

# Continuous After-the-fact Leakage-Resilient eCK-Secure Key Exchange

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# Presentation Outline

- 1 Key Exchange Security Models
- 2 Leakage Resilience
- 3 Continuous After-the-fact Leakage-eCK Model
- 4 Constructing an CAFL-eCK-secure Protocol
- 5 Summary

# Diffie–Hellman-based one-round protocol

long-term key:  $x_A$

**A**

$$a \leftarrow R$$

$$A = g^a$$

$\xrightarrow{A}$

long-term key:  $x_B$

**B**

$$b \leftarrow R$$

$$B = g^{r_B}$$

$\xleftarrow{B}$

$$K = F(x_A, r_A, B, \text{public info})$$

$$K = \hat{F}(x_B, r_B, A, \text{public info})$$

- $a$  and  $b$  are the *ephemeral keys*
- $K$  is the *session key*
- Prominent concrete protocols include MQV, HMQV and UM

# Key Exchange Security Models

- Designed to capture informal security goals.
- Typical elements in a security model:
  - **Adversary Capabilities** - set of adversary operations. Always allows adversary to view protocol runs and alter/inject messages.
  - **Security Game** - the order in which the adversary operations are performed.
  - **Security Definition** - the requirement to win the security game. Usually require the adversary to reliably distinguish session key from a random string.

# Adversarial Capabilities (eCK model)

- Adversary runs the protocol:
  - Send: Adversary can send a message to a protocol session which answers according to the specification.
- Adversary compromise certain secret keys:
  - 1 SessionKeyReveal: Adversary is given the session key of a session.
  - 2 EphemeralKeyReveal: Adversary is given the ephemeral key of a session.
  - 3 Corrupt: Adversary is given the long-term secrets of a principal.
- Adversary asks for the challenge:
  - Test: Adversary is given either the session key or a random string.

# Security Game

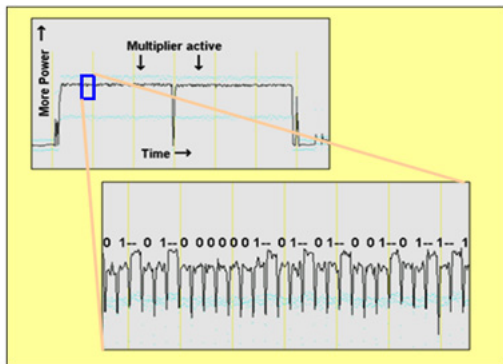
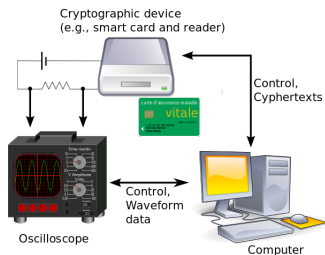
- **Stage 1:** The adversary performs `Send`, `SessionKeyReveal`, `EphemeralKeyReveal` and `Corrupt` operations.
- **Stage 2:** Test operation to any uncompromised test-session.
- **Stage 3:** `Send`, `SessionKeyReveal`, `EphemeralKeyReveal` and `Corrupt` keeping the test-session uncompromised.
- **Stage 4:** Adversary outputs a bit as its guess whether the Test operation output was random (0) or the real session key (1).

The adversary wins the game if it guesses correctly.

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# Side-Channel Attacks

- Leaking information from cryptographic implementations can be used as side-channels to reveal secrets.
- Side-channels: Timing information, Power consumption information, Cache-access pattern, EM-radiation etc.

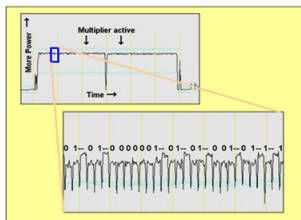
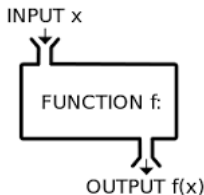


Plots courtesy of Cryptography Research, Inc.



# Leakage-Resilient Cryptography

- Provable security against side-channel attacks.
- Constructing cryptographic schemes in leakage-resilient manner.
  - Model leakage.
  - Prove that even in the presence of certain amount of leakage to an attacker, a cryptographic scheme is secure.
- Adversary gets the leakage of the secret  $x$  using adversary-chosen, adaptive, efficiently computable leakage functions  $f$ .



Plots courtesy of Cryptography Research, Inc.

# Modelling Leakage

Different options have been used.

<b>Leakage function <math>f</math></b>	restricted class of functions (eg hard to invert functions) <i>or</i> arbitrary polynomial time functions
<b>Output amount of the <math>f</math></b>	bounded <i>or</i> continuous leakage
<b>When to apply <math>f</math>?</b>	only before the security challenge <i>or</i> before + after the security challenge (after-the-fact)

Ideally want the leakage function be arbitrary and allow continuous, after the fact leakage.

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# Modelling Leakage in the CAFL-eCK Model

- Follow model of Dziembowski and Faust (2011)
- An arbitrary polynomial time leakage function  $\mathbf{f}$  is used to model the leakage s.t.  $\mathbf{f}(sk) = \textit{leakage}$ .
- Leakage is modelled in a place where computation takes place using long-term secret keys.
- Total leakage amount is unbounded (continuous leakage).
- Allows after-the-fact leakage.

# Adversarial Capabilities of the CAFL-eCK Model

- Adversary run the protocol:
  - $\text{Send}(m, \mathbf{f})$ : Models the capabilities of the adversary who can initiate, delay, modify or insert protocol messages  $m$ . The adversary observes the leakage of the secret key  $\mathbf{f}(sk)$ , whenever a computation takes place in a party.
- Adversary compromise certain secret keys:
  - 1 **SessionKeyReveal**: Adversary is given the session key of a session.
  - 2 **EphemeralKeyReveal**: Adversary is given the ephemeral keys (per-session randomness) of a session.
  - 3 **Corrupt**: Adversary is given the long-term secrets of a principal.
- Adversary asks for the challenge:
  - **Test**: Adversary is given either the real or a random session key.

# Comparison with Earlier Models

Security model	Ephemeral Key	Long-term Key	Combinations	Leakage resilience
eCK (2007)	Yes	Yes	4/4	None
MO (2011)	Yes	Yes	4/4	Bounded, before-the-fact
BAFL-eCK (2014)	Yes	Yes	4/4	Bounded, after-the-fact
CAFL (2014)	Yes	Yes	2/4	Continuous, after-the-fact
CAFL-eCK (now)	Yes	Yes	4/4	Continuous, after-the-fact

- ①  $\text{Corrupt}(U)$  and  $\text{Corrupt}(V)$ .
- ②  $\text{Corrupt}(U)$  and  $\text{EphemeralKeyReveal}(V, U, s)$ .
- ③  $\text{Corrupt}(V)$  and  $\text{EphemeralKeyReveal}(U, V, s')$ .
- ④  $\text{EphemeralKeyReveal}(V, U, s)$  and  $\text{EphemeralKeyReveal}(U, V, s')$ .

## Dziembowski–Faust leakage-resilient storage scheme

For any  $n \in \mathbb{N}$ , the storage scheme (Encode, Decode) efficiently stores an element  $s \in \mathbb{Z}_q^*$  where:

- Encode( $s$ ) :  $s_L \xleftarrow{\$} (\mathbb{Z}_q^*)^n \setminus \{(0^n)\}$ , then  $s_R \leftarrow (\mathbb{Z}_q^*)^n$  such that  $s_L \cdot s_R = s$  and outputs  $(s_L, s_R)$ .
- Decode( $s_L, s_R$ ) : outputs  $s_L \cdot s_R$ .

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- $\text{Decode}(s_L, s_R) : \text{outputs } s_L \cdot s_R$ .

The values  $(s_L, s_R)$  can then be *refreshed* using the following algorithm

- Refreshing  $s_R$ :
  - 1 Choose  $A, B \in (\mathbb{Z}_q^*)^n$  such that  $A \cdot B = 0^m$ .
  - 2 Choose  $M \in (\mathbb{Z}_q^*)^{n \times n}$  such that  $s_L \cdot M = A$ .
  - 3  $s_{R'} = R + M \cdot B$ .
- Refreshing  $s_L$ :
  - 1 Choose  $\tilde{A}, \tilde{B} \in (\mathbb{Z}_q^*)^n$  such that  $\tilde{A} \cdot \tilde{B} = 0^m$ .
  - 2 Choose  $\tilde{M} \in (\mathbb{Z}_q^*)^{n \times n}$  such that  $\tilde{M} \cdot s_{R'} = \tilde{B}$ .
  - 3  $s_{L'} = L + \tilde{A} \cdot \tilde{M}$ .



# Constructing an CAFL-eCK-secure Protocol

- Construct a simple eCK-secure protocol
- Use leakage-resilient storage scheme and its refreshing protocol to convert to a CAFL-eCK-secure protocol.
- If the storage scheme and its refreshing protocol are leakage-resilient, the protocol is CAFL-eCK-secure. Allows 15% continuous leakage of the secret key with  $n = 21$ .

## An eCK-secure Protocol

**A****B****Initial Setup**

$$a \xleftarrow{\$} \mathbb{Z}_q^*, A \leftarrow g^a$$

$$b \xleftarrow{\$} \mathbb{Z}_q^*, B \leftarrow g^b$$

**Message Exchange**

$$x \xleftarrow{\$} \mathbb{Z}_q^*, X \leftarrow g^x$$

$$\xrightarrow{ID_A, X}$$

$$\xleftarrow{ID_B, Y}$$

$$y \xleftarrow{\$} \mathbb{Z}_q^*, Y \leftarrow g^y$$

**Session Key Computation**

$$Z_1 \leftarrow B^a, Z_2 \leftarrow B^x$$

$$Z_3 \leftarrow Y^a, Z_4 \leftarrow Y^x$$

$$sid_A = ID_A, X, ID_B, Y$$

$$K \leftarrow H(Z_1, Z_2, Z_3, Z_4, sid_A)$$

$$Z'_1 \leftarrow A^b, Z'_2 \leftarrow X^b$$

$$Z'_3 \leftarrow A^y, Z'_4 \leftarrow X^y$$

$$sid_B = ID_A, X, ID_B, Y$$

$$K \leftarrow H(Z'_1, Z'_2, Z'_3, Z'_4, sid_B)$$

$K$  is session key

## An eCK-secure Protocol

**A****B****Initial Setup**

$$a \xleftarrow{\$} \mathbb{Z}_q^*, A \leftarrow g^a$$

$$b \xleftarrow{\$} \mathbb{Z}_q^*, B \leftarrow g^b$$

**Message Exchange**

$$x \xleftarrow{\$} \mathbb{Z}_q^*, X \leftarrow g^x$$

$$\xrightarrow{ID_A, X}$$

$$y \xleftarrow{\$} \mathbb{Z}_q^*, Y \leftarrow g^y$$

$$\xleftarrow{ID_B, Y}$$

**Session Key Computation**

$$\boxed{Z_1 \leftarrow B^a}, Z_2 \leftarrow B^x$$

$$\boxed{Z_3 \leftarrow Y^a}, Z_4 \leftarrow Y^x$$

$$sid_A = ID_A, X, ID_B, Y$$

$$K \leftarrow H(Z_1, Z_2, Z_3, Z_4, sid_A)$$

$$\boxed{Z'_1 \leftarrow A^b}, Z'_2 \leftarrow X^b$$

$$Z'_3 \leftarrow A^y, Z'_4 \leftarrow X^y$$

$$sid_B = ID_A, X, ID_B, Y$$

$$K \leftarrow H(Z'_1, Z'_2, Z'_3, Z'_4, sid_B)$$

$K$  is session key

## Using LR-stored secrets for exponentiation

- Let  $s \in \mathbb{Z}_q^*$  be a long-term secret key and  $E = g^e$  be a received ephemeral value. Then, the value  $Z = E^s$  needs to be computed.
- The secret key is encoded as  $s_L, s_R$ . So the vectors  $s_L = (s_{L1}, \dots, s_{Ln})$  and  $s_R = (s_{R1}, \dots, s_{Rn})$  are such that  $s = s_{L1}s_{R1} + \dots + s_{Ln}s_{Rn}$ .
- The computation of  $E^s$  can be performed as two component-wise computations:
  - compute the intermediate vector  $T = (E^{s_{L1}}, \dots, E^{s_{Ln}})$
  - compute the element
 
$$Z = E^{s_{L1}s_{R1}} E^{s_{L2}s_{R2}} \dots E^{s_{Ln}s_{Rn}} = E^{s_{L1}s_{R1} + \dots + s_{Ln}s_{Rn}} = E^s.$$

## An CAFL-eCK-secure Protocol

**A****B****Initial Setup**

$$a \xleftarrow{\$} \mathbb{Z}_q^*, A \leftarrow g^a$$

$$(a_L^0, a_R^0) \leftarrow \text{Encode}(a)$$

Erase  $a$

$$b \xleftarrow{\$} \mathbb{Z}_q^*, B \leftarrow g^b$$

$$(b_L^0, b_R^0) \leftarrow \text{Encode}(b)$$

Erase  $b$

**Message Exchange**

$$x \leftarrow \mathbb{Z}_q^*, X \leftarrow g^x$$

$$\xrightarrow{ID_A, X}$$

$$y \xleftarrow{\$} \mathbb{Z}_q^*, Y \leftarrow g^y$$

$$\xleftarrow{ID_B, Y}$$

**Session Key Computation**

$$T_1 \leftarrow B^{a_L^j}, Z_1 \leftarrow T_1^{a_R^j}$$

$$T_3 \leftarrow Y^{a_L^j}, Z_3 \leftarrow T_2^{a_R^j}$$

$$Z_2 \leftarrow B^x, Z_4 \leftarrow Y^x$$

$$sid_A = ID_A, X, ID_B, Y$$

$$K \leftarrow H(Z_1, Z_2, Z_3, Z_4, sid_A)$$

$$T'_1 \leftarrow A^{b_L^j}, Z'_1 \leftarrow (T'_1)^{b_R^j}$$

$$T'_2 \leftarrow X^{b_L^j}, Z'_2 \leftarrow (T'_2)^{b_R^j}$$

$$Z'_3 \leftarrow A^y, Z'_4 \leftarrow X^y$$

$$sid_B = ID_A, X, ID_B, Y$$

$$K \leftarrow H(Z'_1, Z'_2, Z'_3, Z'_4, sid_B)$$

# Summary

- First concrete construction of strongly secure key exchange with continuous after-the-fact leakage resilience.
- Possible improvements:
  - increase efficiency with regard to randomness and key computation;
  - different leakage resilience models;
  - standard model.