Dialectics and Logic

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1 Introduction

A vigorous debate concerning the relationship between Hegelian and Marxist thought has been taken up again [Mosely1993, Rees1998, Rosenthal1998, Smith & Ollman1998a, Rosenthal1999, Smith1999] with a reevaluation of the dialectic method in Marxism. The central issue in this debate is the importance of a dialectic method of enquiry and presentation for Marxism in general and for Marxist political economy in particular. At one extreme, dialectics is presented as a general logic of development (see [Smith1993, Rees1998]). [Smith & Ollman1998b] argue that "the form of all Marx's arguments are dialectical. Hence, so long as Marxism helps us understand the world we will need to study dialectics in order to improve our understanding of Marxism". Against that Rosenthal offers the most sceptical assessment, arguing that the dialectic method is quite mystical and, worse, "dynamic historicism is not a 'method', but merely a methodological fantasy" [Rosenthal1998, p 33].[1]

Although the discussion focussed on the dialectic method of enquiry and presentation and its application in political economy, the discussion necessarily raised the issue of dialectic logic, as an alternative to, or extension of, formal logic. Generally, in the cited works, 'Hegelian logic' is used to describe Hegel's conceptual framework for his analysis. In this article, I reject the notion that this is a logic at all and investigate more thoroughly

[1] The cited works include a spectrum of other positions on this question. Arthur, for example, puts a convincing argument that Hegel's logic is helpful to Marx's study of Capital only because of the inverted reality of the Capitalist system, saying that Hegelian logic "is indeed relevant — precisely to the peculiar character of a money economy" [Arthur1993].

the relationship between dialectics and formal logic. Thus I support Rosenthal's project of freeing Marxism from some of the more mystical aspects of Hegelian thinking, without committing the errors of the analytic Marxists who threw out the tenets of Marxism as well as those of Hegelianism.

Dialectics and Logic Dialectics and formal logic are sometimes posed as two contrasting forms of reasoning. In this contrast, formal logic is appropriate for reasoning about static properties of separate objects involving no interaction. To deal with change and interaction it is necessary to use the dialectic approach. In some accounts, these two subjects are seen as complementary. Accordingly, formal logic is not wrong, it is just too restricted in its domain of application. Dialectic logic generalises formal reasoning and goes beyond it. To use an analogy, this is like the relationship between the theory of relativity and Newtonian mechanics. Newtonian mechanics can be explained by relativity theory and is fairly accurate, provided you deal only with speeds much slower than the speed of light. And so formal logic is not wrong, provided you restrict yourself to properties which are static and lifeless. Once you start reasoning about change and interaction you must move from formal to dialectic logic (see, for example, [Smith1999, page 232]). Arthur wrote that "Dialectic(s) grasps phenomena in their interconnectedness, something beyond the capacity of analytical reason and linear logic"[Arthur1998]. Trotsky used the metaphor of elementary and higher mathematics to explain the relationship between formal logic and dialectic logic.

There are other expositions of the theory of dialectics which present it in opposition to formal logic. For example, Novack writes

> "The ruling ideas of the ruling class in logical science today are the ideas of formal logic lowered to the level of common sense. All the opponents and critics of dialectics stand upon the ground of formal logic, whether or not they are fully aware of their position or will honestly admit it." [Novack1973, page 28]

The problem with these views is firstly that it is not clear in what sense dialectics is a logic. Also, when formal logic is counter-posed to dialectics, formal logic is usually taken to be the syllogism of Aristotle, even though the subject has advanced considerably since classical times. A further difficulty in considering the relationship between formal and dialectic reasoning is that

in the latter view there are contradictions existing in reality, whereas in the former view this is completely impossible.

Because of these problems, there is a danger that the dialectic approach will seem unscientific and its strengths will be overlooked. In this article I defend dialectical materialism as a great advance over previous *philosophies* and the correct framework for a scientific method of understanding the world, but I reject the notion that dialectics is a *logic*. I investigate the relationship between modern formal logic and dialectics and re-appraise some of the formulations given in the Marxist tradition. I show that formal logic is not a fixed doctrine, but a tool that we use to help us model the reasoning process. In its early history formal logic was a subject that was restricted to static, non-interacting events. But, like other tools, formal logic had to be extended and developed in the course of history. On the other hand I argue that dialectics is not a logic at all, but a philosophical and conceptual framework, much more powerful than its rivals. Thus the two approaches are really dealing with different things and certainly should not be seen as opposing each other.

2 Logic

I propose to define a logic to mean a model of a rational thought process. A thought process is a developing sequence of thoughts and it is rational if the development can be justified. A logic should be able to tell us when it is permissible to make a certain deduction and when it is not. This definition has the disadvantage that it will offend both formal logicians and Marxists. In logic there is much excellent research which has no obvious connection with the problem of modelling human reasoning. And proponents of dialectic logic will perhaps find this definition too restrictive in that it almost certainly rules out a dialectic method of reasoning (see below). But, at least for the purposes of this article, I want a word that describes how we can go from premises to conclusions and the word I use is 'logic'.

Furthermore, formal logic is mainly concerned with the form rather than the content of an argument. If I point a gun at you and demand your money, my argument is persuasive but not logical. A logic is a *formal logic* if there are unambiguous rules that tell us whether a deduction is correct, or at least consistent, or not. A formal logic must in no way depend on contextual knowledge of a particular problem domain, nor on intuition or any factors

which are not clear and explicit. This separation of form from content in logic is criticised by dialectitions and we'll consider these criticisms later. Still, it should be acknowledged that formal logic has great strengths: the process of reasoning is made clear and transparent.

Marxists have made serious criticisms of formal logic but unfortunately the main part of the Marxist literature deals with that form of logic expounded by Aristotle, over 2300 years ago. So here I give a very brief account of some of the key episodes in the development of logic.

Before Aristotle's time it was not thought necessary to formalise the deductive process. Elementary properties of numbers and geometry were taken to be self-evident truths. But following the discovery of irrational numbers at the time of Pythagoras, Greek mathematics entered a crisis [Szabo1978]. Concepts of number and arithmetic, having previously been considered as reliable and beyond all questioning, were shown to be problematic. The Greek philosophers responded partly by adopting geometry instead of arithmetic as a solid foundation of knowledge, but at the same time they no longer trusted their intuition, so they wanted a system of reasoning in which every step in a deduction was clearly justified.

The Aristotelean syllogism was the first great system of formalising the laws of rational thought. At its heart there were three principles.

The law of identity For any object, x, we have x is x.

The law of non-contradiction Nothing is allowed to have the predicate P and simultaneously the predicate not-P.

The law of excluded middle Everything has either the predicate P or the predicate not-P.

Here a predicate is any property that may or may not apply to an individual, e.g. 'mortality' is a predicate that applies to an individual, say Socrates. Thus 'Socrates is mortal' is a basic proposition in Aristotle's system. Based on these three elementary laws there were a number of *syllogisms* which were rules about correct inferences that could be made from given premises. An example of such a syllogism is the following:

Socrates is a man,
All men are mortal,
Therefore Socrates is mortal.

As I mentioned, the basic propositions are predicates applied to single individuals. Aristotle considers relations *between* different objects to be a very problematic field and not really suitable for formalisation[2]. The problem of properties which change in time is not dealt with.

Until relatively recently this form of reasoning remained unchallenged. Indeed Kant [Kant1992] had argued that

> Since Aristotle's time Logic has not gained much in extent, as indeed nature forbids it should. ... Aristotle has omitted no essential point of the understanding; we have only become more accurate, methodical, and orderly.

Yet since Kant, formal logic has undergone revolutionary changes. If formal logic is to be criticized, it must be in its modern form.

Augustus De Morgan was one of the first formal logicians to criticize the Aristotelean syllogism. De Morgan was interested in modelling the laws of rational thought and found the syllogism inadequate in two ways. It was *expressively* inadequate, because it could not express relations between things, only properties of single objects. And it was *deductively* inadequate, because properties of relations could not be deduced using the laws of the syllogism. In 1860 he wrote:-

> Accordingly, all logical relation is affirmed to be reducible to *identity* A is A, to *non-contradiction*, Nothing both A and not-A, and to *excluded middle*, Everything either A or not-A. These three principles, it is affirmed, dictate all the forms of inference, and evolve all the canons of syllogism. I am not prepared to deny the truth of either of these propositions, at least when A is not self-contradictory, but I cannot see how, alone, they are competent to the functions assigned. I see that they distinguish truth from falsehood: but I do not see that they, again alone, either distinguish or evolve one truth from another. [de Morgan1860]

So De Morgan attempted to develop a modern formalism which could overcome some of these limitations. The formalism he chose was an abstract algebra of binary relations. Algebra was increasingly successful in the 19$^{\text{th}}$ century and De Morgan was particularly impressed by the calculus of propositions invented by the Irishman George Boole — what we now call *Boolean algebra*. De Morgan wrote

[2]See Aristotle's *Categories* chapter 7 in [Aristotle1963].

> When the ideas thrown out by Mr Boole shall have borne their full fruit, algebra, though only founded on ideas of number in the first instance, will appear like a sectional model of the whole form of thought. Its forms, considered apart from their matter, will be seen to contain all the forms of thought in general. The anti-mathematical logician says that it makes thought a branch of algebra, instead of algebra a branch of thought. It *makes* nothing; it *finds*: and it finds the laws of thought symbolized in the forms of algebra.

So in the 19th century mathematicians like De Morgan, and later Peirce, Schröder and Tarski, made advances in mathematical logic using an algebraic framework. These algebraic logics made significant advances on Aristotle, notably their basic elements were binary relations (or binary predicates) — properties which relate two objects to each other.

Of even greater significance, though, was the invention of quantifier logic, what we now call first-order logic or predicate logic, by Frege [Frege1972]. And later, Alfred Tarski gave first-order logic a formal and precise semantics. In Frege's quantifier logic you can write down predicates that relate more than one object. For example "sister" is a binary predicate which relates two people to each other. So "Anne is the sister of John" is a basic formula (called an atomic formula). More complex formulas can be built from these atomic formulas in a number of ways. You can *negate* a formula: so "Anne is *not* the sister of John" is a formula. You can form the *disjunction* of two formulas: so "Either Anne is the sister of John *or* x is the sister of Anne" is a formula (the letter x here is called a *variable*). Similarly, you can form the *conjunction* of two formulas by connecting them with the word 'and'. And you can quantify variables: "there exists some x such that x is the sister of John" is also a formula[3]. A conjunction of a formula and its negation is called a contradiction, e.g. "Anne is the sister of John and Anne is not the sister of John" is a contradiction.

There are also methods of deduction in first-order logic. In a Hilbert system, for example, we have a number of axioms and rules of inference. A sequence of formulas each of which is either an axiom or follows from previous formulas in the sequence by one of our rules of inference is called a proof. Incidentally, using just three axiom schemes and one inference rule, it is possible to prove an arbitrary formula from a contradiction. Thus first-order

[3]See [Hodges1991] for an introduction to predicate logic.

logic (indeed, even the less expressive propositional logic) becomes entirely degenerate in the presence of contradictions.

First-order logic is the benchmark for modern logics. It is highly expressive, certainly compared to propositional logic. There are many other logics that have come since. Some of these were developed in response to philosophical criticisms of first-order logic. Intuitionistic logic, for example, rejects the law of the excluded middle. Modal logic has a more sophisticated truth definition in which formulas are not simply globally true or false, their truth depends on your point of view. Recently there has been some interest in paraconsistent logics — logics where contradictions are permitted but where inference is weakened so that it is not possible to deduce an arbitrary formula from a contradiction. The problem of dealing with uncertainty led to so-called fuzzy logic, in which formulas are not just true or false but are assigned values between 1 (true) and 0 (false). Epistemic logic attempts to model belief and knowledge, so you can write things like "A believes that B knows the answer".

3 Dialectics

I do not propose to provide a detailed exposition of the Hegelian and Marxist theories of dialectics (see [Rees1998] for an excellent account). A great advantage of dialectics, as a philosophical framework, is its ability to explain why the world is in a state of flux. It contrasts with other world-views which either deny that change occurs at all (e.g. feudal Christianity with its emphasis on the permanence and stability of nature and society) or those which acknowledge change but argue that it is brought about by external forces (as with many mystical explanations). The Hegelian dialectic attempts to grasp the totality of the system and argues that change occurs as a result of contradictions internal to that system. "Contradiction is the root of all movement and vitality; it is only in so far as something has a contradiction within it that it moves, has an urge and activity." [Hegel1969, 439].

A second achievement of dialectics, at least in its materialist form, is to solve that central problem in philosophy — the relation between thinking and being. Marx took the Hegelian dialectic and placed it on a materialist base. So Marx's view of society was one in which economic contradictions are more fundamental than ideological ones.

To understand the significance of this materialist dialectic we should first

consider the opposing schools of idealism and materialism in philosophy. Many of the enlightenment thinkers saw a strict division between the mental and the physical worlds. For example, Kant argued that we could have no true knowledge of the 'thing-in-itself' as this remained hidden behind the veil of sensory appearances. Famously, Descartes' view was of two separate worlds, the world of ideas and the world of things, which had hardly any interaction with each other at all. Hegel broke from this *Cartesian dualism*, describing the world as a dialectic unity. But for him, history was still the history of ideas, reality was secondary. He wrote, "Once the realm of ideas is revolutionized, actuality does not hold out." (quoted in [Avineri1972]).

Against that, there is a kind of crude, mechanical materialism where our ideas are seen as passive reflections of our environment. This philosophy is there in part in the works of Hobbes and Locke. More explicitly, Helvetius wrote "All our thoughts and will must be the immediate effect or necessary consequence of impressions we have received." (quoted in [Hampson1968, page 126]). Feuerbach is said to have taken materialism to an extreme by arguing "What you eat is what you are"!

Marx applied the dialectic to provide a far richer solution to this problem. He described consciousness and reality as a unity of opposites in which the material is fundamental. Consciousness depends on the physical world and has no independent existence — "But life involves before everything else eating and drinking, a habitation, clothing and many other things." [Marx & Engels1977]. But Marx also argued that humans have the ability to consciously alter their own environment. Thus our ideas have the capacity to bring about a change in the world and in the process we change ourselves. In one of his most powerful and well known extracts, Marx wrote "Men make their own history but they do not make it just as they please; they do not make it under circumstances chosen by themselves but under circumstances directly encountered, given and transmitted from the past."[Marx1954]. History is not imposed on us from outside, it is made up of our own choices and activities. But our objective situation, which is the outcome of previous history, imposes a structure on our choices and activities. "The tradition of all dead generations weighs like a nightmare on the brain of the living."

4 Criticism of dialectics as a logic

The first problem with dialectics, from the point of view of formal logic, is to do with the concept of a contradiction. As we saw, contradiction is fundamental to the Hegelian view: "Contradiction is the very moving principle of the world" [Hegel1830]. The notion of contradictions in reality was emphasised by Marx and Engels, indeed motion itself is taken to be a contradiction:

> "...even simple mechanical change of place can only come about through a body at one and the same moment of time being both in one place and in another place, being in one and the same place and also not in it." [Engels1984, page 137].

But this violates the laws of identity and non-contradiction in classical logic. Hence, this kind of reasoning cannot be a generalisation of formal logic: it is inconsistent with it. In part I believe the argument cited above is just plain wrong: a moving object is at different places at different times, not different places at the same time.[4] But also this problem is to do with the meaning of 'contradiction'. In Hegel and Marx, a contradiction is more or less the same as a negation or an opposition. But in logic, and in common usage, contradiction refers to an absurdity or impossibility.

As an illustration of these different meanings of the word, let's first consider the way contradictions are used in mathematics. There is a well-known mathematical proof that the square root of two is not a fraction using a method of deduction called 'proof by contradiction' or '*reductio ad absurdum*'. Starting from the assumption that the square root of two is a fraction $\frac{p}{q}$ it is possible to deduce a contradiction[5]. In mathematics at least, we do not conclude that $\sqrt{2}$ is some sort of dialectic or contradictory fraction. No, instead we argue that a contradiction is impossible and so we reject our original assumption that $\sqrt{2}$ is a fraction.

[4]Since the time of Zeno, the concept of motion has led to paradoxes, at least apparently. But in the 19^{th} century the problem was solved by mathematicians like Cauchy and Weierstrass who provided a rigorous framework (now called *real analysis*) for the concepts of calculus. In analysis it is sometimes possible to define motion of an object at an instant t by considering the position of the object at other instants t' in a *neighbourhood* of t, and by considering the *limit* of the gradient of the line from t to t'. I would suggest that Marx and Engels must have been unaware of this work which had only recently been published.

[5]The argument is not too complex. Suppose instead, just so that we can deduce a contradiction, that the square root of two is a fraction $\frac{p}{q}$ where p and q are whole numbers. Because of the rule of cancelling we can assume that p and q have no common factor, which

Well, that is mathematics, a very formal subject. But let us consider a political example. There are people around who claim that the US military, whatever it's motives were in the past, is now an organisation commited to humanitarianism and against terrorism. They argue that the US/UK bombing of Afghanistan in 2001 and Iraq in 2003 is explained by these new motives. This argument can be refuted by deriving a contradiction. Bombing a country (Afghanistan) with over seven million dependent on food aid is a serious hindrance to the aid agencies. Blocking access for the aid organisations in a devastated country (Iraq) also contradicts the supposed humanitarian motive. The opposition to terrorism is contradicted by the support provided in the 1980s to the Al Quaida organisation and by the military support given now to Israel. The anti-terrorist motive is also contradicted by acts of terrorism conducted by the US occupying forces in Iraq, for example at Falluja in April 2003. We should not conclude from this that on the one hand the US military is anti-terrorist and pro-humanitarian (because its spokespersons tell us so) and on the other hand it is not (by the above), nor that it 'depends on your point of view'. Neither do we want to allow left wing supporters of the war a way of avoiding this inconsistency by some sophisticated dialectic reasoning. No, we must be absolutely clear: the assumption of a humanitarian and anti-terrorist motive is contradicted by the facts and must therefore be rejected. Proponents of dialectic logic might have difficulty in categorically rejecting these notions, since they accept contradictions.

It should be apparent from this that we have two meanings of a contradiction (see [Rosenthal1998, chapter 8] for a useful discussion of the conflation of these two meanings in Hegel). In mathematics and formal logic, and in common usage, a contradiction is impossible. But in dialectics, contradictions exist all over the place. There are contradictions, or oppositions, that exist and motivate change and there are other contradictions that are really impossible. In order to be clear we must separate these two meanings. This could be done, I suppose, by having two different phrases: an *absurd*

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implies that they are not both even. Then

$$\sqrt{2} = \frac{p}{q} \;\Rightarrow\; 2 = \frac{p^2}{q^2} \;\Rightarrow\; 2 \times q^2 = p^2$$

and hence p must be even. But this implies that p^2 is a multiple of four which implies that q is also even. This contradicts our assumption that $\sqrt{2}$ could be written as $\frac{p}{q}$ in lowest form.

or *logical contradiction* and a *dialectic contradiction*[6]. But it seems easier to reserve the word 'contradiction' for the former meaning and use alternative words and phrases like 'opposition' or 'conflict' for the latter[7].

The second objection to dialectics from formal logic, is that dialectics is often presented as a kind of logic. There are several different ways in which the Hegelian dialectic has been proposed as a logic. *Systematic dialectics*[8] considers patterns of conceptual development where we start from the universal abstract concept and move to a more concrete category (particularisation) driven by contradictions in the abstract concept. From the opposition between the universal abstract and the particularisation, Hegel argues, a more concrete characterisation of the universal is obtained as a synthesis (see, for example, [Reuten1993, pp 90–93])[9]. An illustration of Hegelian conceptual development is given in [Smith1993], where Smith contrasts formal logic to the way Hegel, in the *Philosophy of Right*, deduces one category from another: Hegel argues that the category *property* gives rise to the category *contract* which in turn leads to *crime*. For an individual within society it does not necessarily follow that property implies contract or that contract implies crime, but for the whole of society there is a necessary tendency along these lines.

Historical dialectics identifies dialectics with the idea of the "essentially historical character of social formations, and so (in its 'rational form') with the principle of the nonexistence of transhistorical laws of social reality." [Mattick1993, p 117]. Accordingly, dialectic logic cannot be a formal ax-

[6]Indeed, in [Engels1984] Engels appears to go some way towards making this distinction by his acceptance of different degrees of contradiction: "The idea of an infinite series which has been counted, in other words, the world-encompassing Dühringian law of definite number, is therefore a *contradictio in adjecto*, contains within itself a contradiction, and in fact an *absurd* contradiction."

[7]Interestingly, Rees follows Marx and Engels in arguing that the dialectic law of motion expresses itself in different ways in the mental world and the external, natural world, but still insists that in substance the laws are the same. Yet he does not directly address the question of whether real, logical contradictions can exist in reality [Rees1998, pp 74–78].

[8]The systematic-historical dichotomy for dialectics is criticised in [Kliman1995] in his very concise review of [Mosely1993].

[9]Reuten treats contradiction in a weaker sense than that just outlined above. For him "opposed concepts are applied to the same thing or notion, and in this specific sense these opposites are contradictions." A contradiction of this sort is not cause for concern to formal logicians. After all, a logical theory can easily be inconsistent (self-contradictory). It is the claim that contradictions exist in reality (not in theory) that is hard to accept.

iomatic logic. Sekine argued that dialectics is

> not a strictly formal (abstract-general) logic but rather a formal-substantive (concrete-synthetic) one. In other words it constitutes a teleological rather than a tautological system.... The result of a dialectics investigation must, in other words, stand on its own without depending on any axiom or postulate. [Sekine1998]

Another approach to dialectics uses the terminology of the Aristotelean syllogism but, rather confusingly, has the middle term of a syllogism also representing the totality of that syllogism [Smith1993, pp 28–31]. Elsewhere, it is the contradiction between essence and appearance that is emphasised in the dialectic approach.

But it seems to me that each of these provides a conceptual framework or language to describe phenomena, their interactions and the way they transform. What is not given, though, is a deduction method — a way of determining when a particular conceptual development is justified. As Mattick put it,

> "Even in the best cases [of sequences of concepts in Hegel's *Logic* RH], it must be said, the *necessity*, as opposed to the plausibility or illuminating character, of the transition between categories in the Hegelian dialectic — and hence of its being a *logic* — has not been convincingly made." [Mattick1993, p 125].

At any rate, the notion of a dialectical deduction strikes me as very problematic. Examples of such deductions seem rather shaky. For example, in 1801 Hegel presented a thesis [Hegel1987] in which he showed without need of empirical observation, on the basis of logic alone, that there could not be other than seven planets and in particular that there could be no planet between Mars and Jupiter. Of course the reader knows already that just such a planet was discovered, the minor planet Ceres, before Hegel's work had hit the presses. Another piece of dialectic reasoning is Hegel's deduction of the existence of the monarch [Hegel1996].

Certainly a formal logic which comprised a language and semantics but included no deduction method (or, worse, a logic with a faulty deduction method) would not be taken seriously. The use of dialectics as a form of logic is most unreliable and the available 'dialectic deductions' are not convincing. Furthermore, if we are to allow contradictions to exist in reality, we leave the

door open to relativism — the idea that there is not a single reality whose truth we try to discover, but many. Such conclusions are at variance with our materialist concept of the world. Therefore we should reject the notion that dialectics is a form of logic.

5 The dialectic criticism of formal logic

Now let's reverse the argument and list some of ways that dialectitions have found formal logic to be lacking. The criticisms listed are widely circulated amongst dialectitions, for example the first four are given in [Novack1973, lecture III]. Lets start with the easy ones.

Triviality The first objection is that the theorems of classical logic are no more than definitional extensions of the axiom system. They are therefore empty tautologies which add no new information to that already given by the axioms. However, the computation of the theorems that follow from a set of axioms is far from trivial. A result of Turing can be used to show that there is no algorithm that can tell whether a given statement follows from a set of axioms or not: this problem is undecidable. Thus a first-order proof system can yield non-trivial results.

Determinism Secondly, formal logic is determinist and incapable of handling uncertainty and choice. But there is nothing to stop us expressing choice in first-order logic by using disjunctions. The formula 'p *or* q' denotes that either p is true or q is true (or possibly both). We can even express infinite choice by using a quantifier — the formula 'there exists an x satisfying the predicate P', means that at least one value of x has the property P, though the formula does not specify which value of x to choose. Furthermore, we can incorporate probabilities into first-order logic or adopt a logic like fuzzy logic to handle uncertainty[10].

Static The next objection is that formal logic has no way of dealing with transformation and change. In formal logic, if a predicate P is true of an object x then it will always be true. However, it is not hard to use first-order logic to express change in time by using an extra time

[10]In automata theory, which is closely related to formal logic, there has been a great deal of interest in non-deterministic automata. These are (theoretical) machines that have the capability of making choices. This work is important in complexity theory.

parameter. Instead of saying that 'x has the property P' we say that 'x has the property P at time t'. Here the predicate P has become a binary predicate, relating x to t. In this set up it is perfectly possible for x to have the property P at some time t but not at another time t'.[11]

Another formalism, though not really a logic, that deals with change is the *calculus* of Newton and Leibnitz. This highly successful subject deals with rates of change at instants of time by calculating the gradient of the tangent to a curve. It is certainly possible to express the fact that a quantity $f(t)$ is changing at time t, indeed we can quantify the rate of change of f using the derivative f'.

Thus, at least on the face of it, it is possible to handle non-determinism and change in a formal system.

Non-contradiction and excluded middle Classically, there are only two truth values: every assertion is either true or false and never both. This is the law of excluded middle plus the law of non-contradiction. But in the real world we typically do not find things to be so clear cut. Now there are logics which do not insist on just two truth values (e.g. intuitionistic logic and fuzzy logic), but modelling transitions between, say, life and death, or whatever, in a realistic way would certainly be challenging for any formal system.

Events and Processes In dealing with properties that change over time, it is generally the case that formal logic uses static properties and instantaneous events. This is true of all the formalisms mentioned above, except perhaps fuzzy logic, and of the applications of logic to artificial intelligence and planning. A property p will remain true until at some definite time an event takes place terminating p, thereafter p will be false. So the event of 'waking-up' will commence a period when a person is awake and some hours later the event of 'falling asleep' terminates that period. But when we look more closely, we see that these events are not instantaneous, but more or less protracted processes. Being awake is a process that is initiated not by an instantaneous and indivisible event but by the process of waking-up. Modelling this kind of behaviour in formal logic is certainly problematic.

[11] *Temporal logic* provides an alternative, more modern approach to reasoning about time.

Logical Atomism A more profound criticism of formal logic is that it leads to the view that the world is made up of indivisible objects and elementary properties. In this view, we start from basic entities and then apply elementary predicates to them and build up from these to more complex properties. Certainly in predicate logic the names and variables stand for individuals that have no internal structure and the atomic formulas also cannot be broken down any further. The same holds for all the other formal logics mentioned above. And yet, although it is very useful to use names and predicates in this way, in reality we find that each individual is a 'unity of opposites' containing different parts interacting with each other. Predicates also are not elementary. Consider, for example, the property of 'being alive'. The predicate 'alive' actually describes a very complex property which must be analysed further in order to understand it.

Furthermore, concepts such as 'value' or 'money' are not obtained as an aggregate of individual quantums of value or money, but result from a whole system based on the exchange of commodities. You cannot start from individual coins and notes and from there build up to the concept of money. It is necessary, as in the Hegelian method, to start from the abstract concept and go from there to the particular. Logical atomism is a criticism of formal logic which carries considerable weight.

Reductionism Is it possible to formalise in logic the entire process of rational thought? Within logic there has been a school that gave an affirmative answer to the question. The *logicist* project, which followed a suggestion of Leibnitz and was promoted by Frege then Russell and Whitehead, was an attempt to place all of mathematics, and perhaps all of science, on logical foundations. Frege wrote

> The firmest method of proof is obviously the purely logical one, which, disregarding the particular characteristics of things, is based solely upon the laws on which all knowledge rests [Frege1972, preface, p 103].

He then continues, in the same article, to attempt to demonstrate that arithmetic and probably geometry, differential and integral calculus can be handled by this very rigorous method of deduction. To quote Frege again, 'arithmetic is a branch of logic and need not borrow any

ground of proof whatever from experience or intuition.' This logicist project can be seen as a kind of reductionism in which all knowledge is ultimately reduced to simple logical foundations.

Yet it seems implausible that the whole complexity of nature can be determined by an absolute and unchanging formal logic. The same problem occurs with the laws of thought. If it were really possible for us to discover a logical foundation for the whole of science we would be led to a very strong form of determinism. Not only is the future determined by the past but it is possible, at least in principle, for humans to calculate the future from the past.

It should be added that within formal logic the logicist project received a death blow from Gödel's incompleteness theorem which showed, roughly, that a formal logic must be incapable of proving all the true statements of arithmetic[12]. Thus formal logic cannot always discover the truth of statements even in such a formal field as arithmetic.

Form without content Logic, as I have described it, studies the form of an argument separate from its content. Reuten argues that a general framework for analysing the form of an argument separate to its content is wrong. So, a dialectic argument

> "should not be grounded merely abstractly (i.e. giving the arguments in advance), because this always leads to regress. That which is posited must be ultimately grounded in the argument itself, in concretizing it." [Reuten1993, p 92]

This contrasts with formal logic where the study of the form of an argument *is* studied separately from its content. By insisting that a proposition must be concretely grounded, it seems to me that we move to nonlogical considerations. Similarly, Marx contends that without content, logic can tell us nothing about specific problem domains or specific historical epochs. "... every historical period has laws of its own" [Marx1974, p 28].

[12] More accurately, Gödel proved that in any consistent, recursively enumerable, formal logic, sufficient for arithmetic (i.e. it contains symbols for $0, 1, +, \times$ etc. and suitable axioms for these operations) there would be true statements for which there exist no proof.

This is accepted, but it seems to me that there is still some merit in studying the form of argument (e.g. logical consistency, deducability, etc.), so long as we remember that this is but one aspect of a given scientific investigation. Also, the content of an argument is not quite so separate from logic as just indicated. The choice of axioms in a logical system can represent content specific information. So, recently, Hungarian logicians have been able to develop a logical approach to relativity theory and shed light on that subject by selecting suitable axioms and analysing logical properties of the axioms — consequences of the axioms, independence, etc. [Andréka, Madarász, & Németi1999, Andréka, Madarász, & Németi]. Furthermore model theory, mostly using ordinary first-order logic, is a subject devoted to the interaction between syntax and semantics in logic. Hard problems from other fields, unsolved within their own discipline, have been solved using model-theoretic techniques, for example the *Mordell-Lang conjecture* of algebraic geometry was solved using advanced model theory [Hrushovski1996].

There do appear to be inadequacies in formal logic. I do not claim that formal reasoning cannot be improved to take some of this into account, indeed I believe this could be fruitful research. But it seems most improbable that a formal system (or, for that matter, a dialectical system) could be devised that once-and-for-all captured all the laws of knowledge and development. Such a formal system might well run into paradoxes if it were capable of describing accurately the development of formal logic leading up to itself — the final, comprehensive, formal system. For then it would be possible to write down 'liar sentences' in this language of the form "this sentence is false". As is well known, it is impossible to assign a truth-value to such sentences. At any rate, leaving aside the problem of paradoxes, the existence of a comprehensive formal system acting as the foundation of all knowledge would certainly be a refutation of the dialectic framework in which fundamental transformations leading to quite new processes and laws not previously evident is considered to be typical.

6 A dialectic conception of truth

In the section on formal logic I concentrated mostly on *syntax* — what formulas can you write, what is a proof, etc. etc. The other part of logic is to do with *semantics* — what do the formulas mean, is this formula true or valid

or at least possible, and so on. The study of semantics in logic is part of a wider philosophical problem of defining truth in language. We can separate out two distinct issues: the *definition* of truth, i.e. what do we mean when we say that a statement is true; and the question of how we *discover* the truth. This is an area where dialectics can be particularly illuminating.

Although these problems can seem very abstract and philosophical, they are in fact highly practical. Socialists attempt to study history in order to intervene in our own society and change it for the better. But after we make our intervention, how are we to assess whether it worked? Also, we constantly have to put forward theories that explain the world better than the theories of our opponents in order to combat the political conclusions that they would like to lead us towards. Again, how can we demonstrate the truth of our theories? And yet without a method of testing the truth of our ideas we lose all sense of direction. If we cannot correctly assess our interventions then we have no chance of learning from our mistakes. So we must take this question seriously.

In formal logic and more generally in philosophy there are two key approaches: *correspondence* theories and *coherence* theories. There are many other theories of truth, e.g. the *pragmatic* theory (with some similarities to the Marxist theory), the *redundancy* theory and Tarski's *semantic* theory (see [Haack1978] for an account) and (apart from the redundancy theory) these involve elements of correspondence and coherence.

According to the correspondence theory the formula "Anne is the sister of John" is *true* if the names 'Anne' and 'John' refer to real individuals and the former is the sister of the latter. Correspondence theorists attempt to find a structural isomorphism from a formula into the world, i.e. a mapping from names to objects such that all the predicates are preserved[13]. The strength of the correspondence theory is that it acknowledges an external

[13]An important school that developed broadly within correspondence theory was the school of *logical positivism*. Taking correspondence as the definition of truth, positivists argued that we can learn the truth of a proposition only by verifying it by observation. Logical positivists draw a distinction between *symthetic truth* of the world verified by observation and *analytic truth* which applies to logical and mathematical statements. The latter is regarded to be true by definition, or by virtue of the way language is used. This distinction between analytic and synthetic truth came under attack from Russell and Wittgenstein, who had previously advocated this argument.

Popper's theory of falsification developed out of this school [Popper1963]. Accordingly, assertions are never proved true but can only be accepted as consistent so long as no evidence is found to contradict the assertion. Thus the statement "the speed of light is

world independent of our thoughts and judges our own theories by how well they correspond to that external world. Indeed, any materialist concept of truth must include some kind of correspondence as a definition of truth. But there is a tendency[14] with this definition of truth to lead to a type of *logical atomism* in which the real properties of the world are built up from elementary entities and properties like 'Anne' and 'sister', whereas this assumption is certainly questionable.

And for an account of how we can discover the truth of our ideas, a pure correspondence theory is inadequate. One problem is that although we have direct knowledge of our own ideas and theories we have no *direct* knowledge of reality, only knowledge mediated through experience. It is therefore problematic to establish this correspondence, even for elementary statements. Correspondence theories reflect a kind of Cartesian dualism because you have reality on the one hand and ideas on the other and you can say that the ideas are true if they correspond, but it is hard to see how this correspondence can be demonstrated.

Coherence theories judge the truth of a statement by its relationship to other beliefs, in particular they ask whether a given statement is consistent with a large set of beliefs. One worry with this, however, is that many false beliefs have been widely held by previous societies. Coherence theories have a tendency towards idealism and subjectivism.

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constant" can be tentatively accepted until someone performs an experiment to refute it. A problem with this falsification method is that existential statements like "black holes exist" cannot be falsified (if you fail to see a black hole it doesn't prove that they don't exist), only confirmed. More importantly, the falsification method is a gross over-simplification of the way we discover truth.

Popper used his falsification method to refute Marxism[Popper1945] and so, in the context of the cold war, his theory was given some prominence. According to Popper, Marxism was refuted by the experience of the Soviet Union — a Marxist state in which Marx's predictions failed to occur. But Popper failed to apply even his own simple falsification method correctly. Marxism is distinguished from other socialist theories because it is materialist. The possibilities open to any society are constrained by the economic base. The material conditions required for a socialist society are (i) there must be enough of the basic necessities for everyone and (ii) the majority must belong to a collective class, the *proletariat*. Thus Marxism predicted that a workers' state in Russia, isolated from more advanced economies, would be incapable of developing a socialist economy. This was the clear understanding of all the leading Bolsheviks, before the revolution. Thus the Russian experience is in no way a falsification of Marxism.

[14] Particularly with versions of correspondence theory promoted by Russell and Wittgenstein round about 1920.

Let me outline how a materialist, dialectic approach could set about analysing the problem of truth. I believe that any materialist must accept that there is a single reality and therefore the definition of truth must be some kind of correspondence definition. I depart from the pure correspondence theorists in a number of ways. First, although it is acknowledged that there is but a single reality in the world, I do not accept that it is entirely independent of our thoughts. Particularly when we study our own thought processes and the workings of our own society it becomes clear that what we think is tied up with what actually happens. Secondly, correspondence as a definition of truth faces the problem that our ideas can never exactly correspond to reality. The thought process involves abstracting from reality, the use of words and symbols, and many other simplifications. Our thoughts and ideas are of a different quality to the things they refer to. We therefore should not expect any of our theories to correspond absolutely and exactly to reality. A more sophisticated notion of correspondence taking these considerations into account is therefore needed for a definition of truth. Finally, we are not content with a mere definition of truth, we seek a method of establishing the truth and proving its correctness. This takes us beyond the limits of correspondence theory.

The way we prove the truth of our assertions involves a number of different methods. In fact we are generally interested in the truth of whole theories, not just individual assertions. We can test such a theory using logical consistency, because a theory that contradicts itself cannot be true to reality. And we can test a theory by seeing how it works in practice. To quote Marx,

> "The question whether objective truth can be attributed to human thinking is not a question of theory but a practical question. Man must prove the truth – i.e. the reality and power, the this-worldliness of his thinking in practice. The dispute over the reality and non-reality of thinking which is isolated from practice is a purely scholastic question .... All social life is essentially practical. All mysteries which lead theory into mysticism find their rational solution in human practice and in the comprehension of this practice. [Marx1973]"

To show that our theories correspond to reality we must prove them in practice. Our ideas, which arise from experience, lead us to act in certain ways in order to achieve our goals. If we are scientific, we then compare our plans to

the actual results of our actions and modify our theories if necessary. This means that none of our ideas express an absolute truth in the world. The best that can be hoped for is that we demonstrate that a certain theory correctly expresses the behaviour of some phenomenon when tested in a certain way. Thus the truth we establish is contingent on the circumstances.

This approach to the problem of truth differs from both the correspondence and the coherence theories in that it makes the process of finding the truth an active process in which we simultaneously investigate the world and attempt to change it. To summarise: in the dialectic conception of truth we recognize that the world is neither independent (as with correspondence theories) nor determined (as with coherence theories) by our own thoughts. It is the interaction between our own thoughts and actions, those of countless other individuals and other properties of the world, i.e. it is the way our own activity fits into the wider class struggle that determines history.

7 How logic changes over time

Because of the complexity of nature and the fact that completely new processes and behaviour comes into existence, the dialectic scientist does not accept that *any* logical formalisation can fully capture the thought process or the laws of nature. There is a requirement to establish a correspondence between theory and practice and to adjust the theory when the correspondence fails.

There are situations in which the inadequacies of our formal logic become acute because of new circumstances and new ways of reasoning. In such situations there is a possibility of creating new logics and new formalisms. We saw that the Aristotelean syllogism arose in just such a way, the previous approach to mathematics which relied on intuition became untenable. And Aristotle's system was restricted in its application, dealing with permanent properties of objects taken in isolation from each other. But as the productivity of society developed and at the same time science advanced, this form of logic became inadequate. This provided an impulse to divize new logics. Modern logic is far in advance of the logic of Aristotle and has overcome some of the limitations. But of course any formal system, indeed any human theory, cannot capture the entire complexity of the universe.

Or in mathematics, which comprises a range of formalisms, a similar story can be told. Up to the 17^{th} century, mathematics dealt only with static, dis-

crete quantities (as well as geometry, which was not related to arithmetic or algebra until the time of Descartes). But changes in technology and science meant that it was necessary to handle the notion of instantaneous change. The language of mathematics was quite incapable of doing this. Yet mathematicians were able to revolutionize the subject by inventing a whole new language of mathematics which was capable of expressing instantaneous change: that language was the calculus. This revolution was of course problematic, it took 200 years to come up with a satisfactory explanation which could make sense of it. The point is that just as humans have the ability to invent new tools to deal with new problems, so we are able to create new formalisms to model the reasoning process.

Indeed this type of development in formal logic and mathematics is typical of developments throughout science. Engels explains how in its early phase all of the sciences present a very static view of the world. First of all the world is seen as being literally fixed and unmoving; the Ptolomeic system is a system of eternal repetition; the geology of the earth's surface is unchanging; the species date from the creation; and human nature and human society are also permanent. One by one these assumptions came under attack. First we had the Copernican revolution, then Kant put forward the nebular theory of the evolution of solar systems. This theory implied that the earth itself had evolved and put into question the idea of a fixed geology and thereby questioned the dogma of an unchanging collection of species. Later, these subjects also were revolutionised showing that the surface of the earth and the organisms populating it have undergone a history.[15] Marx's theory of history showed human societies developing in history on the basis of the class struggle. Later, Freud put forward a theory in which the human psychology represented a struggle between different parts.

So formal reasoning is not different in this respect to other sciences, except that it operates at a more abstract level. We are engaged in a process of developing new formalisms to help clarify our reasoning. These logical formalisms are applied and tested in applications in other sciences, in everyday phenomena or in logic and mathematics themselves. As the sciences become more sophisticated and abstract we need to overcome the contradictions internal to our scientific theories and we must demonstrate their rational consistency. For this, the techniques of formal logic and proof are well suited. As we develop new modes of reasoning in science we may find

[15] For a fuller and surprisingly up to date account of this piece of history see [Engels1968].

the logical formalisms to be inadequate. This motivates the development of new logics.

Materialist dialectics is the best philosophical framework within which we can scientifically investigate and discover the properties and laws of our world and how it can be changed. But it is not a substitute for this scientific study. It does not ensure that our investigations will be successful, nor that others who are not thinking dialectically won't come up with advances in the scientific theory. Isaac Newton, for example, held to a Christian philosophy quite distinct from dialectical materialism but was still able to produce some of the greatest advances in science so far. But when science investigates our own thinking and our own society we cannot stand at a distance and look from the outside, and the need for a dialectical, materialist framework becomes more pressing.

Yet dialectics is not magic and will not produce the right answers on its own. Dialectics is not economics: Marx did not write Capital purely on the basis of the dialectic, it was necessary for him to make a detailed study of economic theory and the working of capitalism. Similarly, dialectics is not logic, not in the sense of a system that derives conclusions from premises, and it does not tell us the correct form of logic or the correct way to model rational thought. Formal logic is a tool which can be used to analyse and clarify the reasoning process. For Marxists, who seek to explain how the world works and how it can be changed, a formalisation of the reasoning process can only be helpful. But the material that we use this tool on, human thought, is not immutable. And the tool also must be developed and extended.

8 Conclusion

Strangely enough, logic is a political subject. Starting with the technical side, we have seen that the adoption of predicate logic as the basis of reasoning can lead to logical atomism. This encourages a view of the world in which the key unit is the individual. With one more step we arrive at Margaret Thatcher's conclusion: "there is no such thing as society".

But the other political aspect of logic concerns the function of logic for society. The class that runs our system has no logical explanation about how their system works. When the system is expanding, capitalists fall over themselves in their efforts to expand production even though this inevitably leads to a crisis later on. There is a certain narrow justification for this in

that the capitalist who does this most effectively will be among the ones that survives the crisis, but there is no logic to this for the system overall. When the system goes into crisis, it is not logical for governments and banks to implement austerity measures, to cut spending on welfare and to depress wages when this only exacerbates the tendency to overproduction and crisis. It is not logical to use more resources on means of warfare and destruction than on health and education — at least these policies do not follow from the premises that are usually given.

Of course Capitalism recruits scientists and professors who are experts in formal logic. Yet there is no correspondence between the elaborate theories of logic that exist in the academies and universities and the irrationality of the way the system actually works. To leave these two features separate from each other is to miss the whole point of logic which must surely be to *clarify the process of rational thought in order to help bring about a rational way of doing things.* That is not to say that every aspect of formal logic must have an immediate application to the problems in hand. But if the subject as a whole has no link to significant problems facing society then it risks stagnation and aimlessness.

Marxists, on the other hand, do have a coherent and logical explanation of how the system works, why it is a system in motion and therefore in transition, why it has a tendency towards crises, how crises create opposition and resistance and how in the right circumstances and with the right theories it is possible for us to intervene in the conflicts created by the system in order to change it to a different system based on rational, logical planning. The great strength of Marxism is the clarity of these arguments. It can only weaken our case if there are parts of our argument which are hidden in darkness.

In the current period the system is undergoing profound and rapid changes. The collapse of the Russian empire and the apparent triumph of the free market has not led to a period of peace and prosperity as we were told by the advocates of the system, but to a period of war, instability and economic ruin for large parts of the world. These changes are also creating a realignment on the left. Stalinism, the ideology that held back two generations of socialists, is overthrown. The War on Terrorism is creating a level of opposition to the system not seen for quite some time. All over the world, people will be looking for solutions to the disaster that is Capitalism. Some of these people will have very confused, illogical ideas about how to change things. A prominent theory that will attract many is reformism, the 'one thing at a

time' approach to controlling the system by taking control of the state, bit by bit. The reformist parties are also currently going through a crisis as their ability to deliver reforms is reduced to nil. The reformist argument is quite undialectic. At its heart there is a separation of the economic from the political — let the politicians handle the politics while the trade unionists deal with narrow economic issues and keep a healthy distance between them. The reformists treat the working class as passive beneficiaries of their policies. To improve the attitudes and the lot of workers there is great emphasis on education, but the question of who educates the teachers is left unanswered. Thus change is brought in from outside.

Marxists have a superior explanation in which change results from conflicts internal to the system. We always attempt to overcome the separation of politics and economics — to link the power of workers at the point of production with the ideas and theories needed to overthrow the system. We see the class struggle itself as a far more effective education than that available in the schools and colleges.

In a period of rapid changes we will need to develop our theories and at the same time maintain a dialectic link between theory and practice. If we can ensure that our theory is scientific and logical right to its foundations then our analysis will be more convincing to those we seek to convince and we will also be subject to a logical discipline that will help maintain the correct link between theory and practice and thereby help us intervene in the struggles ahead in a way that makes a decisive difference.

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