Cyclic abduction of inductive safety & termination preconditions

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Joint work with Nikos Gorogiannis (Middlesex)

Part I

Introduction and motivations

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- Refined version: is my program safe and/or terminating, given that it satisfies some precondition?
- Even more refined version: can we find a reasonable precondition under which my program is safe and/or terminating?
- In this talk, we focus on this last question, using inductive definitions in separation logic to describe preconditions.

A simple example

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Most general solution is an acyclic linked list, given by

$$\begin{aligned} x = \mathsf{nil} \quad &\Rightarrow \quad \mathsf{list}(x) \\ x \neq \mathsf{nil} * x \mapsto y * \mathsf{list}(y) \quad &\Rightarrow \quad \mathsf{list}(x) \end{aligned}$$

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- Presently, these tools are limited to a few hard-wired such definitions. . .
- . . . which means they must fail, or ask for advice, when encountering a "foreign" data structure.
- It would be nice if we could automatically infer the definitions of these data structures.

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Our aim is to abduce a precondition or "hypothesis" that would justify the "surprising fact" of program safety / termination.

Overview of our approach

- Our approach builds on the cyclic termination proofs in

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- Tool, CABER, implemented on top of cyclic theorem prover CYCLIST:

  J. Brotherston, N. Gorogiannis, and R.L. Petersen.
  A generic cyclic theorem prover.
  In *APLAS* 2012.

Part II

Cyclic safety and termination proofs

Syntax of programs

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$$\begin{array}{rcl} B & ::= & \star \mid E = E \mid E \neq E \\ C & ::= & \epsilon \mid x := E;\, C \mid x := E.f;\, C \mid E.f := E;\, C \mid \\ & & \texttt{free}(E);\, C \mid x := \texttt{new}();\, C \mid \\ & & \texttt{if}\, B\, \texttt{then}\, C\, \texttt{fi};\, C \mid \texttt{while}\, B\, \texttt{do}\, C\, \texttt{od};\, C \end{array}$$

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  where E ranges over expressions, x over variables, n over field names and j over $\mathbb{N}$.
- A program is given by $\texttt{fields}\ n_1, \ldots, n_k;\ C$ where each n_i is a field name and C a command.

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- (C, s, h) is called safe if there is no computation sequence $(C, s, h) \leadsto^* \mathit{fault}$. And $(C, s, h) \downarrow$ means there is no infinite computation sequence $(C, s, h) \leadsto \ldots$

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Proposition (Safety / termination monotonicity)

If (C, s, h) is safe and $h \circ h'$ defined then $(C, s, h \circ h')$ is safe.
If $(C, s, h)\downarrow$ and $h \circ h'$ defined then $(C, s, h \circ h')\downarrow$.

Preconditions

- Formulas F are given by

$$F \quad ::= \quad E = E \mid E \neq E \mid \mathsf{emp} \mid E \mapsto \mathbf{E} \mid P\mathbf{E} \mid F * F$$

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- Semantics given by standard forcing relation $s, h \models_\Phi F$

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- E.g., if C is $x := E.f; C'$, we have the symbolic execution rule:

$$\frac{x = \mathbf{E}_{\overline{f}}[x'/x] * (F * E \mapsto \mathbf{E})[x'/x] \vdash C'}{F * E \mapsto \mathbf{E} \vdash C} \quad |\mathbf{E}| \geq \overline{f}$$

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(Here, $\overline{f} \in \mathbb{N}$ and $\mathbf{E}_{\overline{f}}$ is the $\overline{f}$th element of $\mathbf{E}$. The variable x' is a fresh variable used to record the "old value" of x.)

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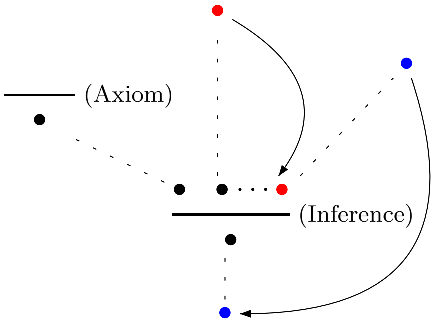
  This gives the unfolding rule:

$$\frac{F * u = \mathsf{nil} \vdash C \qquad F * u \neq \mathsf{nil} * u \mapsto (y, z) * \mathsf{bt}(y) * \mathsf{bt}(z) \vdash C}{F * \mathsf{bt}(u) \vdash C}$$

Cyclic proofs

- A cyclic pre-proof is a derivation tree with back-links:



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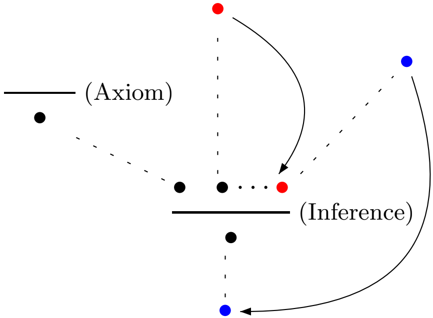

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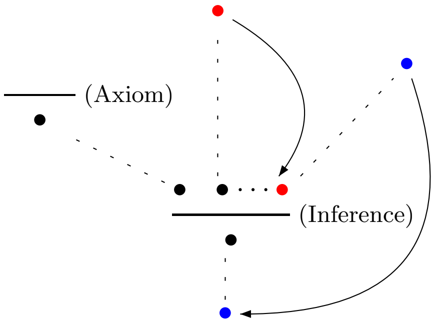

- Safety proof condition: there are infinitely many symbolic executions on every infinite path.
- Termination condition: some inductive predicate is unfolded infinitely often on every infinite path.

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Theorem

Fix rule set Φ, and program C, and suppose there is a cyclic proof $\mathcal{P}$ of $F \vdash C$. Let stack s and heap h satisfy $s, h \models_\Phi F$.

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Proof.

Inductive argument over proofs. □

Part III

Cyclic abduction

Problem statement

- Initial problem: Given program C with input variables $\mathbf{x}$, find inductive rules Φ such that

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  where P is a fresh predicate symbol, and "valid" may have either a safety or a termination interpretation.

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- Our approach: search for a cyclic safety/termination proof of $F \vdash C$, inventing inductive rules as necessary.

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Priority 1: *apply axiom rule*

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We can use a model checker to enforce Principle III.

Worked example: binary tree search

$0 :$ `while` $(x \neq \mathsf{nil})\{$
$1 :$ `if`$(\star)$
$2 : x := x.l$
$3 :$ `else`
$4 : x := x.r \quad \}$
$5 : \epsilon$

$$P_0(x) \vdash 0$$

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4 : x := x.r   }
5 : ε
```

$$x = \mathsf{nil} * P_1(x) \quad \Rightarrow P_0(x)$$
$$x \neq \mathsf{nil} * P_2(x) \quad \Rightarrow P_0(x)$$

$$\dfrac{\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while} \qquad \dfrac{x \neq \mathsf{nil} * P_2(x) \vdash 1}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)$$

Worked example: binary tree search

0 : `while` $(x \neq \mathsf{nil})\{$
1 : `if`$(\star)$
2 : $x := x.l$
3 : `else`
4 : $x := x.r \quad \}$
5 : ϵ

$$x = \mathsf{nil} * P_1(x) \Rightarrow P_0(x)$$
$$x \neq \mathsf{nil} * P_2(x) \Rightarrow P_0(x)$$

$$\dfrac{\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while} \qquad \dfrac{\dfrac{x \neq \mathsf{nil} * P_2(x) \vdash 2 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)$$

Worked example: binary tree search

$0 :$ `while` $(x \neq \mathsf{nil})\{$
$1 :$ `if`$(\star)$
$2 : x := x.l$
$3 :$ `else`
$4 : x := x.r \quad \}$
$5 : \epsilon$

$$\begin{aligned} x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x) \\ x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x) \\ x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x) \end{aligned}$$

$$\dfrac{\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\mathtt{while} \qquad \dfrac{\dfrac{x \neq \mathsf{nil} * P_2(x) \vdash 2 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\mathtt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\mathtt{while}}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)$$

Worked example: binary tree search

```
0 : while (x ≠ nil){
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```

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)
\end{aligned}
$$

$$
\dfrac{
\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
\qquad
\dfrac{\dfrac{\dfrac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2) \qquad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

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x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)
\end{aligned}
$$

$$
\dfrac{
\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
\qquad
\dfrac{\dfrac{\dfrac{\begin{array}{l} x \neq \mathsf{nil} * \\ x \mapsto (y,z) * P_3(x,y,z) \end{array} \vdash 2}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
\qquad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

$0:$ `while` $(x \neq \mathsf{nil})\{$
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$5: \epsilon$

$$
\begin{aligned}
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x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)
\end{aligned}
$$

$$
\dfrac{
  \dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\, x := x.l
      }{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
      \qquad
      x \neq \mathsf{nil} * P_2(x) \vdash 4
    }{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}
  }{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

```
0 : while (x ≠ nil){
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```

$$
\begin{aligned}
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x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$
\dfrac{
  \dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \begin{array}{l} x' \neq \mathsf{nil} * \\ x' \mapsto (x,z) * P_3(x',x,z) \end{array} \vdash 0
      }{
        \begin{array}{l} x \neq \mathsf{nil} * \\ x \mapsto (y,z) * P_3(x,y,z) \end{array} \vdash 2
      }\, x := x.l
    }{
      x \neq \mathsf{nil} * P_2(x) \vdash 2
    }\, \mathcal{A}(P_2)
    \qquad
    x \neq \mathsf{nil} * P_2(x) \vdash 4
  }{
    x \neq \mathsf{nil} * P_2(x) \vdash 1
  }\, \texttt{if}
}{
  \dfrac{x = \mathsf{nil} * P_1(x) \vdash 0 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 0}{P_0(x) \vdash 0}\, \mathcal{A}(P_0)
}\, \texttt{while}
$$

Worked example: binary tree search

0 : `while` $(x \neq \mathsf{nil})\{$
1 : `if`$(\star)$
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$$
\begin{aligned}
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P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$
\dfrac{
  \dfrac{
    \dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon
  }{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\,\mathcal{A}(P_3)
          }{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\,x := x.l
        }{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
        \qquad
        x \neq \mathsf{nil} * P_2(x) \vdash 4
      }{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}
    }{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

0 : `while` $(x \neq \mathsf{nil})\{$
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P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$
\dfrac{
  \dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\,(\text{Frame})
          }{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\,\mathcal{A}(P_3)
        }{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\,x := x.l
      }{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
      \qquad
      x \neq \mathsf{nil} * P_2(x) \vdash 4
    }{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}
  }{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

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P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$
\dfrac{
\dfrac{x = \mathsf{nil} * P_1(x) \vdash 5}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\epsilon\ \texttt{while}
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\,(\text{Frame})
}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\,\mathcal{A}(P_3)
}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\,x := x.l
}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
\quad x \neq \mathsf{nil} * P_2(x) \vdash 4
}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}
}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

```
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x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$\frac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\ \text{(Frame)}$$

$$\frac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\ \mathcal{A}(P_3)$$

$$\frac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\ x := x.l$$

$$\frac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\ \mathcal{A}(P_2) \qquad \frac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 4}\ (P_2)$$

$$\frac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\ \epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\ \texttt{while} \qquad \frac{\dfrac{x \neq \mathsf{nil} * P_2(x) \vdash 2 \quad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\ \texttt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\ \texttt{while}$$

$$\frac{x = \mathsf{nil} * P_1(x) \vdash 0 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 0}{P_0(x) \vdash 0}\ \mathcal{A}(P_0)$$

Worked example: binary tree search

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$1:$ `if`$(\star)$
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$4: x := x.r \quad \}$
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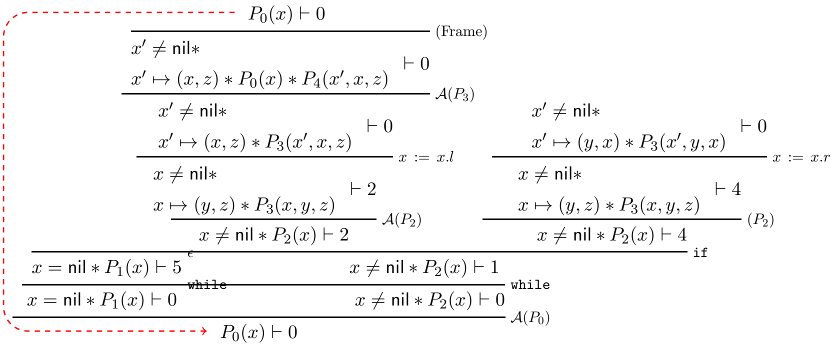

Worked example: binary tree search

```
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4 : x := x.r    }
5 : ε
```

$$
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x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}
$$

$$\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\ \text{(Frame)}$$

$$\dfrac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\ \mathcal{A}(P_3)$$

$$\dfrac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\ x := x.l$$

$$\dfrac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\ \mathcal{A}(P_2)$$

$$\dfrac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}\ x := x.r$$

$$\dfrac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 4}\ (P_2)$$

$$\dfrac{x \neq \mathsf{nil} * P_2(x) \vdash 2 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\ \texttt{if}$$

$$\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\ \epsilon$$

$$\dfrac{x = \mathsf{nil} * P_1(x) \vdash 5}{x = \mathsf{nil} * P_1(x) \vdash 0}\ \texttt{while} \qquad \dfrac{x \neq \mathsf{nil} * P_2(x) \vdash 1}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\ \texttt{while}$$

$$\dfrac{x = \mathsf{nil} * P_1(x) \vdash 0 \qquad x \neq \mathsf{nil} * P_2(x) \vdash 0}{P_0(x) \vdash 0}\ \mathcal{A}(P_0)$$

Worked example: binary tree search

0 : `while` $(x \neq \mathsf{nil})\{$
1 : `if`$(\star)$
2 : $x := x.l$
3 : `else`
4 : $x := x.r \quad \}$
5 : ϵ

$$\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)
\end{aligned}$$

$$\dfrac{\dfrac{\dfrac{x = \mathsf{nil} * P_1(x) \vdash 5}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\epsilon\ \texttt{while} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\,(\text{Frame})}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\,\mathcal{A}(P_3)}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\,x := x.l}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2) \quad \dfrac{\dfrac{\dfrac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}\,(P_3)}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}\,x := x.r}{x \neq \mathsf{nil} * P_2(x) \vdash 4}\,(P_2)}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)$$

Worked example: binary tree search

$0:\texttt{while}\ (x \neq \mathsf{nil})\{$
$1:\texttt{if}(\star)$
$2: x := x.l$
$3:\texttt{else}$
$4: x := x.r \quad \}$
$5:\epsilon$

$$
\begin{aligned}
x = \mathsf{nil} * P_1(x) &\Rightarrow P_0(x)\\
x \neq \mathsf{nil} * P_2(x) &\Rightarrow P_0(x)\\
x \mapsto (y,z) * P_3(x,y,z) &\Rightarrow P_2(x)\\
P_0(y) * P_4(x,y,z) &\Rightarrow P_3(x,y,z)\\
P_0(z) * P_5(x,y,z) &\Rightarrow P_4(x,y,z)
\end{aligned}
$$

$$
\dfrac{
\dfrac{\dfrac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\,\epsilon}{x = \mathsf{nil} * P_1(x) \vdash 0}\,\texttt{while}
\quad
\dfrac{\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\,(\text{Frame})}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\,\mathcal{A}(P_3)}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\,x := x.l}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\,\mathcal{A}(P_2)
\quad
\dfrac{\dfrac{\dfrac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}\,(P_3)}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}\,x := x.r}{x \neq \mathsf{nil} * P_2(x) \vdash 4}\,(P_2)
}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\,\texttt{if}}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\,\texttt{while}
}{P_0(x) \vdash 0}\,\mathcal{A}(P_0)
$$

Worked example: binary tree search

$0:$ `while` $(x \neq \mathsf{nil})\{$
$1:$ `if`$(\star)$
$2: x := x.l$
$3:$ `else`
$4: x := x.r \quad \}$
$5: \epsilon$

$$
\begin{array}{rl}
x = \mathsf{nil} * P_1(x) & \Rightarrow P_0(x) \\
x \neq \mathsf{nil} * P_2(x) & \Rightarrow P_0(x) \\
x \mapsto (y,z) * P_3(x,y,z) & \Rightarrow P_2(x) \\
P_0(y) * P_4(x,y,z) & \Rightarrow P_3(x,y,z) \\
P_0(z) * P_5(x,y,z) & \Rightarrow P_4(x,y,z)
\end{array}
$$

$$\frac{P_0(x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}\ (\text{Frame})$$

$$\frac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_0(x) * P_5(x',y,x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0}\ \mathcal{A}(P_4)$$

$$\frac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_0(x) * P_4(x',x,z) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}\ \mathcal{A}(P_3) \qquad \frac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_0(y) * P_4(x',y,x) \vdash 0}{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}\ (P_3)$$

$$\frac{x' \neq \mathsf{nil} * x' \mapsto (x,z) * P_3(x',x,z) \vdash 0}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}\ x := x.l \qquad \frac{x' \neq \mathsf{nil} * x' \mapsto (y,x) * P_3(x',y,x) \vdash 0}{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}\ x := x.r$$

$$\frac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 2}{x \neq \mathsf{nil} * P_2(x) \vdash 2}\ \mathcal{A}(P_2) \qquad \frac{x \neq \mathsf{nil} * x \mapsto (y,z) * P_3(x,y,z) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 4}\ (P_2)$$

$$\frac{}{x = \mathsf{nil} * P_1(x) \vdash 5}\ \epsilon \qquad \frac{x \neq \mathsf{nil} * P_2(x) \vdash 2 \quad x \neq \mathsf{nil} * P_2(x) \vdash 4}{x \neq \mathsf{nil} * P_2(x) \vdash 1}\ \texttt{if}$$

$$\frac{x = \mathsf{nil} * P_1(x) \vdash 5}{x = \mathsf{nil} * P_1(x) \vdash 0}\ \texttt{while} \qquad \frac{x \neq \mathsf{nil} * P_2(x) \vdash 1}{x \neq \mathsf{nil} * P_2(x) \vdash 0}\ \texttt{while}$$

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Worked example: binary tree search

$0 : \texttt{while}\ (x \neq \mathsf{nil})\{$
$1 : \texttt{if}(\star)$
$2 : x := x.l$
$3 : \texttt{else}$
$4 : x := x.r \quad \}$
$5 : \epsilon$

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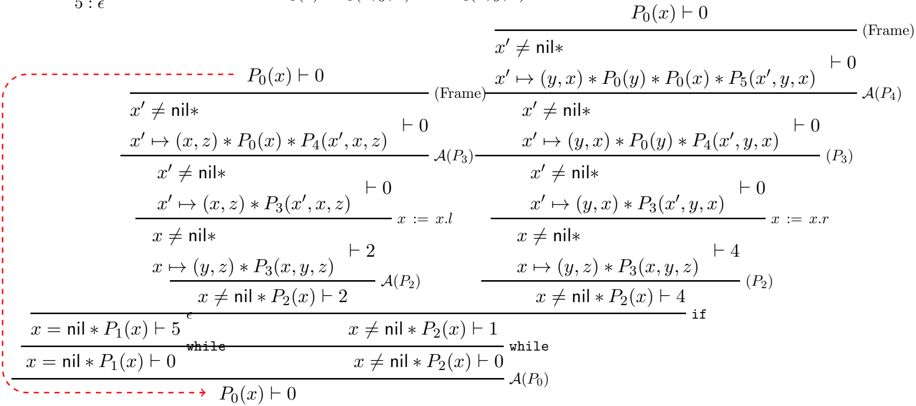

Worked example: binary tree search

```
0 : while (x ≠ nil){
1 : if(⋆)
2 : x := x.l
3 : else
4 : x := x.r    }
5 : ε
```

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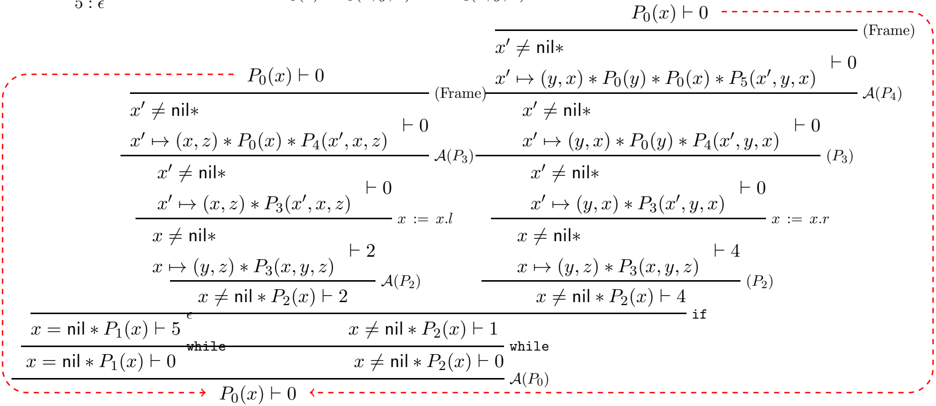

Simplifying inductive rule sets

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(nil-terminated binary tree)

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Simplifying inductive rule sets

- instantiate undefined predicates to emp;
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- inline single-clause predicates.
- remove unsatisfiable clauses (not shown)

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Part IV

Challenges and subtleties

Evaluating solution quality

- Backtracking search can yield different solutions.

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- We can decide whether a predicate is satisfiable

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- Comparing predicates via entailment ($\vdash$) is not practical.
- Currently we use a simple grading scheme for predicate quality.
- We can simplify predicates and replay the proof to improve quality, sometimes.

Experimental results

Program	LOC	Time	Depth	Quality	Term.
List traverse	3	20	3	A	✓
List insert	14	8	7	B	✓
List copy	12	0	8	B	✓
List append	10	12	5	B	✓
Delete last from list	16	12	9	B	✓
Filter list	21	48	11	C	✓
Dispose list	5	4	5	A	✓
Reverse list	7	8	7	A	✓
Cyclic list traverse	5	4	5	A	✓
Binary tree search	7	8	4	A	✓
Binary tree insert	18	4	7	B	✓
List of lists traverse	7	8	5	B	✓
Traverse even-length list	4	8	4	A	✓
Traverse odd-length list	4	4	4	A	✓
Ternary tree search	10	8	5	A	✓
Conditional diverge	3	4	3	B	×
Traverse list of trees	11	12	6	B	✓
Traverse tree of lists	17	68	7	A	✓
Traverse list twice	8	64	9	B	✓

Problem: initial variable assignment

- Consider a local variable assignment $y := x$ at line 0. In the proof we get

$$\frac{y = x * P\mathbf{x} \vdash 1}{P\mathbf{x} \vdash 0}\, y := x$$

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- But there are lots of choices!

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- In principle, we could also use the control flow graph of the program to help us decide what to do.

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- The abstraction problem is inherited from program analysis in general.

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- Here it shows up in the need for lemmas:

$$\frac{\Pi : F * \mathsf{list}(x) \vdash i}{\Pi : F * x \mapsto y \vdash i} \quad x \mapsto y \vdash \mathsf{list}(x) \quad \text{(Cut)}$$

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- Our tool has a limited abstraction capability, mainly based on existentially quantifying variables modified by loops.
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- Cyclist gives us an entailment prover which could be used to prove conjectured lemmas.

Thanks for listening!

Get Caber / Cyclist online (source / virtual machine image):

google "cyclist theorem prover".

J. Brotherston and N, Gorogiannis.
Cyclic abduction of inductive safety and termination preconditions.
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