Short Accountable Ring Signatures Based on DDH

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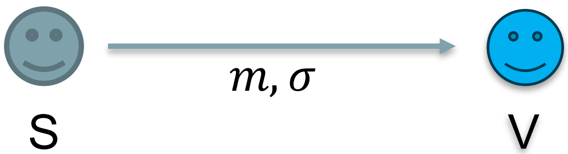
Signature Schemes

Link message to single entity

- Signer
- Verifier



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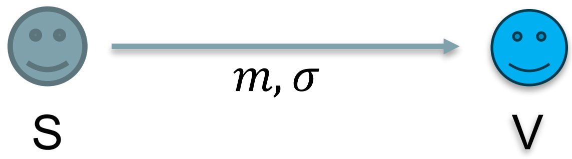

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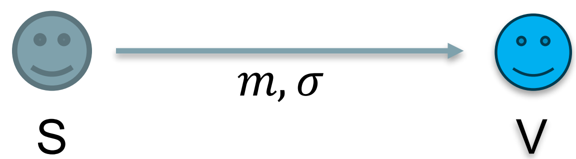

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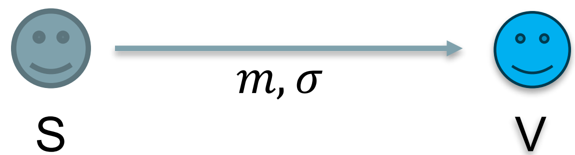

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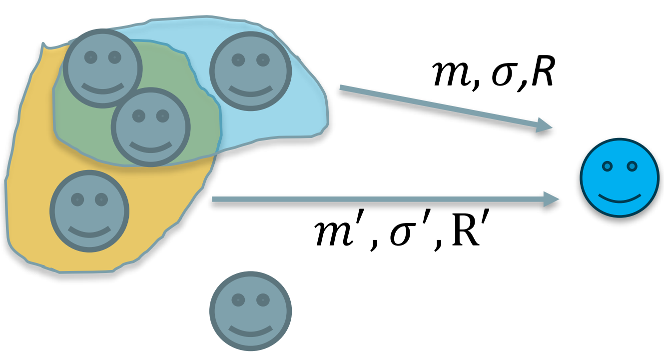

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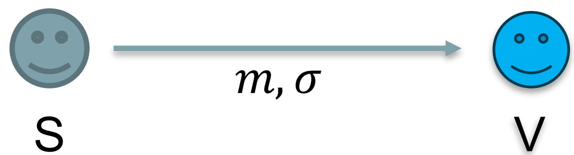

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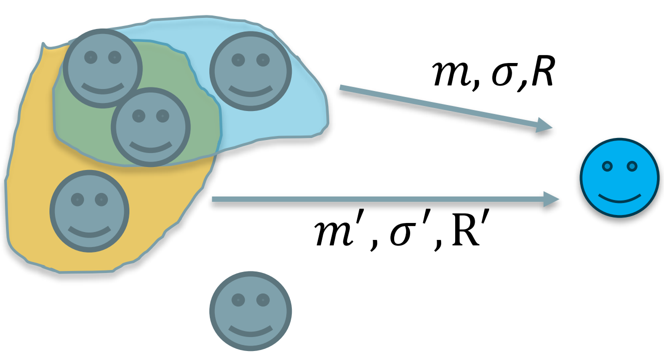

Group Signatures

- Manager
- Users
- Verifier

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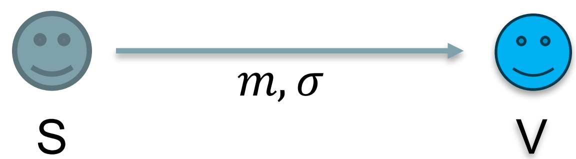

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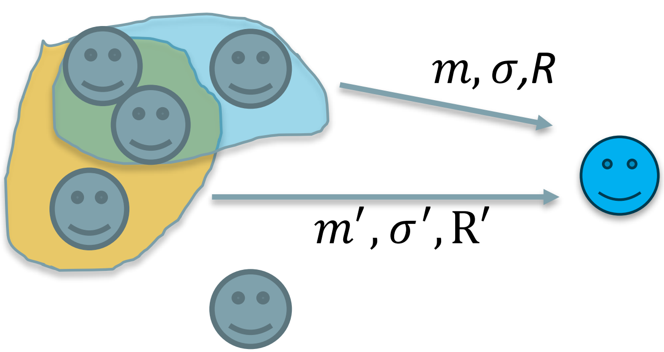

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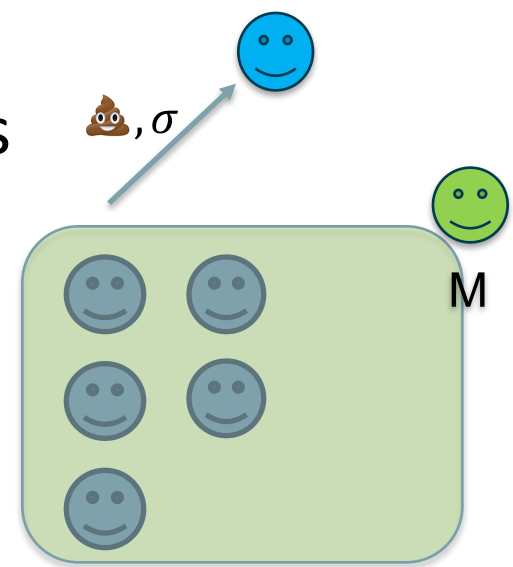

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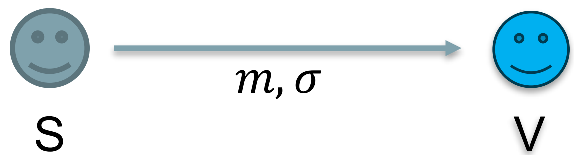

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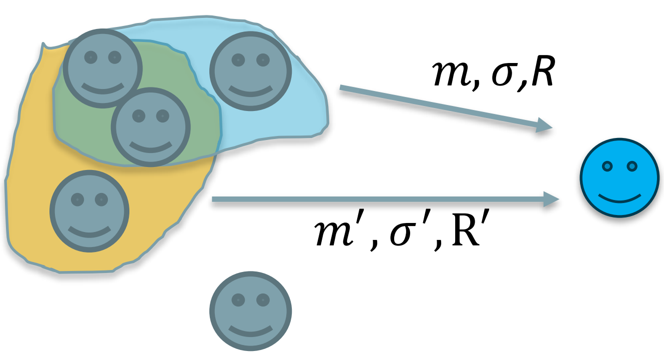

Group Signatures

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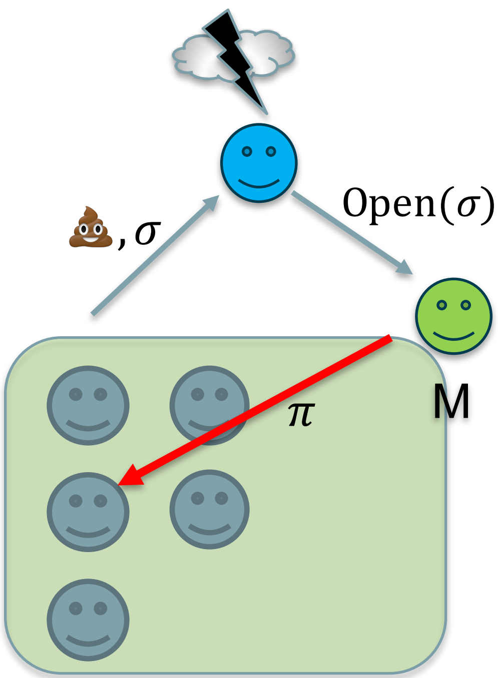
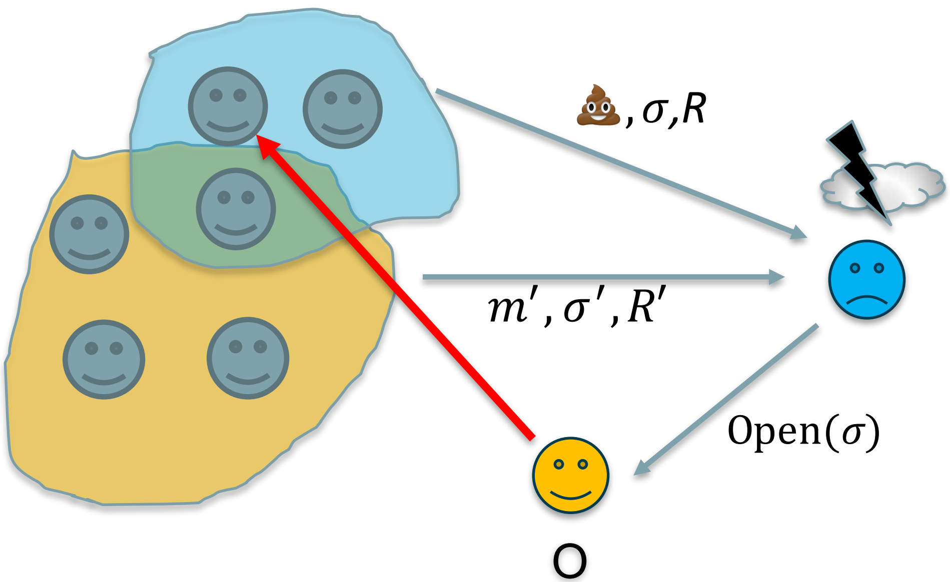


Accountable Ring Signatures [Xu and Yung]

Link message to multiple entities

- Users
- Opener(s)
- Verifier



Accountable Ring Signatures

- Setup, OpenerKeyGen, UserKeyGen
- Sign, Vfy
- Open, Judge

Security:

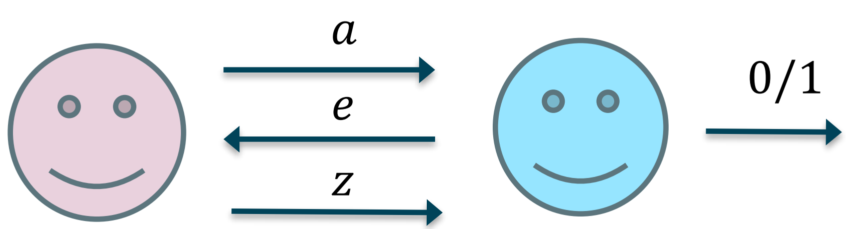
- Correctness
- Full Unforgeability
- Anonymity
- Traceability with Tracing Soundness

Components for Accountable Ring Signatures

- One-way functions (g^x)
- Homomorphic Commitments (Pedersen)
  - $C_{ck}(m_1) \cdot C_{ck}(m_2) = C_{ck}(m_1 \cdot m_2)$
- IND-CPA Encryption (ElGamal)
- Non-Interactive Zero Knowledge Proofs
- Signatures of Knowledge

Σ-Protocols

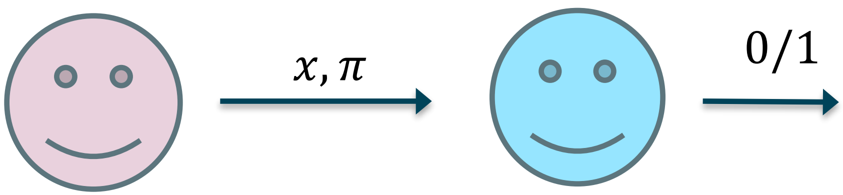
- 3-move protocols for some NP relation R
- Prover demonstrates a statement $x \in L_R$: there exists w s.t. $(x, w) \in R$



- Completeness:  outputs 1 for $x \in L_R$
- n-Special Soundness: n accepting e, z pairs for same x, a: we obtain w
- Special Honest Verifier Zero Knowledge: Transcripts between  and honest  can be efficiently simulated for any challenge e

Non-Interactive Zero Knowledge Proofs

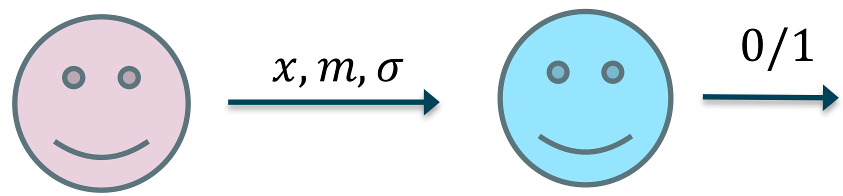
- 1-move protocols for some NP relation R
- Fiat-Shamir: challenge is hash of the transcript



- Completeness:  outputs 1 for $x \in L_R$
- Soundness: If $x \notin L_R$,  almost never outputs 1
- Zero Knowledge: Proofs can be efficiently simulated

Signatures of Knowledge

- 1-move protocols for some NP relation R, given common reference string crs
- Prover demonstrates, w.r.t. message m, *knowledge* of w for statement $x \in L_R$: $(x, w) \in R$



- Extractability: If  produces good signatures, extract w by rewinding
- Straightline f-Extractability: we can extract $f(w)$ without rewinding
- Simulatability: signatures can be efficiently simulated

- Extractor, Simulator is given control of crs creation

Construction

- Setup: Choose discrete log group $\mathcal{G}$, generator g and common reference string crs

- OpenerKeyGen: Create ElGamal keypair, publish pk

- UserKeyGen: Pick secret key sk,
  output verification key $vk = g^{sk}$

Signing

- Choose ring $R = \{vk_0, vk_1, \ldots, vk \ldots, vk_k\}$

- Prove $vk \in R$
- Attach encryption c of vk so opener can trace
- Prove knowledge of $sk = \log(vk)$
- Prove knowledge, correctness of c
- Bind σ to message m via Fiat-Shamir

$$R_{sig} = \left\{ \begin{matrix} (R, c), (sk, r): \\ vk \in R \ \wedge \ vk = g^{sk} \wedge c = E(vk; r) \end{matrix} \right\}$$

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- Choose ring $R = \{vk_0, vk_1, \dots, vk \dots, vk_k\}$
- Prove $vk \in R$

Could prove: $vk = vk_0$ OR $vk = vk_1$ OR … OR $vk = vk_k$

- Linear size: too big for large rings

Use One-out-of-Many proof by Groth and Kohlweiss

- Take $c_i = c/E(vk_i\,; 0)$
- Use modified GK to show one node encrypts 1

GK idea

- We want to open c_l without revealing l
- $c_l = \prod c_i^{\Delta_i}$, where $\Delta_i = 1 \iff i = l$
- Commit to Δ_i. Also commit to blinders a_i
- Given challenge x, reply with $\mathrm{f}_i = \mathrm{x} \cdot \Delta_i + a_i$
- $\prod c_i^{f_i} = c_l^x \cdot \prod c_i^{a_i}$

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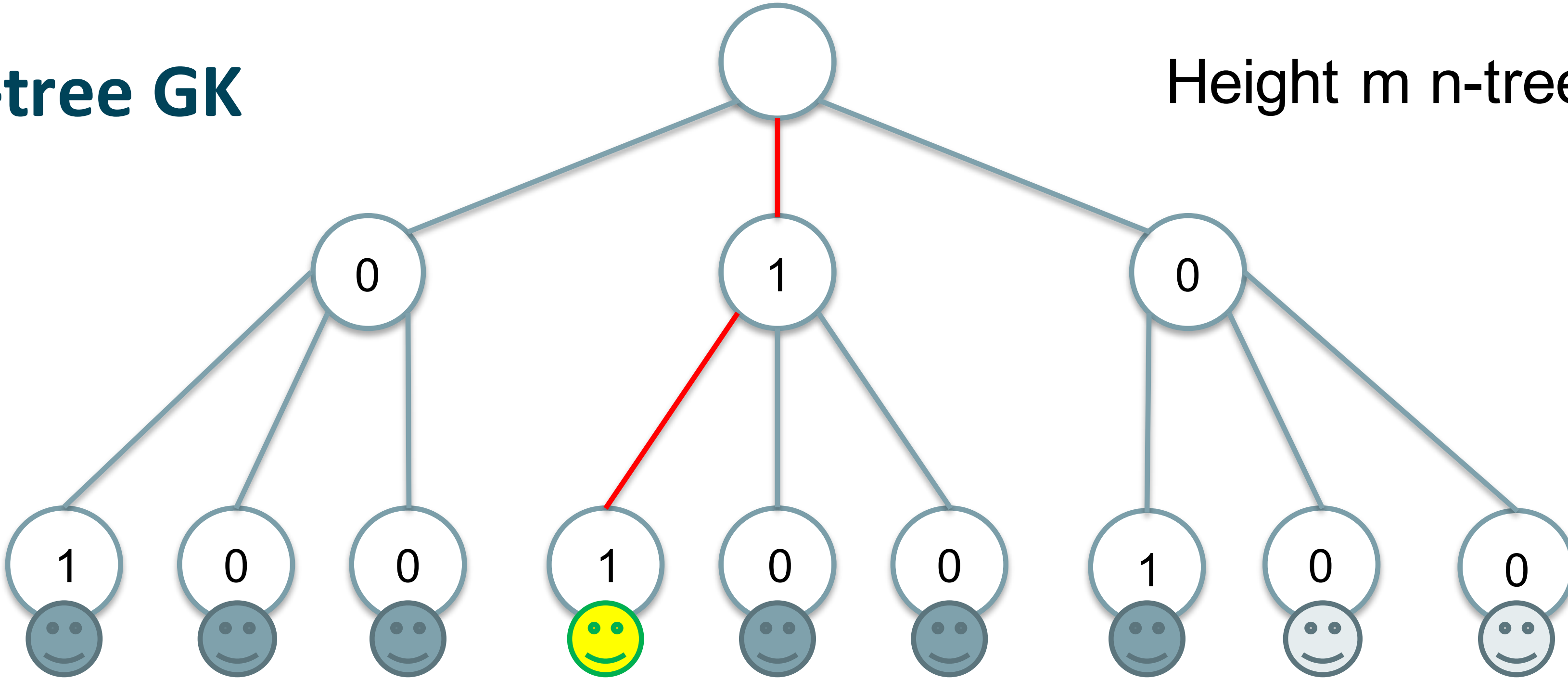
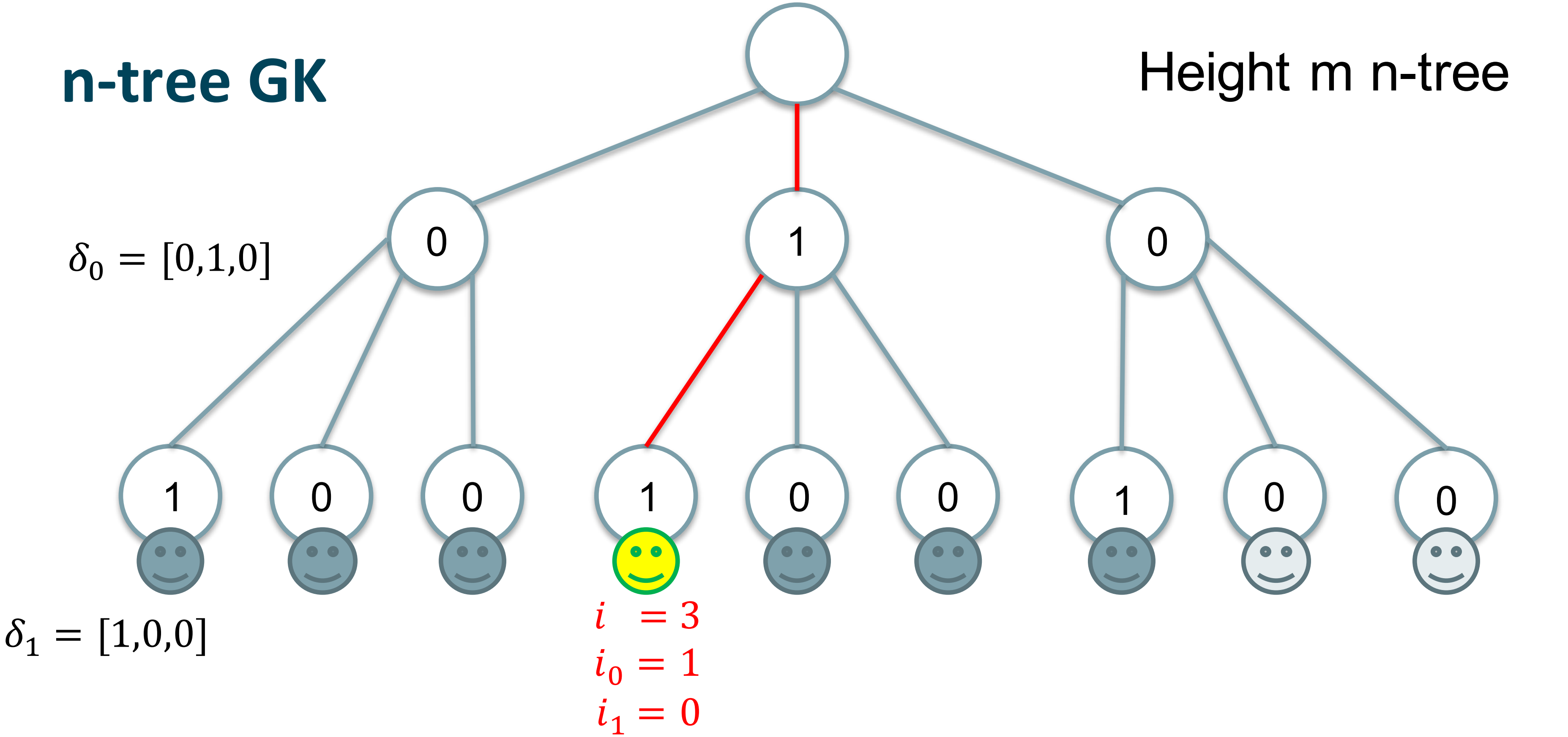
- $G = \prod c_i^{a_i}$ does not depend on x. Rerandomize as G'
- $\prod c_i^{f_i} / G' = c_l^x \cdot E(1; r) = E(1; r')$

n-tree GK



- Split i, Δ_i by level: $i = \sum i_j \cdot n^j$ $\delta i_j, j : \Delta_i = \prod \delta_{i_j,j}$

n-tree GK



- Split i, Δ_i by level: $i = \sum i_j \cdot n^j$ $\delta i_j, j : \Delta_i = \prod \delta_{i_j, j}$
- Commit to $\delta i_j, j$, prove 0/1, for each j exactly one $\delta i_j, j$ is 1

n-tree GK

- Commit to δ_{j,i_j}. Also commit to blinders $a_{i,j}$
- Given challenge x, reply with $\mathrm{f}_{j,i_j} = \mathrm{x} \cdot \delta_{j,i_j} + a_{j,i_j}$
- Let $p_i(x) = \prod f_{j,i_j}$
- Key point: x^m appears only if all δ_{j,i_j} are 1 i.e $i = l$
- $p_i(x) = \Delta_\mathrm{i} x^m + \sum_{k=0}^{m-1} p_{i,k} x^k$ where $p_{i,k}$ depend on l, a_{j,i_j}
- $\prod c_i^{p_i(x)} = c_l \cdot \prod_{k=0}^{m-1} P_k x^k$
- P_k do not depend on x.

n-tree GK

- P_k do not depend on x
- We commit beforehand as G_k
- What is $\prod c_i^{\prod f_{i_j,j}} \prod_{k=0}^{m-1} G_k^{x^{-k}}$?
- If c_l is an encryption of 1, result is encryption of 1
- Otherwise, with overwhelming probability it's an encryption of a value $\neq$ 1, so can't be opened to 1

Opening

- Open
  - Check if σ actually verifies
  - Decrypt ciphertext c attached in signature
  - Prove correctness of decryption in Zero Knowledge
- Judge
  - Check decryption correctness

Simulated Opening & Straightline Extractability

- To prove anonymity, we do an IND-CCA style proof
  - Need to extract vk from sigs
  - Can't see the key
- Adversary can obstruct rewinding
  - Adversary's signatures related to each other
  - Rewinding to open one changes previous $\Rightarrow$ more rewinding
- We need to extract $vk = g^{sk}$ with no rewinding
  - Cheap solution: Attach 2nd encryption of vk to proof [NY]
  - Simulator has 2nd key in simulation
  - Nobody has the key in real world

Efficiency

- $\log N + 12$ Group Elements
- $\frac{3}{2}\log N + 6$ Field Elements
- Competitive vs sRSA/DDH schemes

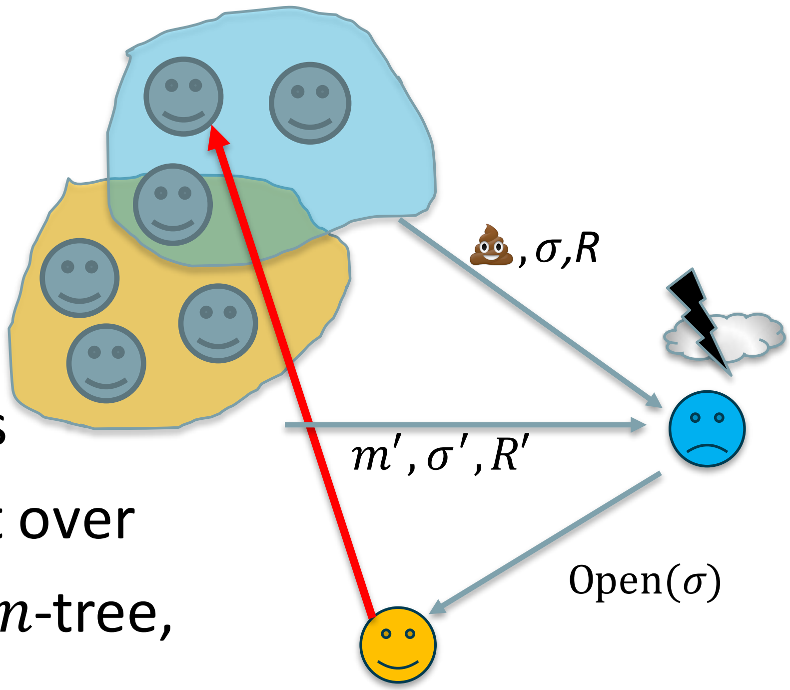
Scheme	$\lvert R \rvert = 128$	$\lvert R \rvert = 1024$	$\lvert R \rvert = 1$**Mi**
[CG05] – 2048 sRSA + d.Log	10 Kib	10 Kib	10 Kib
This – 192 ECC	6.7 Kib	8.1Kib	12.75 Kib
This – 192 ECC	7.8 Kib	9.4 Kib	14.875 Kib

- Linear expos (or worse) to Sign
- Linear expos to Verify

Summary

- Accountable Ring Signatures can be best of both worlds
  - Tracing functionality of Group sigs
  - Free choice of ring
  - Free choice of opener
  - Can derive Ring and Group signatures

- Signature size:
  - Competitive vs sRSA/DDH schemes
  - Better than 50% size improvement over original GK construction: binary → n-tree, mixed Com+Enc



Thanks!