Mathematical Methods, Algorithms and Implementations

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Exercises 2

Discrete Fourier Transforms, and Sampling

The purpose of these exercises is to gain some experience of the practical aspects of sampling

*** This exercise is formally assessed and counts 10% towards your result***

1. Consider the Band-limited function defined by the following MatLab M-File function :

   function res=BLF(t) res=cos(t)+4/5*sin(3*t)-1/6*cos(5*t)+2/7*cos(7*t);

   Determine the Nyquist frequency for this function and sample it

   (a) above
   (b) below

   the Nyquist frequency

   write functions to reconstruct BLF at a higher sampling rate than either a) or b) using

   (a) Time-domain interpolation with a Sinc function
   (b) Fourier domain interpolation by zero-filling

   compare the results to that obtained by directly sampling BLF at the higher sampling rate. Compare the timings in cases a) and b) [In MatLab use the "Tic Toc" function or some other timing function].

   You may find that the reconstructed signal is scaled and time-shifted - explain why and find a correction either in the Fourier domain, or the temporal domain

2. Repeat the above using an unbounded function in the frequency domain such as a Gaussian. In this case the reconstruction can never avoid aliasing.Make estimates of errors as a function of sampling interval

3. Use Fourier zero-filling to implement interpolation for a 2D digital image and compare with any other method (nearest neighbour, linear interpolation)that you have been introduced to in the Machine Vision course.

Notes :

1. Your coursework should consist of program code, results and a short discussion (probably not more than 4 pages).

2. Here is a MatLab routine to read in a bitmap image:

   imread('filename.bmp','bmp')

   Similarly other image formats can be read.

3. To interpolate an image, for example resize it, you can use the imresize function in the Image Processing Toolbox.