging strategy, are used in the analysis to decide the threshold values in the delta move based strategies. The results are in Table 5.5, where the revision frequencies used in delta move based strategies are in the column labelled as *Ratio* for each segment. The *Average No. Per day* and *Equivalent Duration* in minutes show that the current selected threshold values give a roughly same revision number as the frequency setting in the time based strategies. For example, in the ITM option segment, under these delta move based strategies, re-hedging happens when the hedged option's delta has changed by 0.1 percent, 0.3 percent, 0.6 percent, 1.2 percent, 2.8 percent, 15.0 percent and 20.0 percent. Threshold value, 0.1 percent gives the highest frequency and 20.0 percent gives the lowest frequency for ITM options. In the rest of this chapter, these ratio frequency settings are referred to as *Ratio-1* to *Ratio-7* generally for all option moneyness segments.

Frequency	ITM Options			ATM Options			OTM Options		
	Ratio	Average No.Per Day	Equivalent Time (Minutes)	Ratio	Average No.Per Day	Equivalent Time (Minutes)	Ratio	Average No.Per Day	Equivalent Time (Minutes)
Ratio-1	0.1%	117.0	4.9	0.2%	115.7	4.9	0.1%	109.4	5.2
Ratio-2	0.3%	57.5	9.9	0.4%	54.5	10.5	0.3%	54.3	10.5
Ratio-3	0.6%	30.2	18.9	0.7%	30.9	18.5	0.7%	28.6	19.9
Ratio-4	1.2%	18.5	30.9	1.0%	21.4	26.7	1.5%	17.5	32.5
Ratio-5	2.8%	9.8	58.2	2.0%	10.4	54.8	3.0%	10.9	52.4
Ratio-6	15.0%	2.2	263.0	7.5%	2.0	281.5	24.0%	1.9	302.8
Ratio-7	20.0%	1.1	537.0	11.0%	1.1	504.6	35.0%	1.1	529.6

Table 5.5: FTSE 100 Index Futures Trading Opportunities by Delta Changes

5.5.3 GP Based Optimal Hedging

Hedging in practice is the trade-off between residual risk from discrete rebalancing and transaction costs. The hedging error exists when hedging is performed discretely. When rebalancing frequency increases, the residual risk decreases, however the transaction costs increase and vice versa. In Section 5.5.2, the rebalancing frequencies for time based, tick move based and delta move based strategies are prespecified. Rehedging actions are decided outside of the model. In GP based optimal hedging, GP's model induction utility and optimisation are utilised. It is used to find the most sensitive market information variables in hedging, to automatically detect the market changes and give the instruction of rehedging to achieve an objective of minimising the hedging error during the option hedging window. As discussed in Section 5.4.1, GP is used to explore the function form of the control variable *hedging band*. Its working process is given in Figure 5.3.

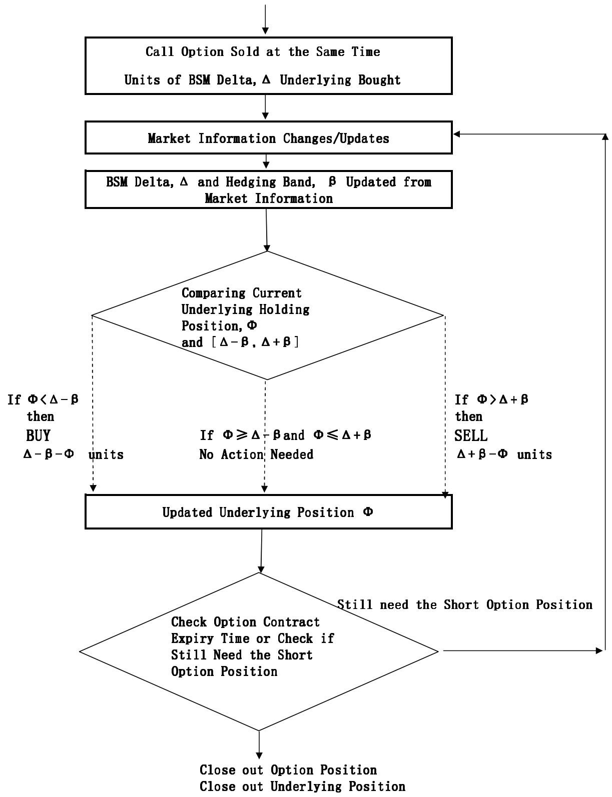


Figure 5.3: GP Based Optimal Hedging Working Process

5.5.4 GP Parameter Settings

In this GP application, the population size for each generation is 2000. There are 50 generations for a run. A big population size and small generation number are used to avoid corner solutions. There are 30 runs for each of three option moneyness segments, ITM, ATM and OTM options. The best individual for each segment is selected from 30 runs. For the new generation, 55 percent of them are filled from crossover, 40 percent are from mutation and the other 5 percent are from the reproduction of the current generation. To avoid the over fitting problem, a relative small tree size, of 5-depth is selected. The terminal set and function set are in the table below. GP does a symbolic regression for the control variable, *hedging band*, which has a function form composed by the elements from the terminal set and function set.

Variables	Expression	Definition
s1	S	Underlying traded price
s2	q	Dividend payment
s3	S/K	Option moneyness
s4	T	Time to maturity in year
s5	σ	BSM implied volatility
s6	r	Risk-free interest rate
s7	Duration	Underlying price change duration
s8	Δ	Option BSM delta
s9	$N^{'}(d_1)$	Component of Gamma calculation
s10	Γ	Option BSM Gamma
s11	S ask	Underlying ask price
s12	S bid	Underlying bid price
s13	log of volume	Log of trading volume
s14	S ask- S bid	Bid-ask spread, the difference of ask and bid price
s15	Bid-ask Spread Change	Bid-ask spread change compared with 1 minute ago

Table 5.6: GP Terminal Set in Delta Hedging

Function	Definition
+	Addition
-	Subtraction
$*$	Multiplication
/	Division
NormC	Normal cumulative distribution function
Exp	Exponential function
rlog	Natural log
sqrt	Square root
sqrt_3	Cube root

Table 5.7: GP Function Set in Delta Hedging

In this application GP is used to solve a path dependent minimisation problem as discussed in Section 5.4.1. The *hedging band* from GP as in Figure 5.3, has two important functions during the hedging process. Firstly, it instructs when to rehedge, i.e. when the quantity of the current underlying held, ϕ, is outside the bandage of BSM delta $\Delta \pm$ *hedging band* (β). Secondly, it instructs how much to rehedge, i.e. adjust the underlying position held to the closest edge of the band, $\Delta \pm \beta$. Each time the market information updates, the investor's overall net portfolio value changes, which is caused by the underlying held, FTSE 100 index futures and FTSE 100 index option held. The portfolio value is also net of accumulated financing costs. When the short option position needs to be closed out, the hedging process finishes and the underlying holding is sold. The final hedging error is then calculated as the summary of all cash flows, including the positive cash flow from selling the final underlying holding, the negative cash flow from closing the option position and the accumulated hedging costs that occurred during the whole hedging window. The accumulated hedging costs during the full path are from the underlying trading and interests charges on financing the trade as in Equation 5.4, where ϕ is the underlying holding, s and p are the underlying price and option price, t is the end of the hedging window, 0 is the beginning of the hedging window, ϕ_0 is the size of the underlying need to buy when the option just sold, it is the BSM Δ at time *0*, *j* indicates the time stamp whenever the market information change in the hedging window, θ is the quantity underlying position that needs to be adjusted and its value is assigned in Equation 5.5 and *int* is the accumulated interest charged daily on accumulated cash balance. In this application the objective is to minimise the hedging error, which could be positive or negative therefore the fitness function is the square root of the mean sum squared error as in Equation 5.6 below, where, $finalHedgingError_i$ is the final hedging error from the *ith* option contract as calculated in Equation 5.4 and *n*, is the option contracts number available in the training dataset. GP's optimisation process is designed to minimise this fitness function.

$$FinalHedgingError_i = \phi_t \times s_t - p_t + (p_0 - \phi_0 \times s_0 + \sum_{j=1}^{t-1} (\theta \times s_j + int)) \tag{5.4}$$

Where

$$\theta = \begin{cases} \Delta - \beta - \phi & \phi < \Delta - \beta \\ 0 \quad\quad if & \Delta - \beta \leq \phi \leq \Delta + \beta \\ \Delta + \beta - \phi & \phi > \Delta + \beta \end{cases} \tag{5.5}$$

$$fitnessfunction = \sqrt{\frac{\sum_{i=1}^{n} FinalHedgingError_i^2}{n}} \tag{5.6}$$

5.5.5 GP Hedging Dataset

Delta hedging an option contract is performed by trading the underlying securities. In an ideal world GP's hedging dataset should be all time stamped high frequency FTSE 100 index futures data. However, the required computation is too high as the BSM delta and *hedging band* need to be updated every time the market information changes. Take one of the option contracts in our dataset for example, it was sold on 5th January 2004 and closed on 19th February 2004. There are 409,877 intraday FTSE 100 index futures traded price records in this time window. GP does a population based optimisation search therefore the computation is incredible large. Due to the computer resource limitation, we only consider the market condition changes when the underlying price has been moved by at least 3-tick size when updating BSM delta and GP *hedging band*. The 3-tick size change has been selected as the filter of market conditions change because the initial results show that the tick based hedging strategies gave better results than the time based strategies. Therefore the highest frequency dataset from the tick move based strategies, which only includes records with at least 3-tick size changes was selected as the underlying intraday dataset. There are 5,967 data records in this 3-tick change dataset (v.s. 409,877 data records in the raw dataset) in the above example.

All hedging strategies including time based, tick move based, delta move based and GP optimal hedging are compared under the consideration of transaction costs by using the corresponding bid or ask price instead of the traded price. GP is trained separately for each of the option moneyness segments.

5.6 Data and Methodology

In this study, data was drawn from market prices on futures and options on the same index, the FTSE 100 index. It consists of all recorded traded prices, volumes, bid and ask quotations and depths from 2nd January 2004 to 31 December 2004. This dataset has been selected because FTSE index futures and option markets are active traded markets and suffer less microstructure effects [113]. The FTSE 100 Dividend Yields and Bank of England LIBOR rates (1 day, 1 week, 2 weeks, 1 month, 3 months, 6 months, 1 year) were obtained from Datastream. The risk-free interest rate term structure was estimated through the Nelson and Siegel interest rate model [71]. The model parameters were obtained by calibrating the model to downloaded Libor rates in 2004.

5.6.1 Option Data

There are in total 1,116 option contracts (472 call option contracts) and 289,186 transaction trading records (139,622 for call option transactions) in this one year options dataset, which is described in Section 3.3. The trading time starts from 8:00 and ends at 16:30. An option contract is characterised by its option type, strike price and maturity date. Call options in hedging tests are segmented by the option moneyness. Table 5.8 gives the basic information for each call option moneyness segment in this one year dataset. The option moneyness and the time to maturity in days used in the calculation are calculated by its first occurrence. In Table 5.8a, the column *Contract No.* shows how many contracts are available for each segment. The columns *Trading Records*, *Unique Trading Day* and *Time to Maturity (Days)* are averaged information for the options under each segment. The column *Trading Records* shows the transactions records available after the first occurrence in the dataset. The column *Unique Trading Day* shows the unique trading days and the column *Time to Maturity (Days)* is the option time to maturity in days.

On average for ITM options we have 183 intraday transactions within 11 trading days. The trading days may not be consecutive. The average time to maturity for them is 115 days. For ATM options we have on average 606 intraday transactions within 38 trading days. The average time to maturity is 179 days. For OTM options, we have on average 244 intraday transaction records for each option within 27 days. The average time to maturity is 239 days.

The hedging window for each option contract in the tests is decided by its first and last transaction time in the dataset. In the majority of cases, the first transaction time is close to the first occurrence time and the last trade is one or two days before its expiry time. Therefore the length of the hedging window of each contract is close to the time to maturity, calculated at its first occurrence time in the data.

It is unfortunate that there are 11 days records missing and only 243 days records available in the one year option dataset. During the hedging process, option traded data is used to estimate the BSM implied volatility (IV) surface as discussed in Section 5.6.2. The IV surface is not estimated for these 11 days due to a missing data issue. IV is an input to calculate the delta ratio. Therefore, only call option contracts with a hedging window not falling in these 11 missing days can be used in the hedging tests. Also, a hedging for option lifetime is attempted, therefore options with time to maturity of less than 30 days are not considered. The option contracts used in the hedging tests are in Table 5.8b.

	Contract	Average		
	No.	Trading Records	Unique Trading Day	Time to Maturity (Days)
ITM	169	183	11	115
ATM	96	606	38	179
OTM	207	244	27	239

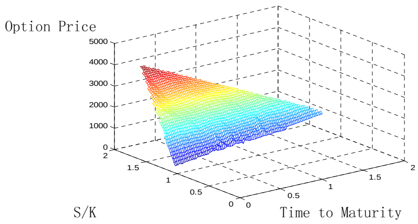
(a) All Available Call Option Contracts

	Call Option Contracts No.		
	In-sample Training	Out-of-Sample Testing	Total
ITM	32	8	40
ATM	23	6	29
OTM	40	10	49

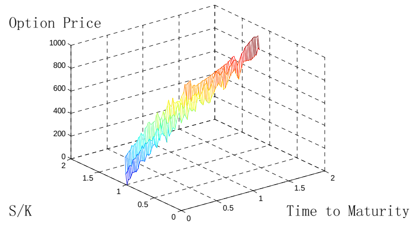
(b) Call Option Contracts in Tests

Table 5.8: FTSE 100 Index Call Option Contracts Statistics by the 1st Occurrence

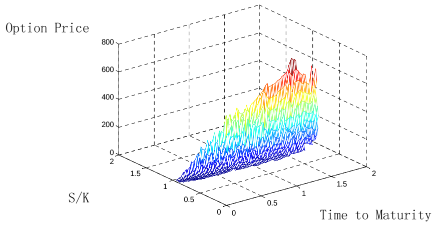
Call option traded prices are graphed separately by option moneyness in Figure 5.4. Here the option moneyness is calculated as the current underlying price divided by the strike price. It is observed that the ITM options in Figure 5.4a are the most expensive ones compared with ATM options in Figure 5.4b and OTM options in Figure 5.4c. The average traded price, average BSM implied volatility and average delta are also given in Table 5.9 for all available call options and for each option segment. In general, in this dataset, the ITM options have the highest traded price, highest BSM implied volatility and highest delta.



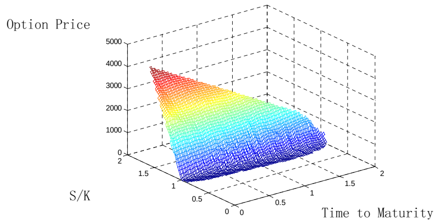
(a) ITM Call Options



(b) ATM Call Options



(c) OTM Call Options



(d) Call Options

Figure 5.4: FTSE 100 Index Call Options Price

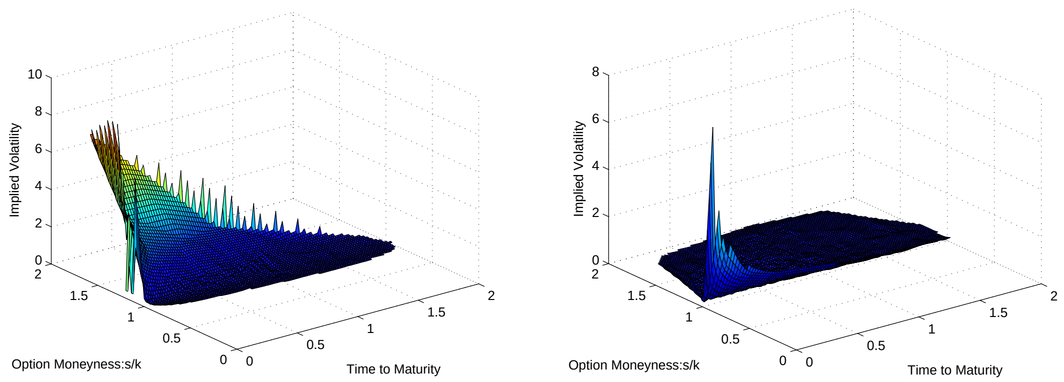
	Average Price	Average BSM IV	Average Delta
All Calls	223	0.586	0.453
ITM	676	1.713	0.657
ATM	125	0.332	0.471
OTM	38	0.152	0.189

Table 5.9: FTSE 100 Index Options Average Information

5.6.2 Implied Volatility Surface

Volatility is a key input in BSM delta ratio calculation, where the volatility of the underlying asset during the period from now until the hedged option contract expires is needed. In the BSM, the option price is determined by the underlying price, time to maturity, strike price, risk-free rate, and volatility. All these inputs except volatility are observable from the market. The implied volatility is backed out from the BSM option pricing formula to let the BSM theory price match the market option price. It is well known that options of the same underlying asset and the same time to maturity but different strikes have different implied volatility estimates.

In the BSM the volatility is assumed as constant as in Equation 5.2. This assumption in reality is not possible. In this application the Black-Scholes model implied volatility is used for the volatility input in the BSM delta ratio. Figure 5.5 gives the BSM implied volatility by option moneyness and time to maturity for all traded options, where each point in the figure is from an option transaction. As in the graphs, the implied volatilities for call options are higher than those for put options. For both types of options ITM options have higher implied volatilities than both ATM and OTM options. In general, the longer the time to maturity, the smaller the implied volatility.



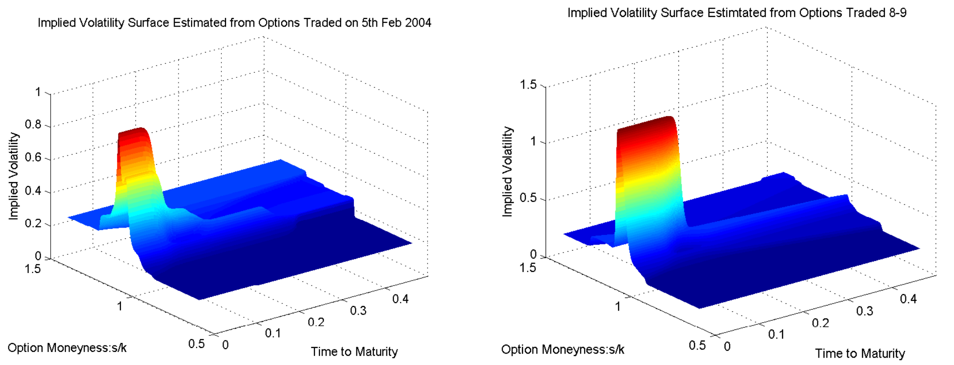
(a) Call Options (b) Put Options

Figure 5.5: BSM Implied Volatility of FTSE 100 Index Traded Options

During the hedging process, implied volatility is required for different option moneyness (the ratio of current underlying price to strike price) and time to maturity (current time to contract expiry time) and this implied volatility can only be estimated from traded options before hedging. However, the option with specific moneyness and time to maturity may not exist in the trading dataset at the required hedging time. Therefore in this study, we estimated the volatility surface for FTSE 100 option contracts where the implied volatility of a contract was a function of option's moneyness and time to maturity. Such a volatility surface giving an implied volatility for any given strike and maturity must be inferred from the available option trades. To do this, the discrete data from traded options was interpolated.

This study takes the approach of Cont and da Fonseca (2002) [94] using a two-dimensional kernel density smoothing method to interpolate between the Black-Scholes volatility surface points, which are calculated directly from traded options.

The scaling factors, which control how the surface is smoothed, were estimated daily by a cross validation method [122]. During the hedging process, the implied volatility surface for time t was estimated from all options traded one hour before t and the scaling factors used were from all available options traded on the previous day of time t. One hour previous options were chosen to estimate the implied volatility surface. This one hour selection has ensured that the freshest information can be used in the estimation and enough data points can be used to give a robust estimation. Figures 5.6 shows the interpolated implied volatility surfaces estimated from traded options on the 5th February 2004. Figure 5.6a is from all traded options on that day. Figure 5.6b is the implied volatility surface estimated by one hour's traded options. It is observed from Figure 5.6a that the implied volatility from the estimated surface for ITM options is the highest during that trading day, which is consistent with the observation in the implied volatility directly backed out from the traded options in Figure 5.5.



(a) From Option Traded from 8:00 to 16:30 (b) From Option Traded from 8:00 to 9:00

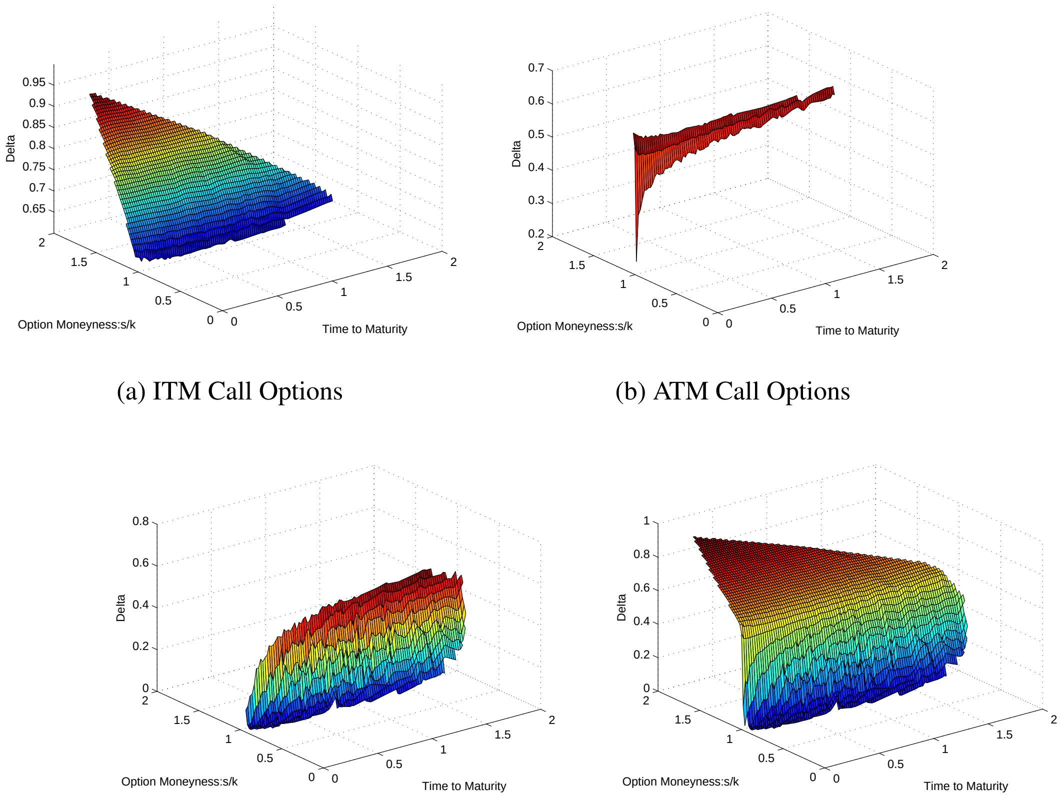
Figure 5.6: IV Surface Estimated from Traded Options on 05-Feb-2004

5.6.3 BSM Delta

The delta ratio for a European call on an asset with dividend payment is in Equation 5.2. The delta ratio for a European put option is in Equation 5.7, where, Δ_{put} is delta for a put option with dividend, d_1 is in Formula 5.3, $N(x)$ is in Equation 5.2, T is the time to maturity of the option and q is dividend payment.

$$\Delta_{put} = e^{-qT}(N(d_1) - 1) \tag{5.7}$$

The BSM delta calculated for call options is summarised in Table 5.9. The volatility used in the delta calculation is the implied volatility. Figure 5.7 gives delta for call options and also for segments, ITM, ATM and OTM options. Each point in the figure is calculated from an option transaction in the dataset. Delta for call options is positive and in the range of zero to one. It gives an implied probability that the option expires in-the-money. As seen in Figure 5.7, within three options segments, delta for ITM options is the highest and delta for OTM options is the lowest.



(a) ITM Call Options (b) ATM Call Options

(c) OTM Call Options (d) Call Options

Figure 5.7: Delta for FTSE 100 Index Call Options

The above figures give delta for different segments call options traded in 2004.

5.6.4 Futures Data

There are in total 26,271,084 transaction records in the FTSE futures dataset. Only contracts with a maturity date closest to the next delivery date, which are the most actively traded contracts, are included in the calculation. After exclusions are applied, there are 2,902,544 trading records included in hedging tests to calculate the spot prices, which are used in implied volatility and delta calculations. The FTSE 100 futures market starts trading at 8:00 in the morning and ends at 17:30 in the evening. Figure 5.8 gives the FTSE 100 futures daily last traded price. The graph shows that from the beginning of 2004 until mid August the market price trend is flat and decreases slightly. After that the market has a mild increasing trend until the end of 2004.

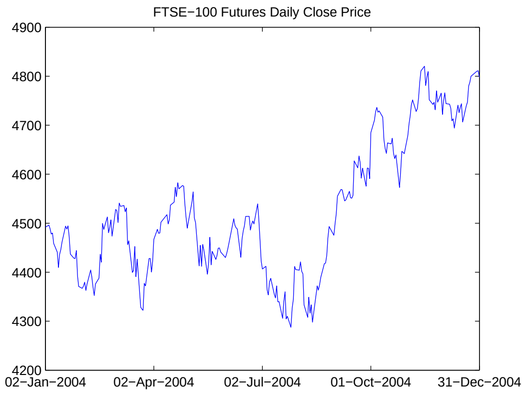
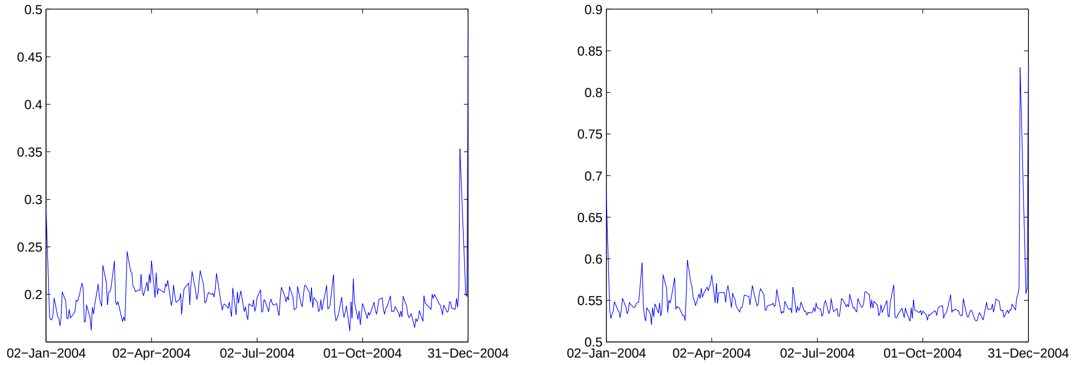


Figure 5.8: FTSE 100 Index Futures Daily Close Price

The intraday information is given in the graphs below. Figure 5.9a shows the average absolute price changes from trades. On the first day 2nd January 2004, the average price change is 0.29, which is less than 0.5, the tick size. This is because nearly half of the trades have zero price change compared with the previous one. Figure 5.9b gives the same information excluding trades with zero change. In this graph the average price changes are all above 0.5. On the first day 2nd January 2004, the average price change is 0.68. Figure 5.10 shows the daily average time duration of trades. On the first day, the average trading duration is 6 seconds. It is clear that at the beginning and end of the year, the trading duration is much longer and the size of the price changes is bigger.



(a) Average Absolute Price Change

(b) Average Absolute Price Change Excluding Zero changes Trades

Figure 5.9: Average Absolute FTSE 100 Index Futures Traded Price Changes

Figure 5.9a gives the average absolute FTSE 100 futures traded price change. Figure 5.9b excludes the trades that have no price changes

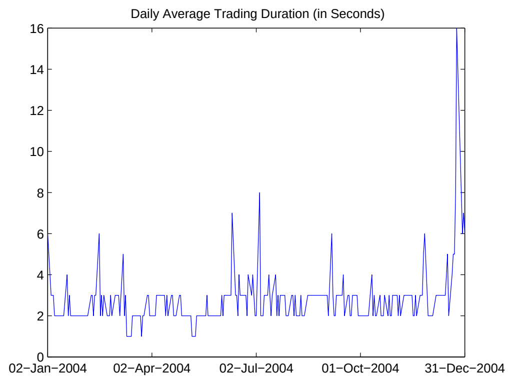


Figure 5.10: FTSE 100 Index Futures Trading Duration

Futures Bid-Ask Information

In the hedging tests, bid and ask prices are used for the underlying trading calculation to consider transaction costs. 5,492,171 bid quotations and 5,455,936 ask quotations are in the one year FTSE futures dataset. All transaction information is sorted by time stamp. Traded prices and bid-ask prices are linked together within the same time stamp. In case there is no bid or ask for a time stamp trade, the most recent bid or ask is used. In most cases, there are multiple bids or asks at one time stamp, and here the simple average number is used. The statistics of the bid-ask spreads in Tables 5.10 and 5.11 were calculated based on the raw bid and ask information.

B-A Sp.	≤ 0	0-0.25	0.25-0.35	0.35-0.4	0.4-0.45	0.45-0.5	0.5-0.75	0.75-1	> 1
% Total	0.12%	3.16%	1.88%	1.29%	1.26%	57.16%	14.92%	17.24%	2.98%

Table 5.10: FTSE 100 Index Futures Bid-ask Spread Distribution

The distribution of the bid-ask spreads is in Table 5.10. Within the band of ≤ 0 0.05 percent of bid-ask spreads are negative. A detailed investigation found that this is caused by the prices mismatch, ie. at one time stamp there are multiple traded/bid/ask prices. It was decided to use the most recent bid-ask spread for these negative bid-ask spreads time stamps as they are caused by the bid-ask spread calculation method. This only accounts for a very small percentage of the full data therefore the impact on the overall testing results is assumed to be very low. The median of the bid-ask spread is 0.5, which is the tick size. It indicates that the bid-ask spread is very narrow in this market. The mean and standard deviation in Table 5.11 show some trading trends. The broadest bid-ask spreads, in general, are from the end of the day and are very volatile. After that the lunch time bid-ask spread is the second broadest during a day's trading though its volatility is smaller than the opening time.

Hour Interval	8	9	10	11	12	13	14	15	16	17
Mean	0.639	0.621	0.623	0.638	0.647	0.63	0.598	0.583	0.602	0.71
Std	0.295	0.239	0.239	0.252	0.262	0.248	0.217	0.197	0.232	0.362

Table 5.11: FTSE 100 Index Futures Bid-ask Spread Statistics by Trading Hour

5.7 Empirical Testing Results

There are three types of exogenous delta hedging strategies including time based strategies, tick move based strategies and delta move based strategies, and one type of GP based optimal control hedging strategies, which are examined separately for 40 ITM call options, 29 ATM call options and 49 OTM call options in this application. The hedging error is actually the portfolio profit and loss, which can be positive or negative. Following [95] and [105], this application compares the performance of the alternative hedging strategies in the mean-variance framework. The mean and standard deviation are reported for all strategies under a two-step testing framework without bid-ask transaction costs (Tables 5.12 and 5.15) and with bid-ask transaction costs (Tables 5.13 and 5.16). For delta move based strategies and GP optimal control hedging strategies there are in-sample training and out-of-sample testing. All hedging strategies are therefore compared in the full sample (Tables 5.12 and 5.13) and the out-of-sample (Tables 5.15 and 5.16) separately.

There are three objectives in this application. The first is to examine BSM delta hedging using high frequency data to find the relationship between rebalancing frequency and hedging error in real market data. The second is to assess the bid-ask transaction cost impact on delta hedging. The third is to compare the performance of different hedging strategies including the GP based optimal control hedging strategies. The testing results are therefore discussed separately.

5.7.1 BSM Delta Hedging Examined with High Frequency Data

According to the BSM as long as the hedging portfolio is re-balanced continuously the portfolio will be perfectly hedged with a zero hedging error under the assumption of no transaction costs. In this chapter, the BSM delta hedging is examined with high frequency data. To assess the transaction costs impact, the tests have been done in two steps, without transaction costs and with transaction costs. The rehedging frequency is tested as low as one day and as high as up to 5 minutes. Delta hedging aims to keep the value of the investor's total portfolio positions unchanged with respect to the underlying price change. According to the BSM, we should be able to see a trend that when rehedging frequency increases, the mean of the hedging errors converges to zero and the standard deviation of the hedging errors decreases too. The testing results from the time based strategies, tick move based strategies and delta move based strategies without transaction costs assessed in the full dataset are graphed in Figure 5.11. Results from the same tests with transaction costs considered are in Figure 5.12.

Tests without Transaction Costs

In Figures 5.11, where the left vertical axis is the mean of the hedging errors, the right vertical axis is the standard deviation of the hedging errors and the horizontal axis is the hedging frequency setting. In the horizontal axis, the frequency increases from right to left. Therefore we expect to see the mean of the hedging errors converge to zero and its volatility (the standard deviation) for different frequencies decrease from right to left. The results are also in Table 5.12.

The trend lines given in each graph show that this hedging error and rehedging frequency relationship does hold for the ATM call option segment across all three strategies (refer Figures 5.11b, 5.11e, 5.11h).

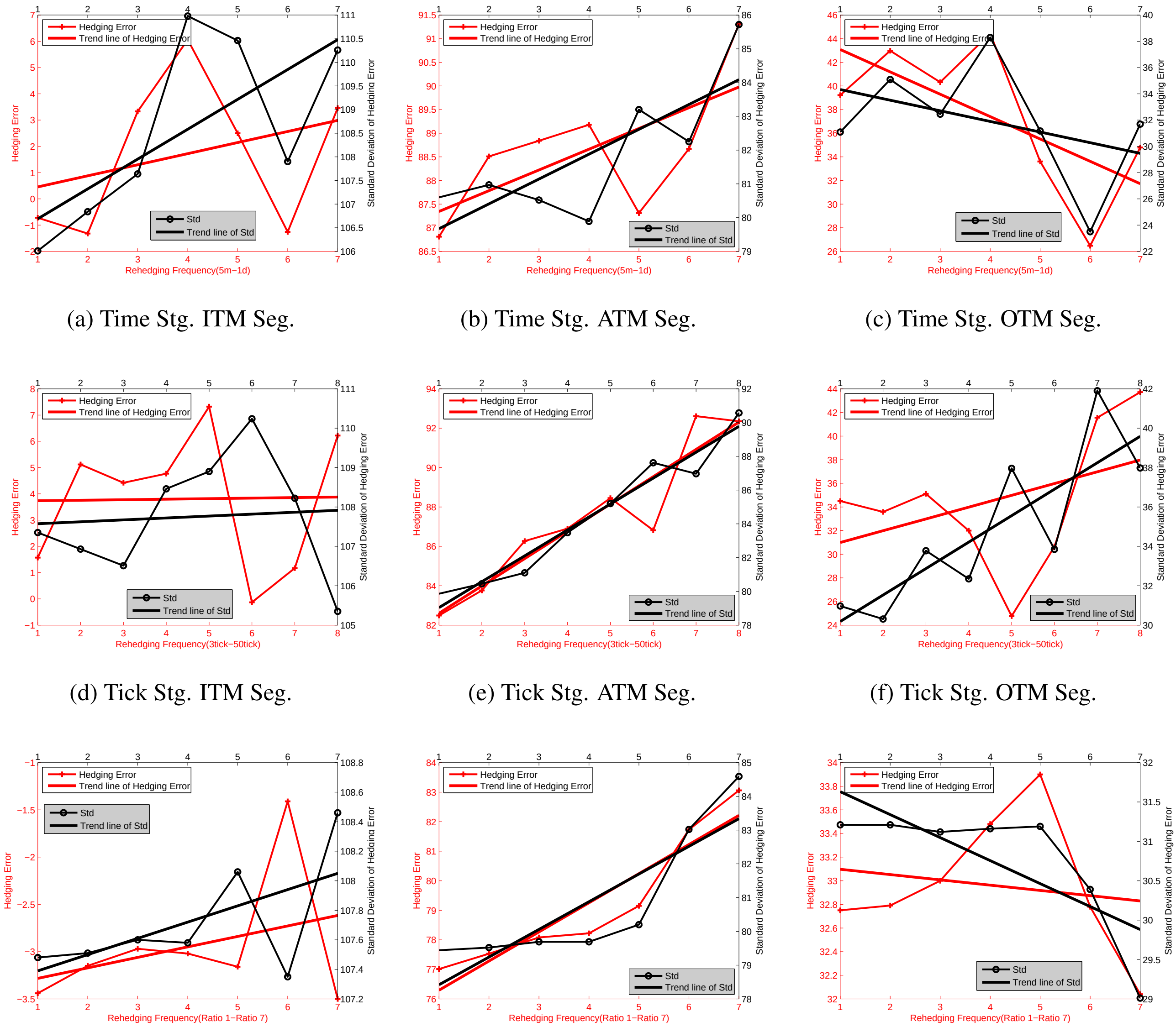
For ITM options, this expectation only keeps for time based strategies (Figure 5.11a). The trends of the mean and standard deviation of hedging errors from a tick move based strategy (Figure 5.11d) are nearly flat. For delta move based strategies the hedging loss increases although the standard deviation of the hedging error deceases as rehedging frequency increases.

For OTM options, this trend only keeps in tick move based strategies in Figure 5.11f. For time based strategies and delta move based strategies, when rehedging frequency increases the mean of the hedging errors increases and the standard deviation increases.

Among all three hedging strategies, the results from tick move based strategies nearly give the expected trend in all segments except for the ITM segment, in which the trends are rather flat.

Without transaction costs the investor makes money for ATM options and OTM options through all strategies while for ITM options, the investor loses money in delta move based strategies and two time based high frequency setting strategies (5-minute and 10-minute strategies).

Although the BSM delta hedging expectation of the relation between hedging error and rehedging frequency does not hold all the time, in the majority of cases when rehedging frequency increases, the risk decreases and the hedging return decreases (or hedging loss increases). However, for OTM options in the time based strategies and delta move based strategies, when rehedging frequency increases the risk increases and hedging return increases. We do see a risk rewarded trend hold for all scenarios. When the hedging return increases, the risk represented by the standard deviation increases and when the hedging return decreases or the hedging loss increases the risk decreases.



(a) Time Stg. ITM Seg. (b) Time Stg. ATM Seg. (c) Time Stg. OTM Seg.

(d) Tick Stg. ITM Seg. (e) Tick Stg. ATM Seg. (f) Tick Stg. OTM Seg.

(g) Delta Stg. ITM Seg. (h) Delta Stg. ATM Seg. (i) Delta Stg. OTM Seg.

Figure 5.11: Hedging Errors from Different Strategies without Transaction Costs

The horizontal axis is the rehedging frequency, with 1 being the highest one and 7/8 the lowest. The left vertical axis gives the mean hedging error and the right axis gives the standard deviation. The linear trend is given in each figure. It indicates that for each hedging strategy in each call option segment the higher the rehedging frequency the lower the hedging error and the lower the volatility.

The absolute hedging errors for each strategy are also calculated. In terms of the relationship of hedging errors and rehedging frequency, the results from hedging error and absolute hedging error are very similar therefore the results from the absolute errors are not reported.

Frequency		5-m	10-m	20-m	30-m	1-h	5-h	1-d
ITM	Mean	-0.72	-1.32	3.33	6.06	2.50	-1.26	3.46
	Std	106.01	106.84	107.64	110.98	110.46	107.90	110.26
ATM	Mean	86.81	88.51	88.84	89.18	87.31	88.67	91.31
	Std	80.60	80.97	80.52	79.89	83.20	82.25	85.72
OTM	Mean	39.22	42.98	40.32	44.44	33.61	26.48	34.79
	Std	31.09	35.09	32.45	38.29	31.17	23.50	31.69

(a) Time Based Strategies

Frequency		3-tick	5-tick	10-tick	15-tick	20-tick	30-tick	40-tick	50-tick
ITM	Mean	1.57	5.12	4.42	4.77	7.32	**-0.13**	1.17	6.22
	Std	107.35	106.93	106.51	108.46	108.90	110.24	108.22	**105.35**
ATM	Mean	82.49	83.76	86.27	86.90	88.45	86.82	92.61	92.35
	Std	79.86	80.45	81.10	83.48	85.20	87.62	86.97	90.57
OTM	Mean	34.50	33.58	35.11	32.00	24.76	30.64	41.56	43.72
	Std	30.97	30.31	33.78	32.35	37.96	33.85	41.90	37.98

(b) Underlying Tick Move Based Strategies

Frequency		Ratio-1	Ratio-2	Ratio-3	Ratio-4	Ratio-5	Ratio-6	Ratio-7
ITM	Mean	-3.44	-3.15	-2.97	-3.02	-3.16	-1.41	-3.50
	Std	107.48	107.51	107.60	107.58	108.06	107.35	108.46
ATM	Mean	77.01	77.53	78.08	78.22	79.15	81.73	83.06
	Std	79.44	79.52	79.69	79.69	80.20	83.02	84.59
OTM	Mean	32.75	32.79	33.00	33.48	33.90	32.78	32.04
	Std	31.21	31.21	31.12	31.16	31.19	30.39	29.01

(c) Delta Move Based Strategies

GP	ITM (GP-in)	ATM (GP-at)	OTM (GP-out)
Mean	1.31	**72.85**	**9.06**
Std	112.17	**79.23**	**15.33**

(d) GP Optimal Strategies

Table 5.12: Full Dataset Hedging Errors without Tran. Cost
The strategy gives the minimum absolute mean hedging errors and the minimum standard deviation is the best hedging strategy, which are highlighted in **bold** for each option segment. Refer to Table 5.5 for frequency threshold values in delta move based hedging strategies.

Tests with Transaction Costs

The test results with transaction costs considered are in Figures 5.12, where the axis definition is the same as in Figure 5.11. In the horizontal axis, from right to left the frequency increases, therefore we expect to see the mean of the hedging errors converge to zero and its volatility from different frequencies decrease from right to left. The results are also in Table 5.13.

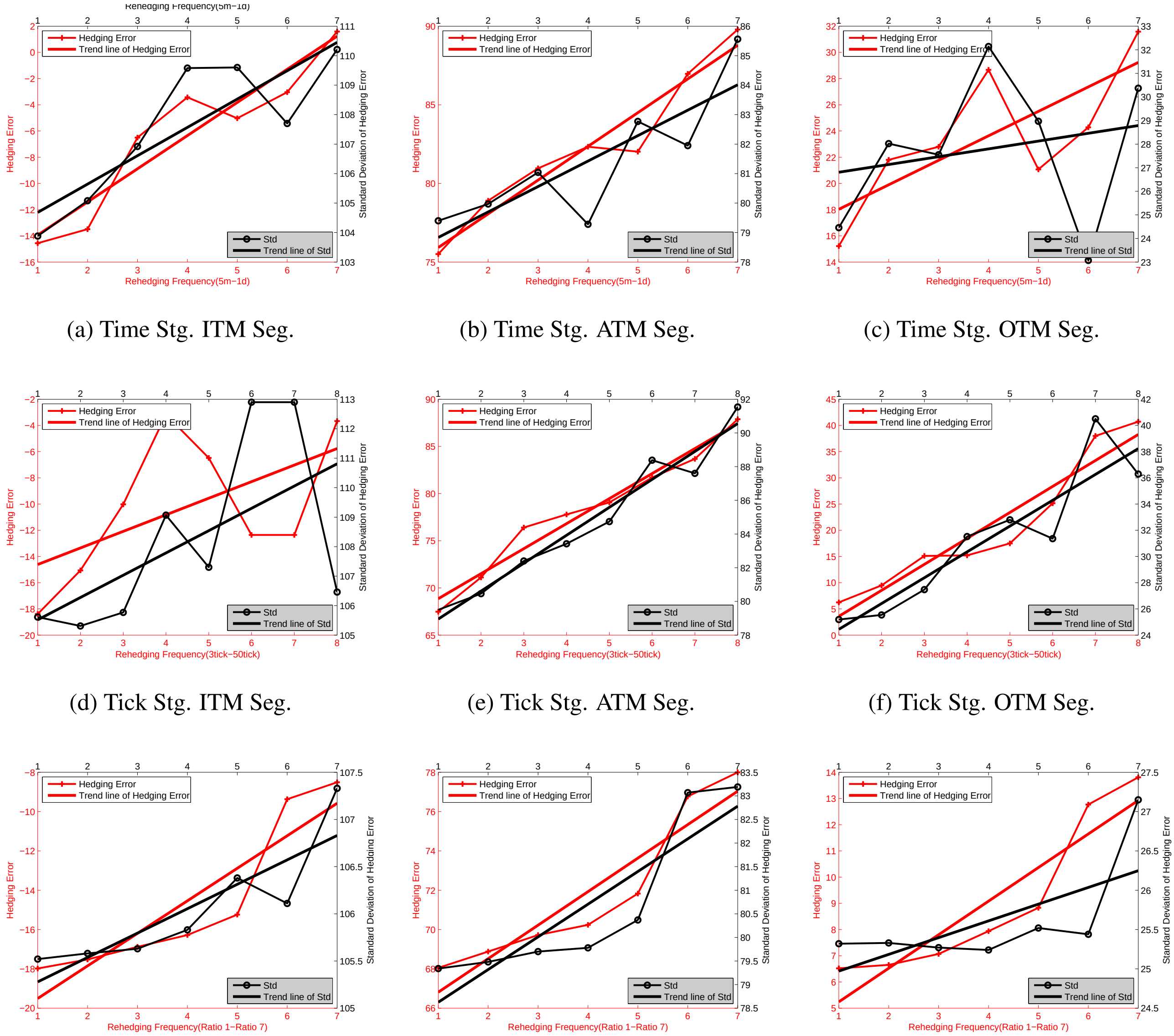
When transaction costs are considered, the expected hedging error and rehedging frequency relation from the BSM actually holds for ATM and OTM options as in Figure 5.12 and across all three hedging strategies. When rehedging frequency increases, the mean of the hedging errors decreases and the standard deviation decreases.

For ITM options, the mean of the hedging errors is negative in all three hedging strategies except that the mean of the hedging errors in the time based strategy with 1-day rehedging frequency is slightly above zero. In this option segment, when rehedging frequency increases the hedging error (loss) increases and the risk decreases.

When transaction costs are included, the hedging errors for ATM options and OTM options are still positive. When hedging frequency increases, the volatility of the hedging error decreases, i.e. the risk decreases but the hedging returns also decrease. This result is intuitive as it indicates that greater risk leads to greater rewards.

The big difference between the result without transaction costs and with transaction costs is that in the second case, when rehedging frequency increases, the risk decreases for all of the option segments and across all rehedging strategies; while in the first case without transaction costs, for OTM options, in time based strategies and delta move based strategies, when rehedging frequency increases, the risk increases.

The general conclusion from both steps in the tests is the risk-return trade-off; when risk represented by the standard deviation increases, the hedging return increases (or the hedging loss decreases).



(a) Time Stg. ITM Seg. (b) Time Stg. ATM Seg. (c) Time Stg. OTM Seg.

(d) Tick Stg. ITM Seg. (e) Tick Stg. ATM Seg. (f) Tick Stg. OTM Seg.

(g) Delta Stg. ITM Seg. (h) Delta Stg. ATM Seg. (i) Delta Stg. OTM Seg.

Figure 5.12: Hedging Errors from Different Strategies with Transaction Costs

The horizontal axis is the rehedging frequency, with 1 being the highest one and 7/8 the lowest. The left vertical axis gives the mean hedging error and the right axis gives the standard deviation. The linear trend is given in each figure. It indicates that for each hedging strategy in each call option segment the higher the rehedging frequency the lower the hedging error and the lower the volatility.

Frequency		5-m	10-m	20-m	30-m	1-h	5-h	1-d
ITM	Mean	-14.56	-13.49	-6.50	-3.43	-5.03	-3.04	**1.59**
	Std	103.88	105.08	106.92	109.58	109.60	107.70	110.21
ATM	Mean	75.51	78.90	80.96	82.33	82.02	86.97	89.79
	Std	79.40	79.97	81.04	79.28	82.77	81.95	85.56
OTM	Mean	15.22	21.80	22.80	28.67	21.07	24.28	31.58
	Std	24.46	28.02	27.55	32.14	28.97	23.07	30.37

(a) Time Based Strategies

Frequency		3-tick	5-tick	10-tick	15-tick	20-tick	30-tick	40-tick	50-tick
ITM	Mean	-18.39	-15.06	-10.02	-3.17	-6.48	-12.36	-12.36	-3.67
	Std	105.61	105.31	105.77	109.07	107.30	112.90	112.90	106.46
ATM	Mean	67.48	71.11	76.40	77.79	79.06	81.70	83.68	87.88
	Std	79.50	80.46	82.40	83.42	84.74	88.38	87.60	91.54
OTM	Mean	6.24	9.49	15.11	15.20	17.50	25.15	38.00	40.71
	Std	25.19	25.54	27.47	31.52	32.80	31.36	40.51	36.29

(b) Underlying Tick Move Based Strategies

Frequency		Ratio-1	Ratio-2	Ratio-3	Ratio-4	Ratio-5	Ratio-6	Ratio-7
ITM	Mean	-17.98	-17.52	-16.88	-16.28	-15.24	-9.37	-8.51
	Std	105.52	105.58	105.63	105.83	106.38	106.11	107.33
ATM	Mean	68.05	68.88	69.72	70.24	71.83	76.77	78.00
	Std	79.34	79.48	79.70	79.78	80.37	83.07	83.19
OTM	Mean	6.52	6.65	7.07	7.94	8.83	12.77	13.81
	Std	25.32	25.33	25.27	25.24	25.52	25.44	27.15

(c) Delta Move Based Strategies

GP	ITM (GP-in-cost)	ATM (GP-at-cost)	OTM (GP-out-cost)
Mean	-15.82	**63.92**	**4.62**
Std	**97.65**	**78.39**	**12.96**

(d) GP Optimal Strategies Trained with Transaction Costs

Table 5.13: Full Dataset Hedging Errors with Tran. Cost

The strategy gives the minimum absolute mean hedging errors and the minimum standard deviation is the best hedging strategy, which are highlighted in **bold** for each option segment. Refer to Table 5.5 for frequency threshold values in delta move based hedging strategies.

5.7.2 Bid-ask Transaction Costs Impact Analysis

On comparison of the two-step results, when transaction costs are included for each option segment and for all hedging strategies, the average hedging return decreases and negative hedging loss increases. Table 5.14 below gives the transaction cost impact by looking at the average hedging errors before and after transaction costs consideration.

The top panel of Tables 5.14a, 5.14b and 5.14c gives the difference of mean hedging errors from tests with transaction costs and tests without transaction costs, which is derived by subtracting the mean of hedging errors without transaction costs from the mean of hedging errors with transaction costs. The bottom panel gives the percentage impact by dividing top panel numbers by the corresponding mean of hedging errors from tests without transaction costs. The high ratios of transaction costs impact on ITM options in the bottom panels are due to small negative hedging errors without transaction costs.

For all three hedging strategies, the transaction costs for OTM options are the most expensive in terms of monetary number for all hedging strategies in the same frequency setting except for 20-tick, 30-tick, 40-tick and 50-tick frequency setting in tick move based strategies, where the most expensive one is from ITM options. For example, for time based strategies with a 5-minute frequency setting, the transaction impact in monetary number is -13.84, -11.30 and -23.99 for ITM, ATM and OTM options. The transaction cost impact on the OTM option is more than the other segments, therefore the transaction costs for the OTM option are the most expensive.

For the highest frequency level of the tick move based strategies where rehedging is triggered when the underlying price has changed by a 3-tick change, the transaction costs cause different changes for different option segments; for OTM options, the mean of hedging returns decrease by 81.9 percent from 34.5 to 6.24; for ITM options, the mean of the hedging returns decrease by 1271 percent from 1.57 to -18.39 and for ATM options, the mean of the hedging returns decrease by 18.2 percent from 82.49 to 67.48.

As expected within the same rehedging strategies, when rehedging frequency increases, the transaction cost impact also increases.

	5-m	10-m	20-m	30-m	1-h	5-h	1-d
	Difference of average Hedging Error with Tran. Costs and without Tran. Costs						
ITM	-13.84	-12.17	-9.84	-9.50	-7.53	-1.78	-1.87
ATM	-11.30	-9.61	-7.88	-6.85	-5.29	-1.70	-1.53
OTM	-23.99	-21.18	-17.52	-15.76	-12.55	-2.21	-3.21
	Difference %average Hedging Error without Tran. Costs						
ITM	1918.55%	919.34%	-295.11%	-156.63%	-300.94%	141.70%	-54.07%
ATM	-13.01%	-10.85%	-8.87%	-7.68%	-6.06%	-1.92%	-1.67%
OTM	-61.18%	-49.28%	-43.46%	-35.47%	-37.33%	-8.33%	-9.23%

(a) Time Based Strategies

	3-tick	5-tick	10-tick	15-tick	20-tick	30-tick	40-tick	50-tick
	Difference of average Hedging Error with Tran. Costs and without Tran. Costs							
ITM	-19.96	-20.18	-14.44	-7.94	-13.80	-12.24	-13.53	-9.88
ATM	-15.01	-12.65	-9.87	-9.11	-9.39	-5.12	-8.92	-4.47
OTM	-28.26	-24.09	-20.00	-16.81	-7.26	-5.49	-3.57	-3.01
	Difference %average Hedging Error without Tran. Costs							
ITM	-1271.76%	-394.38%	-326.51%	-166.36%	-188.46%	9774.94%	-1158.70%	-158.97%
ATM	-18.20%	-15.11%	-11.44%	-10.49%	-10.62%	-5.90%	-9.64%	-4.84%
OTM	-81.90%	-71.75%	-56.96%	-52.52%	-29.30%	-17.91%	-8.58%	-6.88%

(b) Tick Move Based Strategies

	Ratio-1	Ratio-2	Ratio-3	Ratio-4	Ratio-5	Ratio-6	Ratio-7
	Difference of average Hedging Error with Tran. Costs and without Tran. Costs						
ITM	-14.54	-14.38	-13.91	-13.27	-12.08	-7.97	-5.02
ATM	-8.96	-8.65	-8.36	-7.98	-7.32	-4.95	-5.06
OTM	-26.23	-26.14	-25.92	-25.54	-25.08	-20.00	-18.23
	Difference %average Hedging Error without Tran. Costs						
ITM	422.60%	457.09%	468.97%	439.75%	382.48%	565.81%	143.36%
ATM	-11.64%	-11.16%	-10.70%	-10.20%	-9.25%	-6.06%	-6.10%
OTM	-80.10%	-79.73%	-78.56%	-76.29%	-73.97%	-61.03%	-56.90%

(c) Delta Move Based Strategies

Table 5.14: Transaction Costs Impact

5.7.3 Hedging Strategies Performance Comparison

In Section 5.7.1, the performances of different strategies in the full dataset are reported. In this section the out-of-sample results are reported in Tables 5.15a (without transaction costs) and 5.16 (with transaction costs considered). This is to assess the out-of-sample performance of GP optimal hedging strategies and delta move based strategies.

Frequency		5-m	10-m	20-m	30-m	1-h	5-h	1-d
ITM	Mean	-34.10	-33.76	-33.62	-29.43	-30.89	-41.24	-31.54
	Std	127.93	127.96	127.15	138.60	137.87	133.87	139.76
ATM	Mean	46.44	47.79	49.14	49.18	45.11	46.82	48.36
	Std	108.46	108.67	109.20	108.55	111.29	110.47	116.63
OTM	Mean	27.36	28.70	26.65	28.42	23.43	22.01	21.66
	Std	28.42	29.35	27.01	34.49	24.05	20.02	35.41

(a) Time Based Strategies

Frequency		3-tick	5-tick	10-tick	15-tick	20-tick	30-tick	40-tick	50-tick
ITM	Mean	-35.20	-32.21	-35.32	-35.18	-27.62	-28.39	-31.56	-31.97
	Std	129.91	128.95	126.15	130.27	128.97	134.57	130.45	**125.74**
ATM	Mean	42.09	44.14	44.35	42.73	46.09	44.92	53.69	45.37
	Std	**107.19**	110.27	110.97	111.72	112.56	118.22	121.12	122.42
OTM	Mean	25.95	24.66	25.42	29.01	22.80	22.67	23.88	25.94
	Std	32.79	31.60	33.17	30.09	31.54	33.52	36.26	35.21

(b) Underlying Tick Move Based Strategies

Frequency		Ratio-1	Ratio-2	Ratio-3	Ratio-4	Ratio-5	Ratio-6	Ratio-7
ITM	Mean	-38.99	-38.81	-38.69	-38.80	-39.13	-39.99	-42.34
	Std	131.29	131.23	131.43	131.15	131.11	133.53	130.81
ATM	Mean	36.96	37.40	37.98	38.12	38.82	40.70	43.28
	Std	107.63	107.64	107.86	107.83	108.77	113.42	115.03
OTM	Mean	23.38	23.46	23.66	24.12	24.43	23.84	23.03
	Std	33.14	33.10	32.89	33.17	33.46	32.58	33.47

(c) Delta Move Based Strategies

GP	ITM (GP-in)	ATM (GP-at)	OTM (GP-out)
Mean	**-25.67**	**36.22**	**9.47**
Std	148.97	109.92	**16.87**

(d) GP Trained without Transaction Costs

Table 5.15: Out-of-sample Hedging Performance without Tran. Cost

The strategy gives the minimum absolute mean hedging error and the minimum standard deviation is the best hedging strategy, which are highlighted in **bold** for each option segment. Refer to Table 5.5 for frequency threshold values in delta move based hedging strategies.

Frequency		5-m	10-m	20-m	30-m	1-h	5-h	1-d
ITM	Mean	-43.87	-42.89	-38.79	-36.64	-36.39	-42.71	**-32.76**
	Std	125.50	125.31	130.67	136.49	136.86	133.50	139.70
ATM	Mean	36.00	39.06	39.39	42.77	40.22	45.30	46.96
	Std	107.91	108.41	111.49	108.78	111.35	110.13	116.57
OTM	Mean	9.88	13.83	11.21	17.78	15.12	20.34	19.31
	Std	16.80	19.91	21.94	28.56	17.10	18.80	33.93

(a) Time Based Strategies

Frequency		3-tick	5-tick	10-tick	15-tick	20-tick	30-tick	40-tick	50-tick
ITM	Mean	-69.24	-68.66	-63.24	-59.82	-60.12	-56.51	-62.21	-55.00
	Std	136.99	132.27	137.48	140.57	132.91	147.58	138.19	139.39
ATM	Mean	28.31	30.10	33.60	34.58	36.41	38.29	42.32	40.98
	Std	108.44	111.45	112.07	111.39	115.00	118.64	121.74	124.62
OTM	Mean	4.69	7.48	12.50	17.09	15.04	21.00	19.07	24.64
	Std	20.67	21.19	21.91	29.15	29.75	33.24	36.48	39.38

(b) Underlying Tick Move Based Strategies

Frequency		Ratio-1	Ratio-2	Ratio-3	Ratio-4	Ratio-5	Ratio-6	Ratio-7
ITM	Mean	-46.77	-46.48	-46.03	-45.83	-45.14	-44.98	-45.68
	Std	128.60	128.57	128.49	128.28	128.43	127.54	126.40
ATM	Mean	28.88	29.74	30.62	30.80	32.39	37.79	38.44
	Std	108.07	108.00	108.63	108.44	109.63	114.72	113.63
OTM	Mean	4.99	5.16	5.64	6.41	7.22	11.66	12.97
	Std	20.98	20.98	20.92	21.17	21.88	22.60	25.63

(c) Delta Move Based Strategies

GP	ITM (GP-in-cost)	ATM (GP-at-cost)	OTM (GP-out-cost)
Mean	-40.83	**21.78**	**3.78**
Std	**124.15**	**104.65**	**14.64**

(d) GP Trained with Transaction Costs

Table 5.16: Out-of-sample Hedging Performance with Tran. Cost

The strategy gives the minimum absolute mean hedging error and the minimum standard deviation is the best hedging strategy, which are highlighted in **bold** for each option segment. Refer to Table 5.5 for frequency threshold values in delta move based hedging strategies.

Best Strategy	Full Sample		Out-of-sample	
	Mean	Std	Mean	Std
ITM	30-tick	50-tick	GP-in	50-tick
ATM	GP-at	GP-at	GP-at	3-tick
OTM	GP-out	GP-out	GP-out	GP-out

(a) Results without Transaction Costs

Best Strategy	Full Sample		Out-of-sample	
	Mean	Std	Mean	Std
ITM	1-d	GP-in-cost	1-day	GP-in-cost
ATM	GP-at-cost	GP-at-cost	GP-at-cost	GP-at-cost
OTM	GP-out-cost	GP-out-cost	GP-out-cost	GP-out-cost

(b) Results with Transaction Costs

Table 5.17: Best Hedging Strategy

The hedging objective here is to minimise the hedging error, which is either positive or negative. Perfect hedging should achieve a zero hedging error. Under the mean and volatility framework, the best strategy is the one that gives the lowest absolute mean hedging error and the lowest standard deviation.

By this selection criteria, the best strategy without the transaction costs is selected for different option segments and different testing samples from the results of Tables 5.12 and 5.15. The performance comparison result according to the selection criteria is in Table 5.17a. The best strategies with transaction costs considered are in Table 5.17b, which is based on the information from Tables 5.13 and 5.16.

There are 23 candidates, 7 time based strategies, 8 tick move based strategies, 7 delta move based strategies and one GP based optimal strategy for ITM, ATM and OTM options. The best strategies for mean and standard deviation are reported separately as for some cases the minimum absolute mean and minimum standard deviation are from different strategies. It is shown in Table 5.17 that GP optimal control hedging strategies perform better than other strategies in most cases.

Tests without Transaction Costs

For the OTM call option segment, GP gives the consistent best performance not only for full dataset but also for out-of-sample testing dataset. As in Table 5.15, for OTM call options, in the out-of-sample tests, the second best strategy for mean (21.66) is from the 1-day frequency time based strategy and the second best strategy for standard deviation (Std) (20.02) is from the 5-hour frequency time based strategy. *GP-out* improved the mean hedging error by 56.3 percent to 9.47, and improved the Std by 15.7 percent to 16.87.

For ATM call options, GP gives the best performance for the full dataset, while it only gives the best result in terms of the mean of hedging errors in out-of-sample tests. The 3-tick frequency tick move based strategy gives the lowest Std (107.19) for out-of-sample tests, where GP gives the Std of 109.92. The best performance strategy improved the Std from GP by 2.5 percent. The second best strategy for the mean (36.96) of hedging errors in out-of-sample tests is from the highest frequency delta move based strategy. *GP-at* improved the mean hedging error by 2 percent to 36.22.

For the ITM call option, the 50-tick frequency tick move based strategy gives the lowest Std (125.74) in full dataset tests and in the out-of-sample tests, where the Std from GP is 148.97. The best performance strategy improved the Std from GP by 15.6 percent. *GP-in* gives the best performance in terms of the mean (-25.67) of hedging errors, although it does not give the best performance in the full dataset tests. The second best strategy is the 20-tick frequency tick move based strategy, which gives a mean -27.62. *GP-in* improved the mean hedging error by 7.1 percent to -25.67.

Tests with Transaction Costs

For ATM and OTM call option segments, GP gives the consistent best performance not only for full dataset but also for out-of-sample testing dataset. As in Table 5.16, for ATM call options, in the out-of-sample tests, the second best strategy for the mean (28.31) is from the 3-tick frequency tick move based strategy and the second best strategy in terms of the Std (107.91) is from the 5-minute frequency time based strategy. *GP-out-cost* improved the mean hedging error by 23.1 percent to 21.78, and improved the Std by 3.0 percent to 104.65. For OTM call options, the second best strategy for mean (4.69) is from the 3-tick frequency tick move based strategy and the second best strategy for standard deviation (Std) (16.80) is from the 5-minute frequency time based strategy. *GP-out-cost* improved the mean by 19.4 percent to 3.78, and improved the Std by 12.9 percent to 14.64. The reason for GP's success is that it can take in more market conditions and trigger the rebalancing whenever needed while the rehedging frequencies are fixed in prespecified frequency strategies.

For ITM options, the time based strategy with a revision frequency of 1-day is the best one. For this option segment, the investor loses money by writing an option and hedging it. As revision frequency increases, the transaction costs increase therefore the hedging loss increases. It is interesting to note that the GP optimal hedging strategy and delta move based strategies perform well for ATM and OTM options but perform worse for ITM options. However time based strategies perform well for ITM options while they perform worse for ATM and OTM options. We think this is because the gamma for the ITM option is low and correspondingly the delta ratio does not change too much.

5.8 Market Condition Analysis in Delta Hedging

GP is used to find the most sensitive market information variables in hedging, to automatically detect the market change and to give the instruction of rehedging to achieve an objective of minimising the hedging error. This process is realised through the control variable (*hedging band*) function form fitting, which is introduced in the GP working process and discussed in Section 5.5.3. When the current underlying held is outside the tolerance range of the BSM delta ratio, which is defined by the *hedging band*, the rehedge action is triggered and the size of the current underlying held will be adjusted to the closest edge of this tolerance range. When the current underlying held is within the tolerance range of the BSM delta ratio, there is no action needed. In this section, the relationships of market condition variables selected by GP and the *hedging band* are analysed, which reveals how the outperforming results are achieved.

GP is used in this chapter to investigate the optimal hedging strategies for different option segments in two scenarios with and without transaction costs considered. In both scenarios, it achieves good performances, which demonstrates the learning ability of GP in different circumstances. In reality, the bid-ask transaction costs cannot be avoided. Therefore this section focuses on the scenario with transaction costs considered. GP optimal strategies for ATM and OTM option segments give consistent best results among 23 different hedging strategies. The relationships of the control variable and selected market conditions in these two strategies are investigated in the sections below.

5.8.1 Market Conditions and GP *hedging band* for ATM Options

The control variable *hedging band* (HB) for the ATM option takes a function form in Figure 5.13a. There are three variables selected by GP for ATM options, which are the trading durations (*s7*), the logarithm trading volume (*s13*) and the bid-ask spread (*s14*).

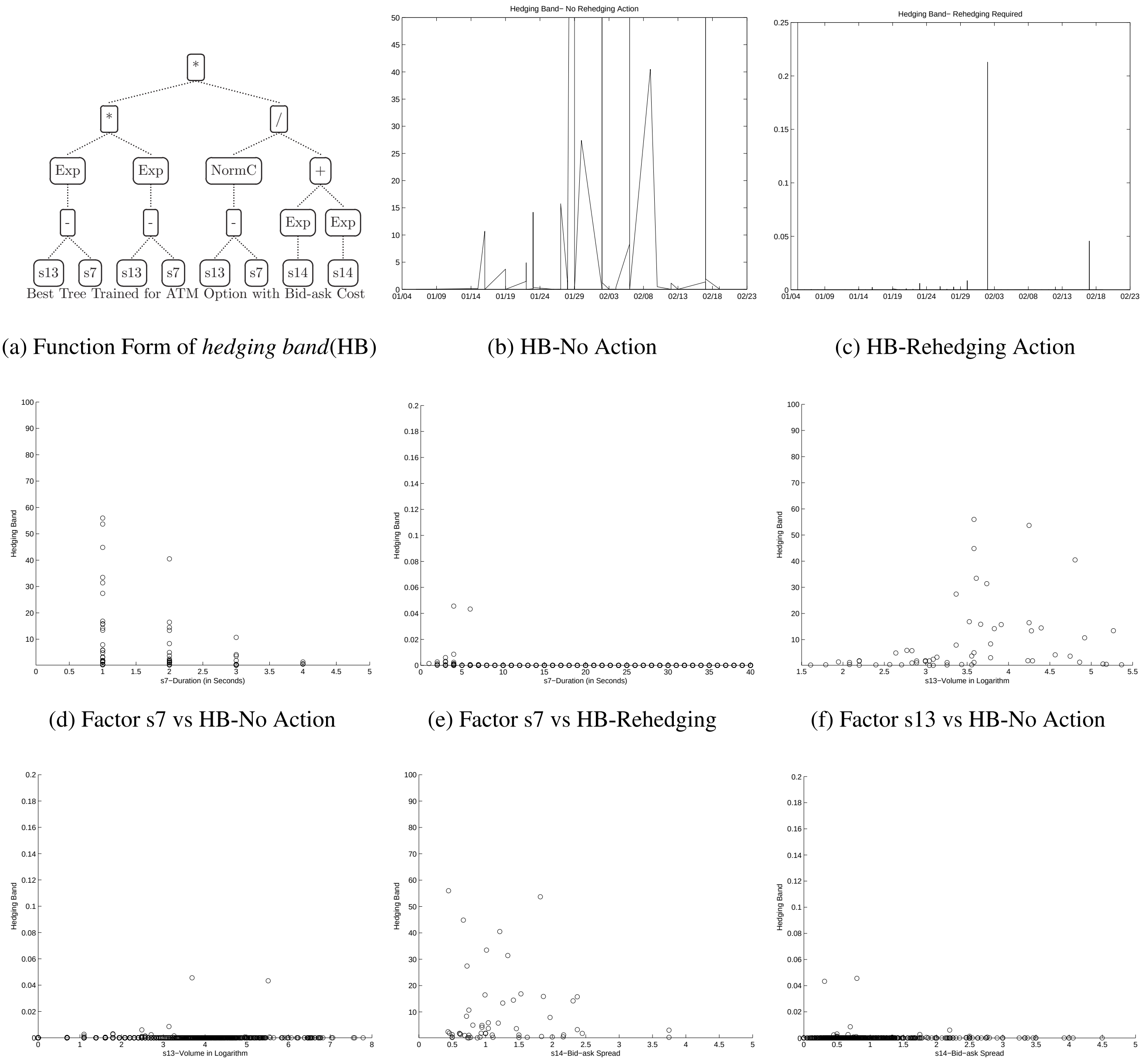
The ATM option hedged during the time period of 04-Jan-04 to 23-Feb-04 is used in this analysis. The HB is graphed in Figures 5.13b and 5.13c separately segmented by no-action and rehedging required. It is observed that the HB is large in Figures 5.13b when no-action is required and small in Figures 5.13c when rehedging is required. Figure 5.13c provides all HBs which trigger rehedging actions during the GP optimal hedging process. The rehedging actions are not evenly distributed in the hedging window, which is different to the time based hedging strategies. It depends on the selected market variables.

The relationships of selected variables and the HB are also analysed separately for no-action required (in Figures 5.13d 5.13f and 5.13h) segment and rehedging required segment (in Figures 5.13e 5.13g and 5.13i). The nonlinear relationships with the HB are observed for all three variables.

For the trading duration variable, when it is small the HB is large in Figure 5.13d and no-action is required. When it is more than 5 seconds the HB is small and rehedging is triggered in Figure 5.13e. There is a negative nonlinear relationship between the trading duration and the HB captured by GP.

For the logarithm trading volume variable, the relationship is not very clear. From Figure 5.13f, the no-action required HB is only from the range of 1.5-5.5. While in Figure 5.13g, the rehedging HB is also associated with the logarithm trading volume out of this range.

For the bid-ask spread variable, the relationship is also not very clear. From Figure 5.13h, for the no-action required HB, the majority of cases are associated with a bid-ask spread in the range of 0.5-2.5. While in Figure 5.13i, the rehedging HB is also associated with the bid-ask spread out of this range.



(a) Function Form of *hedging band*(HB) (b) HB-No Action (c) HB-Rehedging Action

(d) Factor s7 vs HB-No Action (e) Factor s7 vs HB-Rehedging (f) Factor s13 vs HB-No Action

(g) Factor s13 vs HB-Rehedging (h) Factor s14 vs HB-No Action (i) Factor s14 vs HB-Rehedging

Figure 5.13: *Hedging Band* and Market Conditions for an ATM Option

s7:the trading durations; s13:the logarithm trading volume; s14 the bid-ask spread.

It is observed in Figures 5.13e 5.13g and 5.13i that the HB is zero in most cases when rehedging actions are triggered. In these cases, the BSM delta hedging is actually used. Through the captured market conditions, GP is able to decide when to use BSM delta and when not to.

5.8.2 Market Conditions and GP *hedging band* for OTM Options

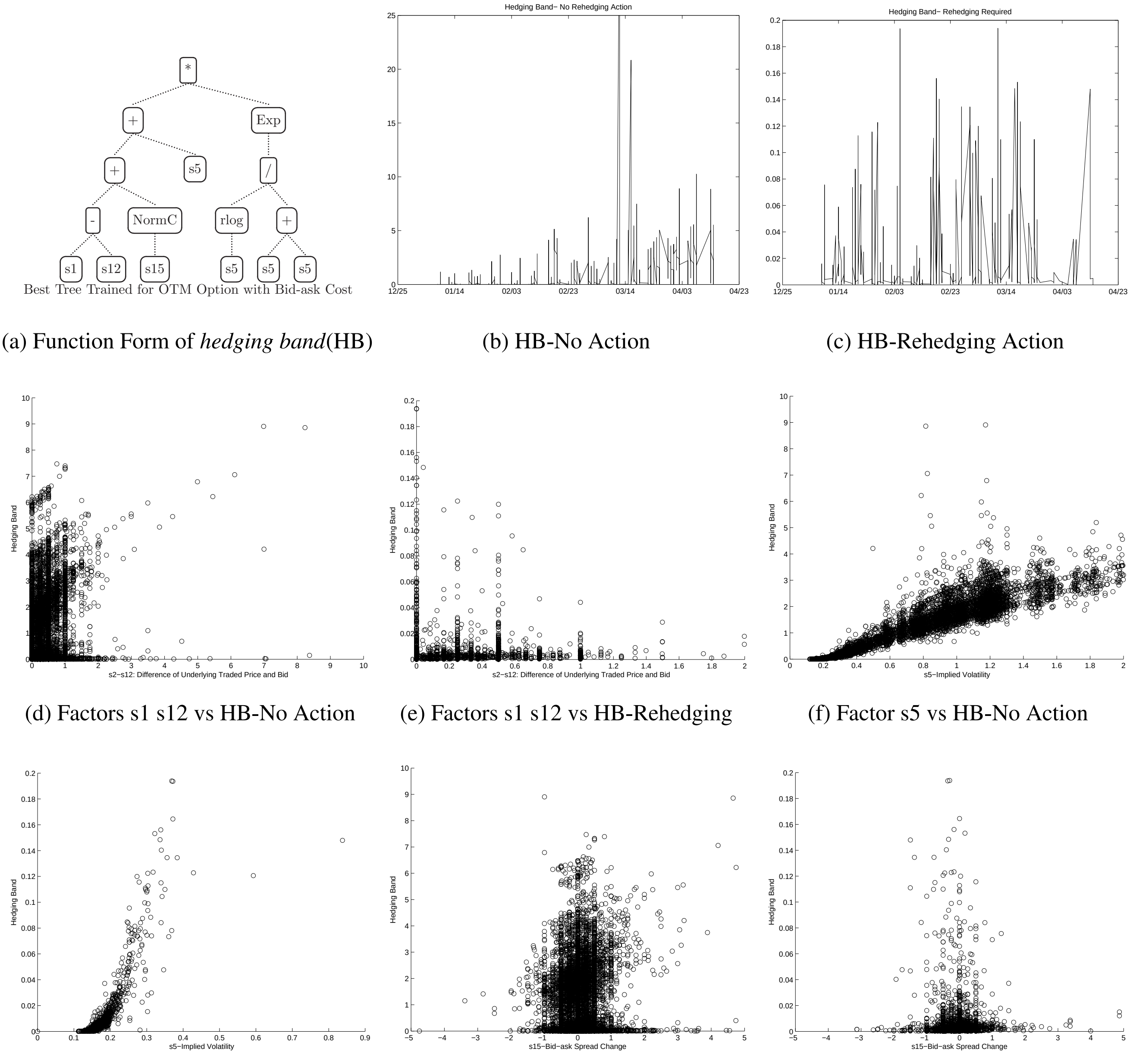
The control variable *hedging band* (HB) for the OTM option takes a function form in Figure 5.14a. There are four variables selected by GP for OTM options, which are the underlying traded price (*s1*), the BSM implied volatility (*s5*), the underlying bid price (*s12*) and the bid-ask spread change compared with one minute ago (*s15*). GP has chosen a good way to use the underlying traded price and the bid price. The difference between the traded price and bid price is used as a new combined variable. Therefore, the underlying traded price and bid price have been analysed together in the following section.

The OTM option hedged during the time period of 10-Jan-04 to 20-Apr-04 is used in this analysis. The HB is graphed in Figures 5.14b and 5.14c and is separately segmented by no-action and rehedging required. It is observed that the HB is large in Figures 5.14b when no-action is required and small in Figures 5.14c when rehedging is required. Each point in Figure 5.14c represents a actual rehedging action required during the GP optimal hedging process. The rehedging actions are not evenly distributed in the hedging window, which is different to the time based hedging strategies. It depends on the selected market variables.

The analysis of the relationships of selected variables and HB is also segmented by no-action required (in Figures 5.14d 5.14f and 5.14h) and rehedging required (in Figures 5.14e 5.14g and 5.14i). The nonlinear relationships are observed from all three variables.

For the variable, the difference of the traded price and bid price, the HB with rehedging required is only associated with this variable from the range of 0-2 and an inverse relation is given in Figure 5.14e. In Figure 5.14d, when the variable is less than 2, it has an inverse relation with HB with no-action needed. When this variable is more than 2, a positive relationship with HB with no-action needed is observed.

For the BSM implied volatility, in Figure 5.14f, a positive and near linear relationship exists between it and where HB no-action is required. In Figure 5.14g, a nonlinear positive relationship is observed between BSM implied volatility and HB with rehedging required. Also, the HB with rehedging required is only associated with the BSM implied volatility less than 0.4. This indicates that when the implied volatility is extremely high, above 0.4, there is no rehedging required. When the implied volatility is within 0.4, the higher the implied volatility the less amount of the size of the underlying that needs to be adjusted. This is because the implied volatility has a positive relation with the HB and the optimal GP hedging strategy only requires the size of the underlying held to be adjusted to the nearest edge of the tolerance of the BSM delta ratio defined by HB.



(a) Function Form of *hedging band*(HB) (b) HB-No Action (c) HB-Rehedging Action

(d) Factors s1 s12 vs HB-No Action (e) Factors s1 s12 vs HB-Rehedging (f) Factor s5 vs HB-No Action

(g) Factor s5 vs HB-Rehedging (h) Factor s15 vs HB-No Action (i) Factor s15 vs HB-Rehedging

Figure 5.14: *Hedging Band* and Market Conditions for an OTM Option

s1: the underlying traded price, s5: BSM implied volatility, s12: the bid price, s15 the change of bid-ask spread.

The variable of the bid-ask spread changes, in Figure 5.14i, associated with the HB with rehedging required has a negative nonlinear relationship with the HB when it is positive; and has a positive nonlinear relationship with the HB when it is negative. The relationship of the variable and the HB is symmetrical about the line, on which the variable equals zero. A similar pattern is noted in Figure 5.14h where the HB does not require rehedging. For the negative bid-ask spread changes, there is a positive nonlinear relationship between the HB and the variable and for the positive bid-ask spread changes, there is a negative nonlinear relationship. However, the relationship of the variable and the HB with no-action is not symmetrical about the line, on which the bid-ask spread change is zero.

There is a dramatic difference between GP strategies for ATM and OTM segments when rehedging is required. For the ATM option segment, in the majority of cases the HB is zero, while, for the OTM segment in most cases the HB is positive. For the ATM segment, in zero-HB cases, when rehedging is triggered, the underlying holding is adjusted to the BSM delta ratio. For the OTM segment, when rehedging is triggered with positive HB, the underlying holding is adjusted to a ratio, which is in a range around the BSM delta ratio.

5.9 Chapter Summary

According to the BSM, as long as the hedging portfolio including the underlying stock, is re-balanced continuously with Δ re-calculated continuously, the overall portfolio will be perfectly hedged with a zero hedging error. However, in real world financial markets this is not the case as the assumptions in the BSM do not hold.

In this chapter, BSM delta hedging is examined in one year high frequency FTSE 100 index futures and options dataset by three rehedging strategies, which include rehedging at uniform time intervals, rehedging when the underlying asset moves by a fixed number of ticks and rehedging based on a change in the delta of the option. All tests are done in two steps. In the first step, transaction costs are not considered. In the second step, transaction costs are taken into account in all hedging strategies by using the corresponding bid and ask futures prices when trading the underlying security to maintain a hedged portfolio.

The empirical tests conducted in this research indicate that when delta hedging rebalancing frequency increases, the hedging return decreases and the corresponding risk decreases. This trend holds clearly for all option segments and across all three tested hedging strategies after transaction costs are considered. In terms of the monetary amount, the bid-ask transaction costs for OTM options are the most expensive ones compared with other option segments. Therefore the transaction costs impacts the OTM more than the ITM and ATM options.

A GP based hedging strategy is also examined, in which a control variable, *hedging band* is used to decide the rehedge frequency and the new hedging ratio, based on the BSM delta hedge ratio and market conditions. It has been trained with a fitness function to minimise hedging error. As expected it gives the lowest hedging error and lowest hedging risk for ATM options and OTM options compared with other hedging strategies. The relationships of market conditions and this control variable are analysed through the function form returned from GP. Future research could focus on using a GP based hedging strategy with a joint objective function of maximising hedging return whilst minimising hedging risk.

This chapter has made a few contributions, which are listed below.

- This chapter gives an up-to-date literature review on option delta hedging. Section 5.3 presents an up-to-date survey of relevant literature on theoretical and empirical delta hedging studies. Firstly, the theoretical works of delta hedging under the discrete hedging and transaction costs are reviewed. Then the different methodologies measuring the statistics of the discrete hedging errors are summarised. Theoretical works of model misspecification on hedging are also reviewed. Studies of the hedging error in the presence of microstructure effects are also covered. Empirical analyses under a controlled environment and through the use of real data are also summarised and the research gap in delta hedging is revealed.

- This chapter empirically assesses Black-Scholes delta hedging by high frequency data. According to the Black-Scholes option pricing model, as long as the hedged portfolio composed by an option and a certain amount of underlying assets indicated by the delta ratio is rebalanced continuously, the portfolio earns the risk-free rate. The availability of high frequency data makes this assessment possible. This dissertation has made the first effort to empirically assess delta hedging theory using high frequency data. The result demonstrates that for at-the-money options and out-of-the-money options the option writers who hedge their positions on a high frequency basis do generate returns greater than the risk-free rate although the hedge portfolio also bears some risk. When the rehedging frequency increases, the risk and the return decrease. The risk rewarding relationship is not the same across different option segments.

- This chapter empirically assesses the bid-ask spread transaction cost impact on delta hedging. Designed deterministic delta hedging methods, where the rebalancing decisions are not based on market conditions are carried out on the futures transaction price as well as on the bid and ask prices. In the second case, when buying the futures, the ask price is used and when selling the bid price is used. The hedging results without transaction cost and with transaction cost are compared. The bid-ask spread transaction cost has impacted the hedging result differently for different option moneyness segments.

- This chapter proposes an optimal GP delta hedging strategy, which makes a hedging decision based on market conditions. An optimal GP delta hedging strategy is proposed by considering all relevant market conditions including bid-ask spread, trading volume, average trading duration and implied volatility. In this strategy rebalancing frequency and the new hedge ratio are driven by market conditions. The out-of-sample results show that this optimal GP delta strategy can outperform the other strategies with a significantly lower risk when minimum risk is required.

- This chapter empirically assesses the relationships of delta hedging and market conditions. In the proposed GP optimal delta hedging strategy, control variable *hedging band* is used to instruct the rehedging frequency and rehedging ratio, which is a nonlinear combination of market conditions. The relationships of delta hedging and the selected market conditions are analysed dynamically for ATM options and OTM options through the fitted function form of *hedging band*.

Part III

Conclusions

Chapter 6

Conclusions and Future Work

Financial derivative is a financial instrument whose value depends on or derives from the value of other underlying securities. The exchange-traded derivative markets have irreplaceable roles in finance. Financial derivatives are used for shifting risk, arbitrage between different markets, speculation and changing the nature of an investment without incurring transaction costs. The importance of the derivative market is demonstrated by its significant growth in monetary size.

Risk modelling in derivative markets is crucial. The risk and rewarding relations in these leveraged markets are not straightforward as in the conventional financial markets. Investors can open a derivative product position with little money, however the potential loss involved could be unlimited. More importantly, in extreme events, risk management can reduce the cost of financial distress [2]. Risk management in the derivative markets covers a broad range. This thesis has examined one aspect of the risk in the derivative market, the risk from the underlying price changes.

In this chapter, the thesis summary is given in Section 6.1 and the limitations of this thesis are highlighted in Section 6.3. Suggestions for future work complete the chapter in Section 6.4.

6.1 Thesis Summary

One of the lessons learned form the latest financial crisis of 2007-2009 is that a financial decision should be made with environmental changes in mind and should adapt accordingly [1]. This thesis empirically investigates the question of how to model the risk in derivative markets by considering market conditions. In detail, this thesis has examined two risk management problems in financial derivatives markets, realised volatility modelling and option delta hedging. GP, a population based model induction methodology is used to tackle these two problems. In order to make a financial decision by considering market conditions, the following questions have been asked.

1. What are the characteristics of derivative products and what are most important factors in the financial derivative market environment?
2. How to apply GP efficiently to two financial problems?
3. How to forecast realised volatility by considering all related market conditions?
4. How to get an optimal intraday delta hedging strategy by considering market conditions?

The first question is addressed in Chapter 3 where the derivative products characteristics and the trading environment are analysed through a high frequency dataset including one year FTSE 100 futures and options transactions and quotations. The characters of futures contracts and options contracts are analysed. The prevailing derivative market theory is assessed against real data. The dynamic trading environments for futures and options are reviewed initially. Important indications are drawn from this analysis for the following empirical studies.

GP methodology and its current financial applications in finance are reviewed in Chapter 2. GP's working process is explained. A demonstration example of applying a new form of GP in option pricing is given. GP's applications in different finance areas including asset price/return prediction, financial trading, volatility modelling, discovering arbitrage opportunity and portfolio management are then reviewed. Key parameter settings in these financial applications are summarised. These make preparations for the two financial risk modelling studies conducted in the following chapters, which fulfil the third and the fourth research aims.

The third research aim is addressed in Chapter 4. The literature review of financial volatility modelling is given first. Realised volatility is a measure of volatility, which bridges high frequency data and the daily volatility. With this concept volatility can be modelled directly. The realised volatility is estimated based on FTSE 100 index futures data. One-day-ahead RV is forecasted based on market conditions by GP. The out-of-sample model performances are compared among GP models and benchmark models. Market conditions are analysed by the best individual from a GP. The results indicate that the best volatility modelling strategy dynamically changes and adapts to its modelling environment.

The empirical delta hedging is in Chapter 5. A literature review of delta hedging in terms of discrete rehedging, transaction cost impacts and in the presence of market microstructure effects is given first. It is identified that the empirical work of delta hedging compared with its theoretical work is behind. The BSM delta hedging theory is assessed by three deterministic rehedging frequency strategies including rehedging at uniform time intervals (e.g. every five minutes), rehedging when the underlying asset moves by a fixed number of ticks (e.g. every 10 ticks), rehedging based on a change in the delta of the option (e.g. if the delta changes from 0.5 to 0.52) through the one year high frequency dataset. An optimal rehedging strategy by GP is proposed, where the rebalancing decision is made on a nonlinear function of a number of liquidity and volatility measures related to the conditions of the market. The out-of-sample performance results show that the GP based optimal delta hedging strategy gives the minimum risk. The reason for GP's success is that it can utilise surrounding market conditions.

6.2 Contributions

In this thesis, through the gathering of the literature on relevant domains, two research gaps are recognised. The dynamic trading environment of FTSE 100 futures and options markets is analysed. Two financial problems, realised volatility modelling and optimal delta hedging strategy are examined empirically by a GP methodology with the market conditions considered. A number of key insights are generated. This section reviews the contributions of this thesis in the following sections.

6.2.1 Literature Review Contributions

There are three literature review contributions in this thesis. The first is to give an up-to-date literature review of GP applied in finance. In Chapter 2, the GP's applications in financial modelling are reviewed and summarised. GP is the methodology used to study the risk modelling issues in derivative markets. These reviewed financial applications, especially their GP parameter settings give references regarding how to efficiently apply GP in this thesis.

In Chapter 4, the up-to-date literature review of realised volatility modelling is given which covers volatility concepts, up-to-date traditional volatility modelling approaches and the relationships of volatility and market conditions (including trading volume, number of transactions, bid-ask spread and price range) and empirical volatility modelling works. The research gap in RV modelling is identified from the literature reviewed.

In Chapter 5, the up-to-date literature review of option delta hedging is provided, which covers different aspects of delta hedging including the theoretical works of delta hedging under the discrete hedging and transaction costs, the different methodologies measuring the statistics of the discrete hedging errors, theoretical works of model misspecification on hedging, studies of the hedging error in the presence of microstructure effects, empirical analyses under the controlled environment, and empirical analysis by real data. The research gap in delta hedging is revealed through a review of the theoretical and empirical delta hedging research works.

6.2.2 Methodology Contribution

A new form of GP is designed for the option pricing problem where the probabilities of crossover and mutation are adapted dynamically during the GP run. Compared with GP with a static parameter setting, the adaptive GP is able to provide an improved result. This adaptive form GP is suitable for complex problems where local optimum exist and demonstrate the potential ability of GP in solving complex problems. This is a methodology contribution.

6.2.3 Empirical Analysis Contributions

Derivative Data Analysis Contributions

Chapter 3 provides the analysis of the characteristics of derivative products and their intraday trading environment in the stock index futures market and stock index options market. FTSE 100 index futures and options markets belong to financial exchange trade derivative markets, where the trading environment is different to conventional financial markets. The analysis gives the products' characteristics and the statistical summary of the traded price, volume, quotation price and quoted depth for both futures and options.

The related financial theories including put-call-parity, spot-futures-parity, futures and options co-movements have been examined in this one year dataset. The analysis in the selected samples shows that financial theories do not always hold at an intraday level. This indicates that market conditions are important considerations in financial risk modelling besides financial theories.

RV Analysis Contributions

The relation of market conditions and realised volatility are analysed dynamically by the fitted RV function form. The factor of average trading duration has been shown to have a consistent nonlinear relation with RV.

Option Delta Hedging Analysis Contributions

Chapter 5 empirically assesses Black-Scholes delta hedging by high frequency data. The assumptions in the Black-Scholes option pricing model include that underlying returns are martingales and investors are rational. In the BSM world, as long as the hedged portfolio composed by an option and a certain amount of underlying assets indicated by the delta ratio is rebalanced continuously, the portfolio earns the risk-free rate. The assessing result demonstrates that for at-the-money options and out-of-the-money options the option writers who hedge their positions on a high frequency basis do generate returns greater than the risk-free rate although the hedge portfolio also bears some risk. When the rehedging frequency increases, both the risk and the return decrease. The risk rewarding relationship is not the same in different option segments.

Chapter 5 also empirically assesses the bid-ask spread transaction cost impact on delta hedging. The empirical delta hedging tests are done separately without and with considering the bid-ask transaction costs. In the first case, the traded futures prices are used. In the second case, when buying the futures, the ask price is used and when selling the bid price is used. The impact of transaction costs is assessed by comparing two tests results.

Another empirical contribution from Chapter 5 is that the relationships of delta hedging and market conditions are analysed through the fitted function form of the control variable, *hedging band*.

6.2.4 Modelling Contributions

Two risk modelling issues are examined in this thesis. The first is the RV forecasting and the second is the option delta hedging. There are two specific modelling contributions in these two topics and one general modelling contribution is made.

Chapter 4 adaptively forecasts one day horizon realised volatility by the market conditions. Traditionally, volatility is modelled by its lagged information in GARCH and ARCH theoretical framework. In the proposed method, RV is adaptively forecasted with a number of market conditions considered. The one day out-of-sample is compared with benchmark models including ARMA, GARCH, HAR and a stepwise linear regression. In the first three benchmark models only RV lagged information is used. All potential market condition factors provided to GP are also used in the stepwise linear regression. GP's outstanding performance indicates that the relations between RV and market conditions are nonlinear. Though stepwise regression takes account of market conditions, it failed to give an improved result than the models that only took the lagged RV information.

Chapter 5 proposes an optimal GP delta hedging strategy by taking account of the market conditions including bid-ask spread, trading volume, average trading duration and implied volatility. In this strategy rebalancing frequency and the new hedge ratio are driven by market conditions. The out-of-sample results show that this optimal delta strategy can outperform the other strategies with a significantly lower risk when minimum risk is required. The reason for this is that the proposed optimal delta hedging strategy is able to take account of market conditions.

This thesis demonstrates that by using relevant market conditions, improved modelling results are achieved based on theoretical domain knowledge. In practice, financial modelling considering relevant conditions could be potentially highly rewarded. From this point of view, the thesis makes a general contribution to financial modelling.

6.3 Limitation

Risk management in derivatives markets is a very broad concept. Take the exchange-traded options for example, the risk aspects can be summarised by Greek letters, which are quantities representing the sensitivity of the price of options to a change in affecting factors. A sample of these affecting factors includes the underlying price, the volatility of the underlying asset, the option time value, interest rate and the option delta. One thesis however, cannot cover all aspects of this topic. This thesis studied underlying price change related risk management issues in derivative markets. More specifically, it makes contributions in one-day horizon RV forecasting and option delta hedging in the presence of market microstructure effects. There are a number of areas worthy of further research that fall outside the scope of this work.

Realised volatility in this thesis is estimated by summing up the squared five-minute logarithm returns. There is also an increasing interest in controlling the microstructure effects to construct RV by ultra high frequency data [126] and [127]. The modelling approach in this thesis can be extended to a RV constructed on ultra high frequency data.

Due to time limitations, the put option contracts are not used in the delta hedging strategy tests. Although in theory there is a put-call-parity, in the presence of microstructure effects in real data, the relation between call options and put options may not be as expected. This limitation should be considered in the future work.

The delta hedging strategy considered in this thesis is for an option lifetime hedging. In reality, practitioners also open and close an option position on a daily base. It will be useful to extend the current study to an intraday hedge for an option position where the hedging error results will be checked daily.

GP is a promising modelling methodology for the finance area. It has been developed for more than two decades. Researchers have devoted themselves to the development of advanced functions of GP, including automatic defined functions(ADF), multi-objective GP and adaptive forms of GP listed as examples. In this thesis only the basic form of GP is applied in the two risk modelling studies. Modelling financial risk problems by advanced GP should be considered in the future work.

6.4 Future Work

Building upon this research, there are numerous avenues for future work. A few of them are outlined as follows.

For the RV modelling work, there are two future works planed.

1. The results of RV forecasting in Chapter 4 indicate that the forecasted information from benchmark models including HAR and ARMA does not overlap with the GP's forecasting. In the future, forecasting results from other conventional models including HAR and ARMA could be used as explaining inputs in GP to further improve the model performance. The linear relation picked up from stepwise regression could also be used in GP. The relation of market information variables and RV should be examined further in a longer time period dataset to give a robust conclusion.

2. As discussed in Section 6.3, there is a RV estimation method, which can utilise the ultra high frequency data (more frequently than 5-minute data) with the microstructure effects controlled. In the future RV should be constructed in this method. RV from different estimation methods should be compared.

There are a few directions proposed for the delta hedging modelling.

1. As discussed in Section 6.3, only the call option contracts were used in the delta hedging tests. In the future the same tests should be extended to put option contracts. Hedging results from put options should be compared with the current results from the call options.

2. The optimal GP delta hedging strategy proposed in Chapter 5 is to minimise the hedging error. As expected it gives the lowest hedging error for at-the-money options and out-of-the-money options compared with other hedging strategies and the strategy responds to market conditions. Future research could focus on the use of a GP based hedging strategy with a joint objective function of maximising hedging return whilst minimising hedging error.

3. The market conditions dataset should be extended to other relevant factors, such as RV of the underlying security. It can be used as a market condition factor to be considered in the optimal GP delta hedging strategy.

4. As discussed in Section 6.3, practitioners normally close an option position on a daily base. The current result from option lifetime hedging may not apply in the case of intraday hedging. The delta hedging tests should be extended to intraday hedging.

On the methodology side, advanced GP should be tested in future risk modelling work. Potentially improved results can be expected. The advanced GP includes the new form of GP proposed in Section 2.2, Automatic Defined Function in GP, Parallel GP and multi-objective GP among others.

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