Linear-Time Zero-Knowledge Proofs for Arithmetic Circuit Satisfiability

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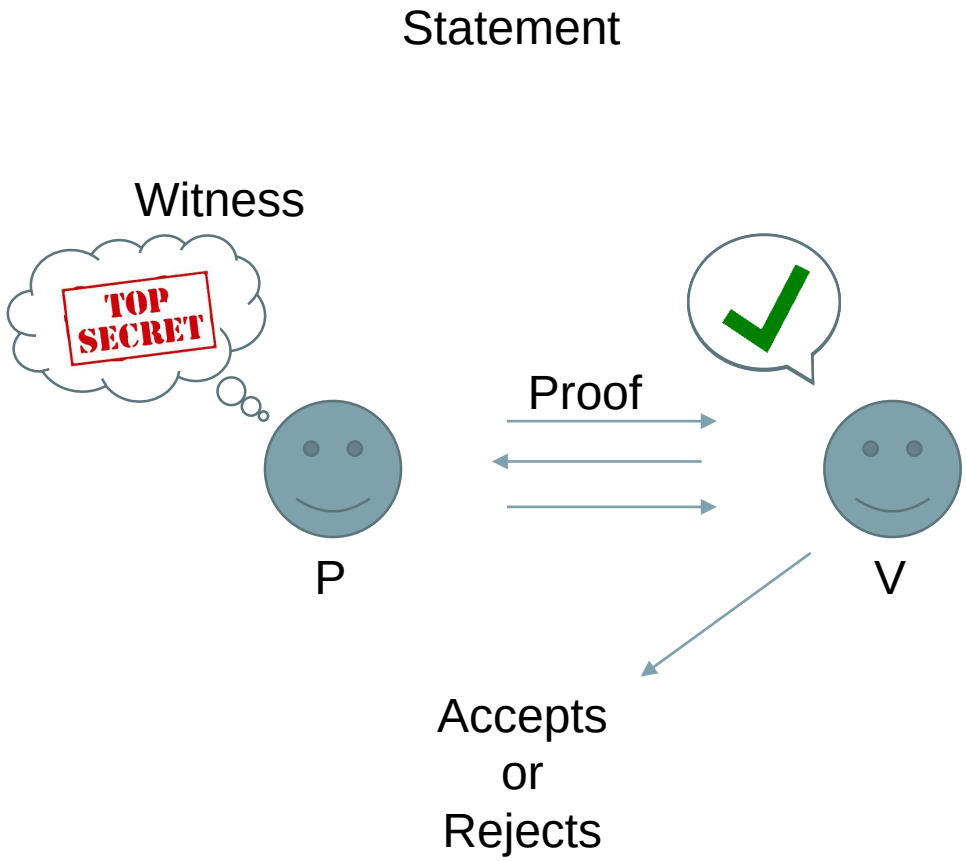
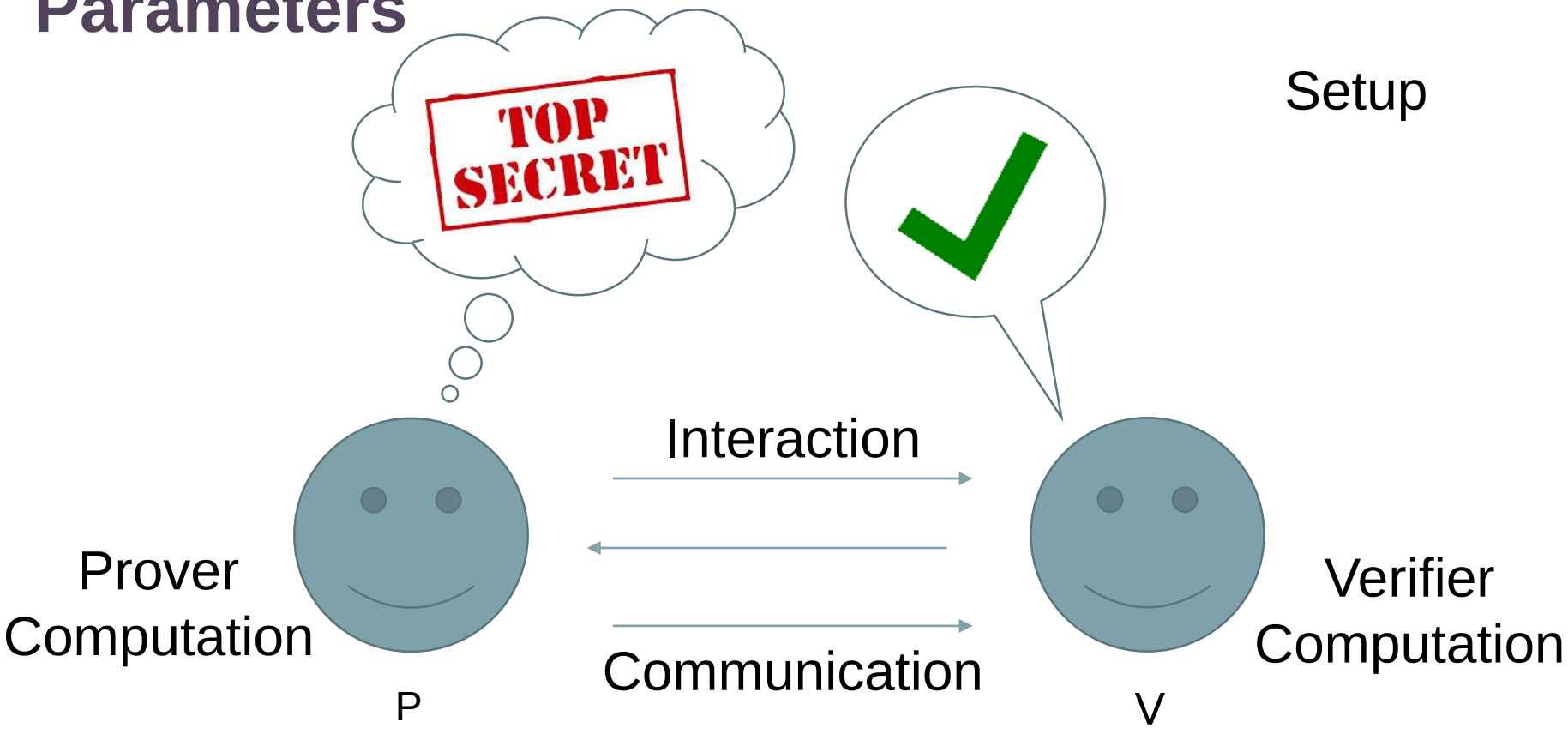
Zero Knowledge Proofs

- Completeness
- Soundness
- Zero-Knowledge

- Proof of Knowledge
- Interactive
- Public-coin



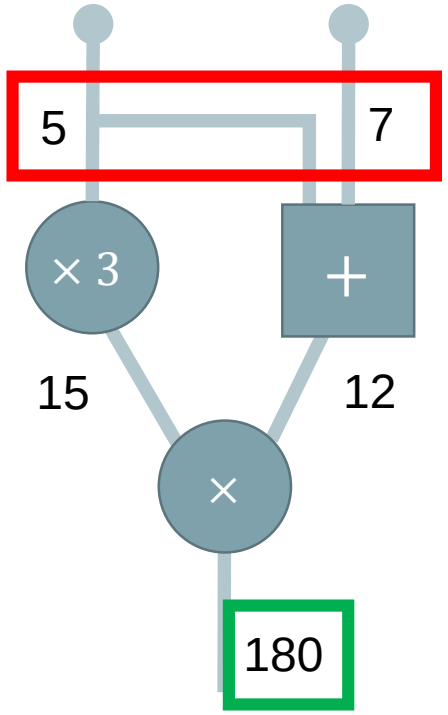
Parameters



Goal: constant computational overhead for the Prover

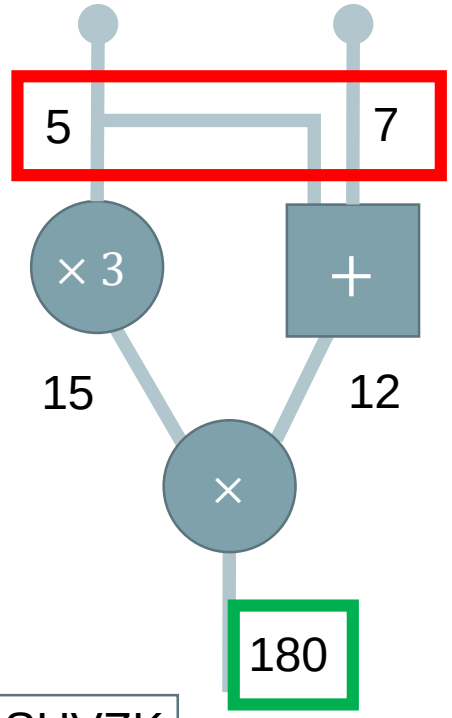
Arithmetic Circuits

- Prover knows inputs
- Publicly known outputs
- Check inputs give the correct outputs
- Do valid inputs exist? NP-Complete



Results

- Security parameter λ
- Finite field F, 2^{λ} elements
- Arithmetic circuit,
  N = poly(λ) gates
- Zero-knowledge arguments
  and proofs

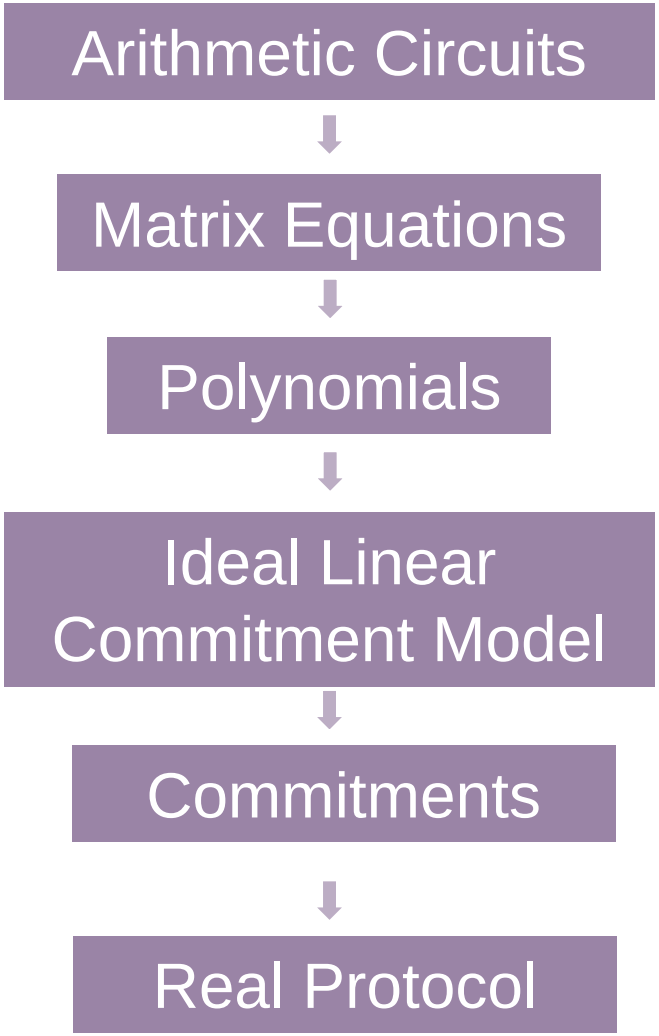


Statistical SHVZK

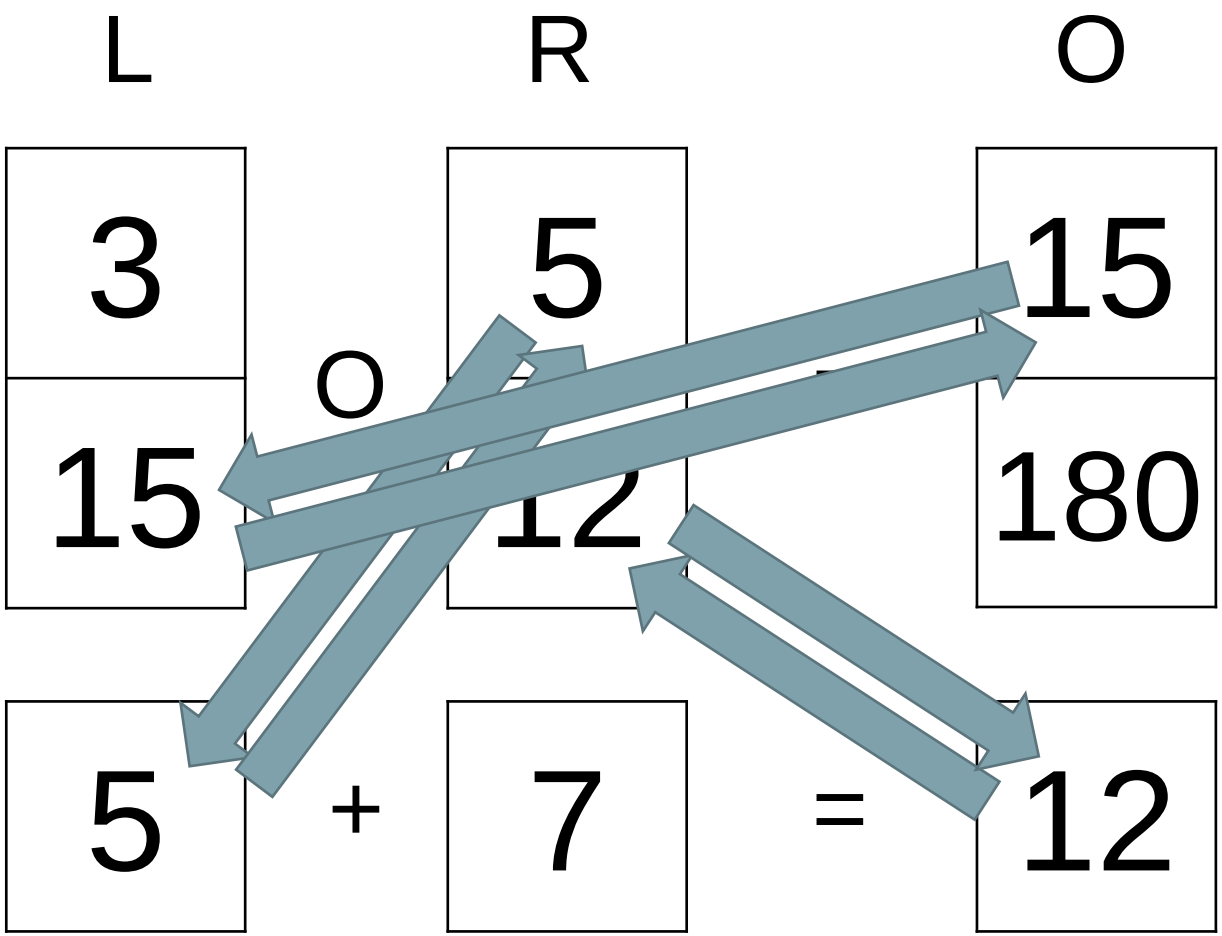
Prover	Verifier	Comm.	Rounds	Assumption
O(N) multiplications in F	o(N) multiplications in F	poly(λ)√N elements of F	O(loglog N)	lt-CRHF
O(N) multiplications in F	o(N) multiplications in F	O(N) elements of F	O(loglog N)	lt-OWF

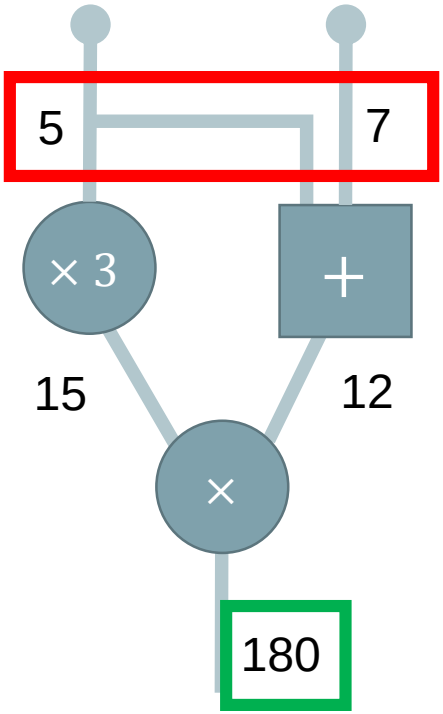
Statistical Soundness

Overview

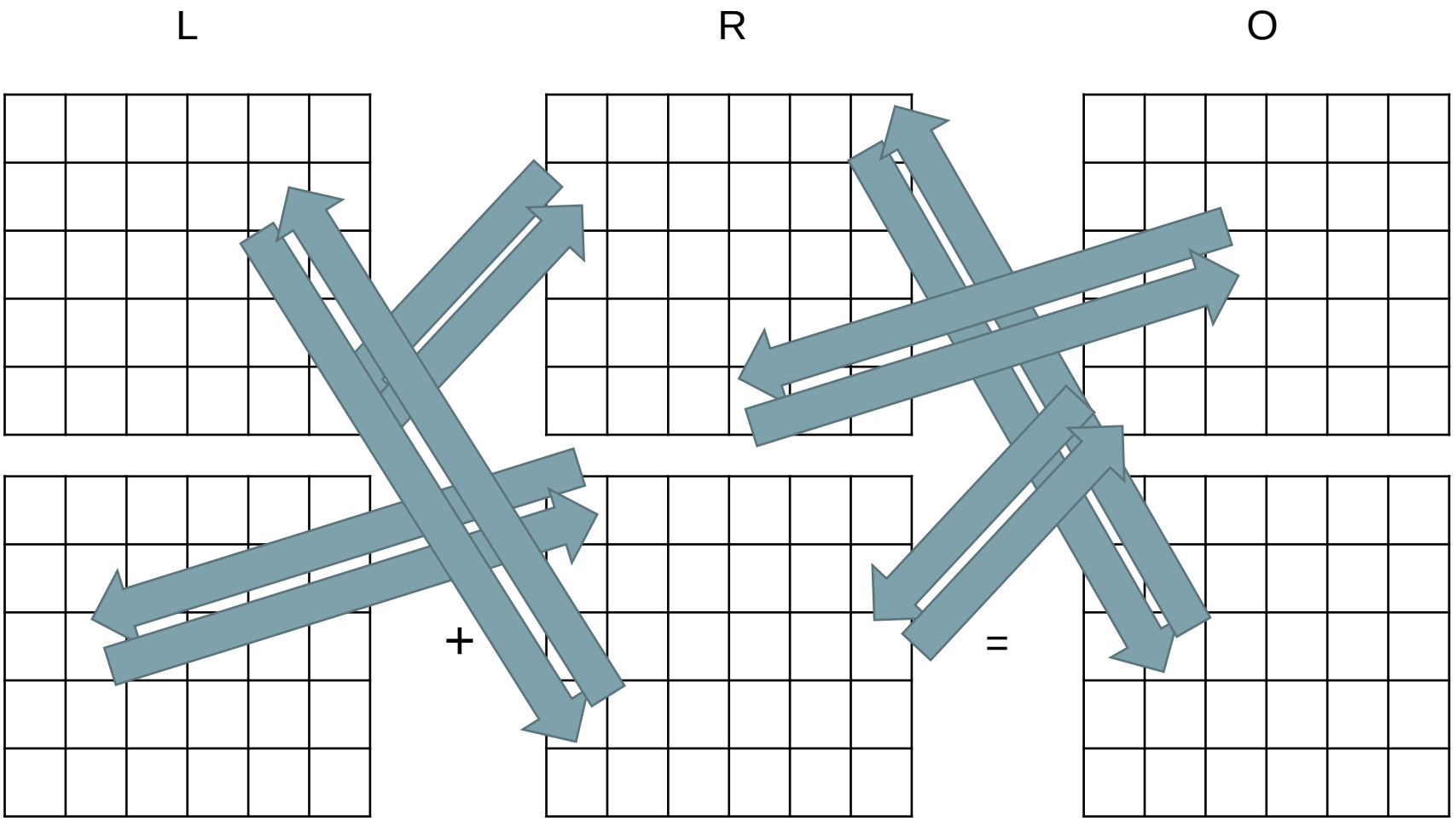

Arithmetic Circuits
Matrix Equations
Polynomials
Ideal Linear Commitment Model
Commitments
Real Protocol

High Level Structure

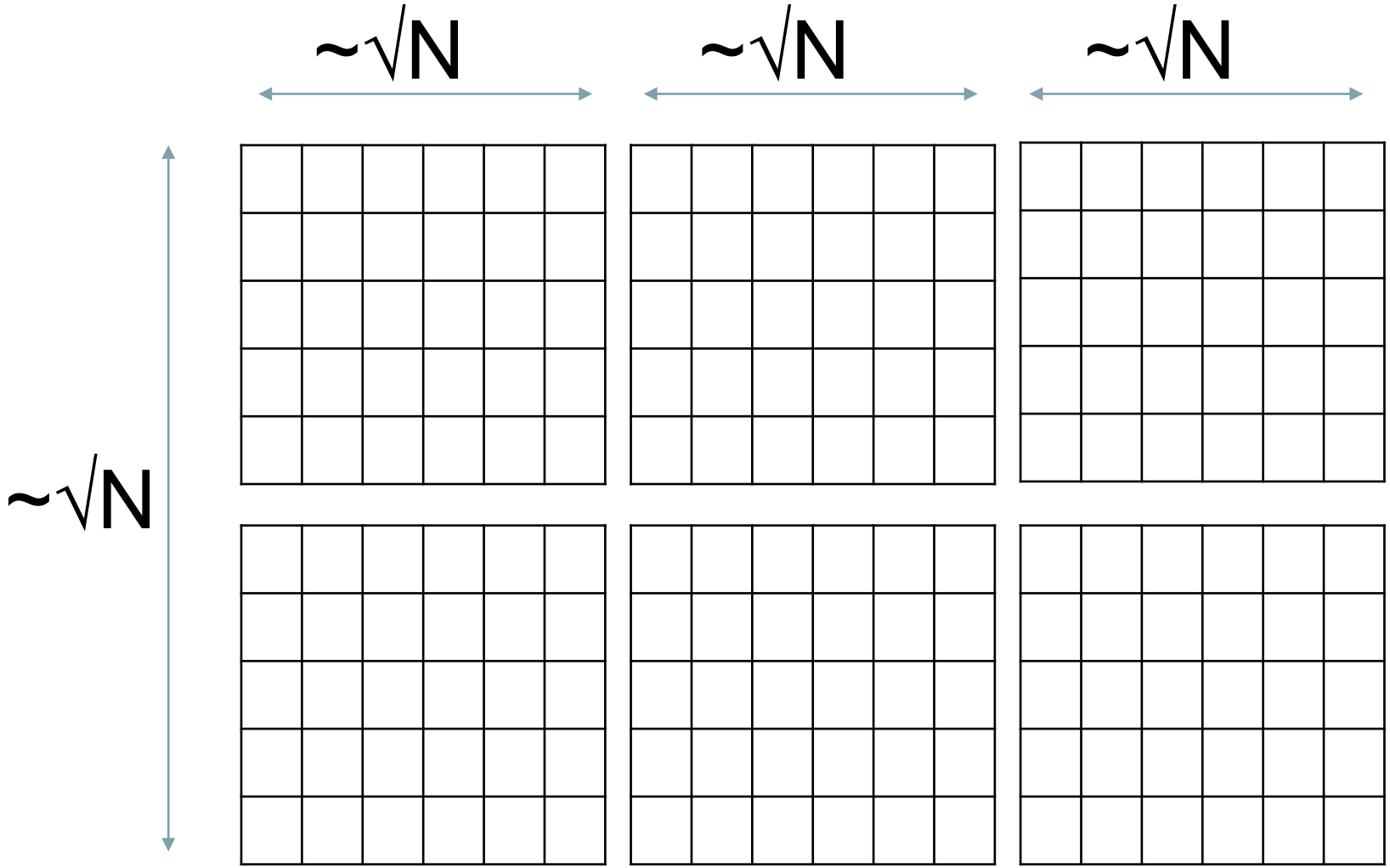




High Level Structure



Matrix Dimensions

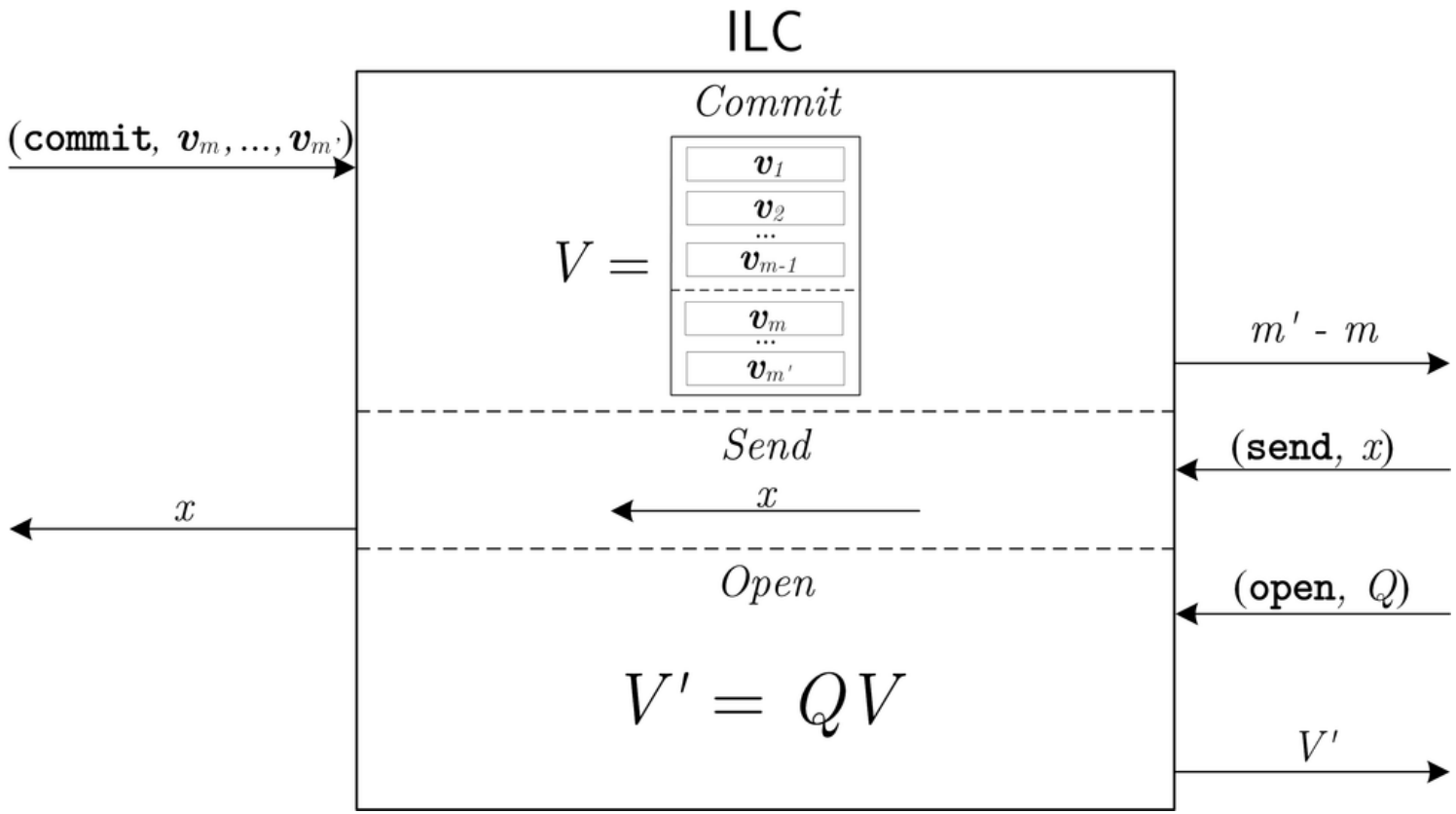


Previous Arguments

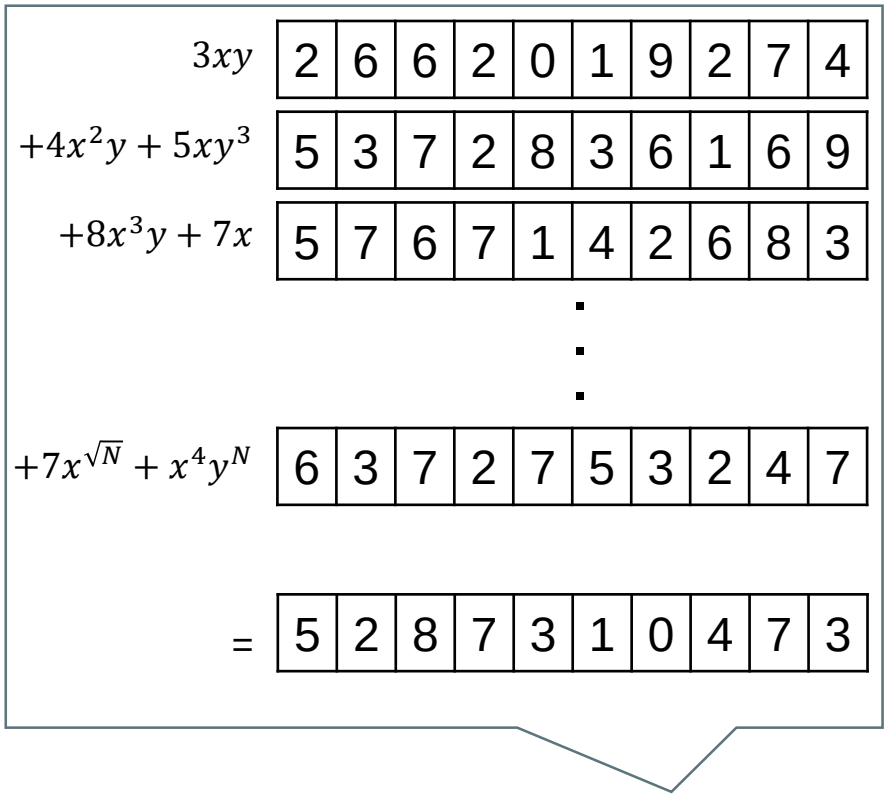
- Other protocols commit to vectors ([G09], [S09])
- Random challenge x
- Prover opens linear combinations
- Check openings are correct
- Embed AC-SAT into coefficients

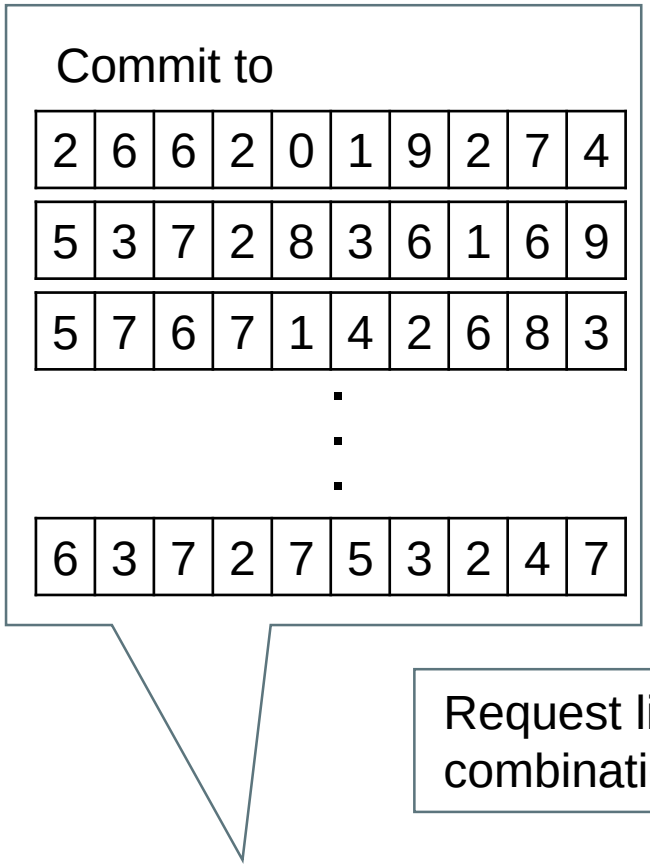
$3x$	2	6	6	2	0	1	9	2	7	4
$+4x^2$	5	3	7	2	8	3	6	1	6	9
$+8x^3$	5	7	6	7	1	4	2	6	8	3
$+7x^4$	6	3	7	2	7	5	3	2	4	7
=	5	2	8	7	3	1	0	4	7	3

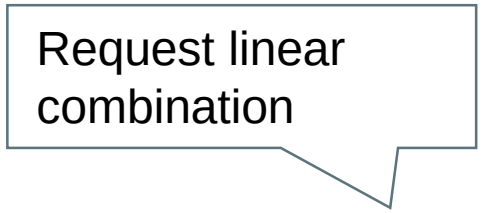
Ideal Linear Commitment Model



Sub-linear Verifier





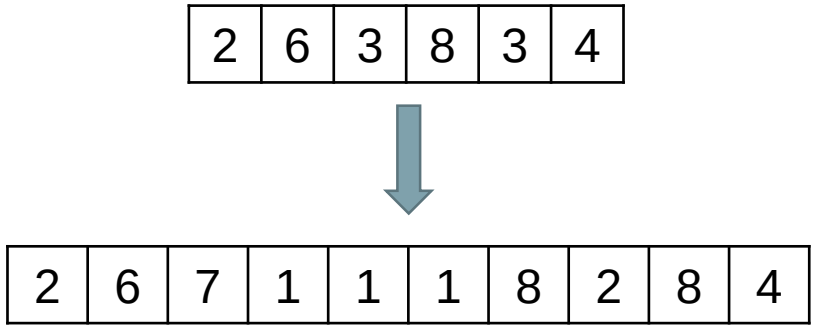


Commitment Ingredients

- Linear error-correcting code
- Example: [DI14]

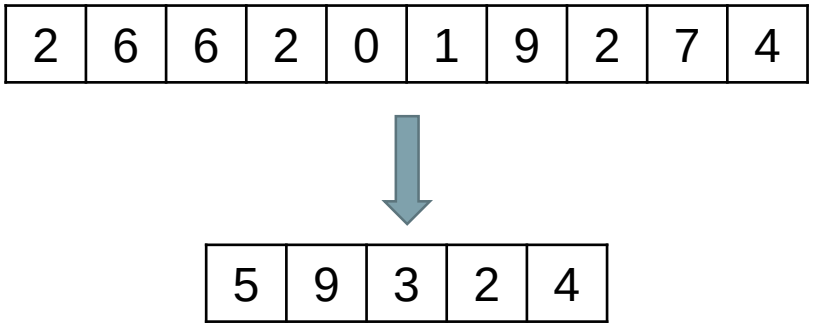
- Randomise for zero-knowledge

Linear code
Linear-time encoding
Linear Minimum Distance



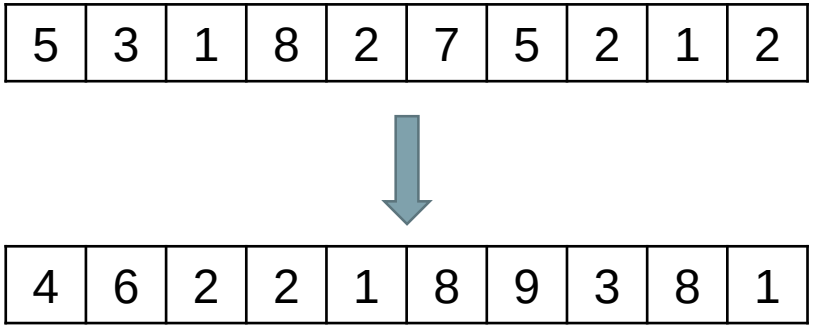
Commitment Ingredients

- **Hiding:**
- Collision-resistant hash function
- Example: [AHIKV17]



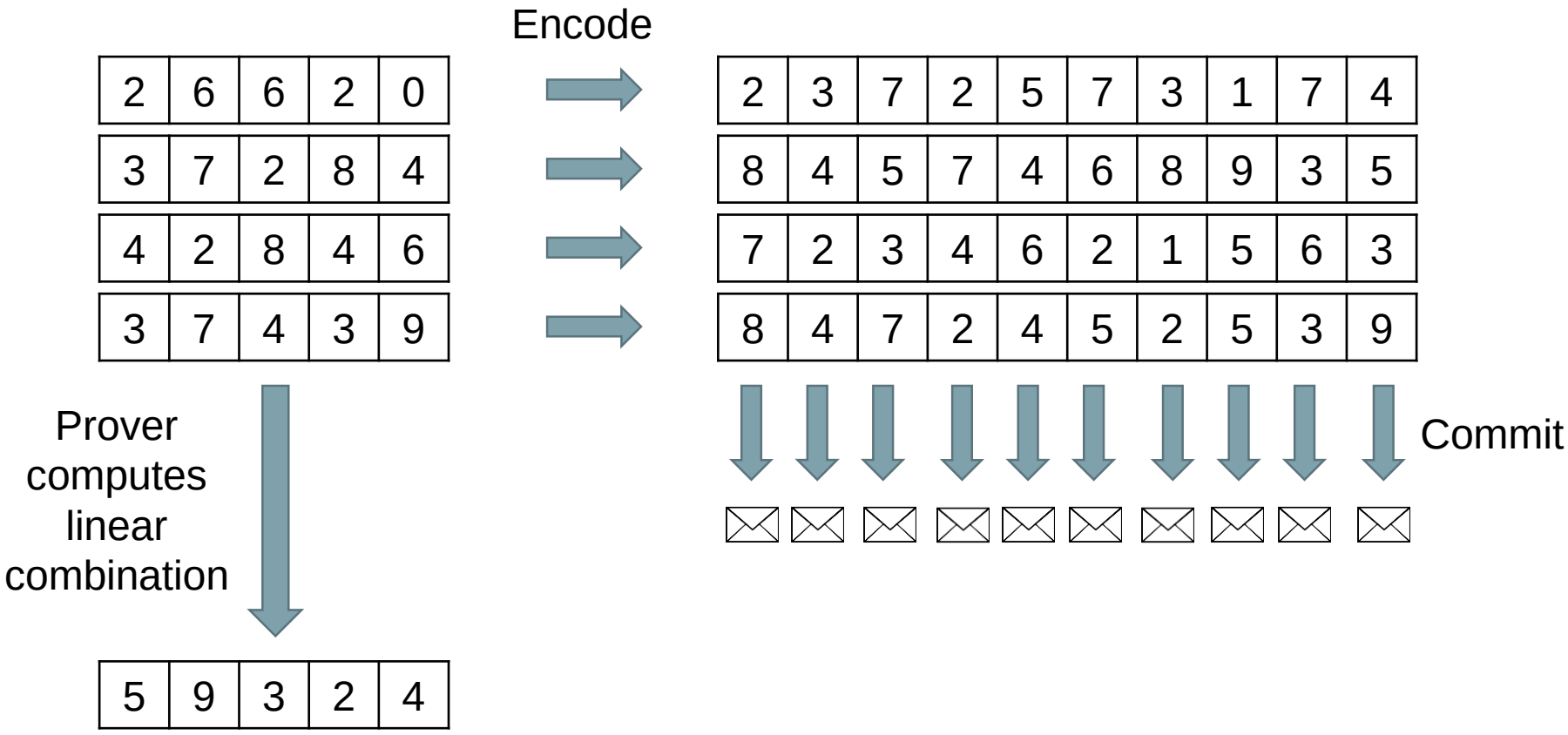
Linear-time computable

- **Binding:**
- One-way function
- Example: [IKOS08]

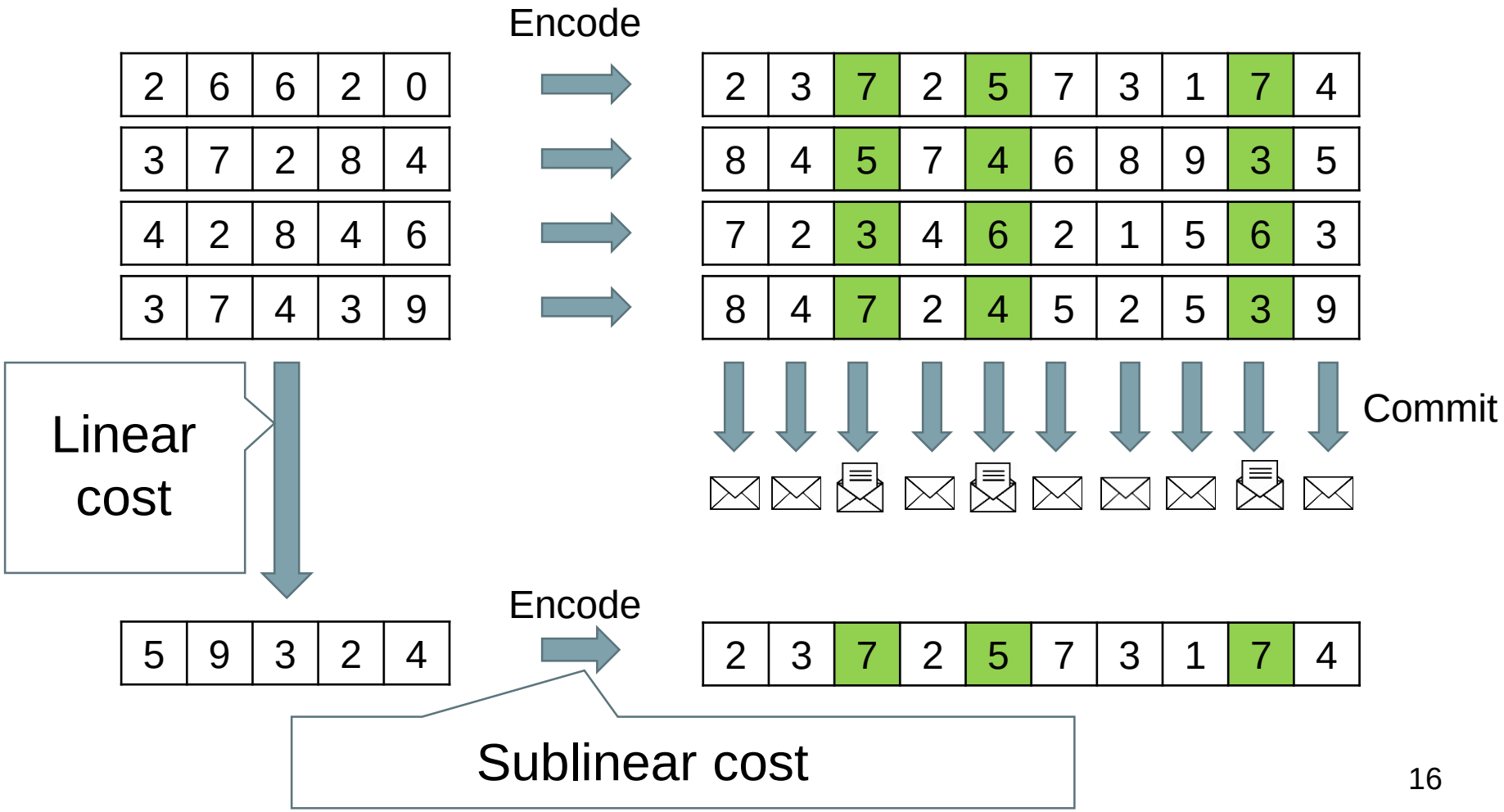


Linear-time computable

Opening Commitments



Opening Commitments



Ideal Protocols to Real Protocols

Arguments using [AHIKV17]

- Hiding commitments

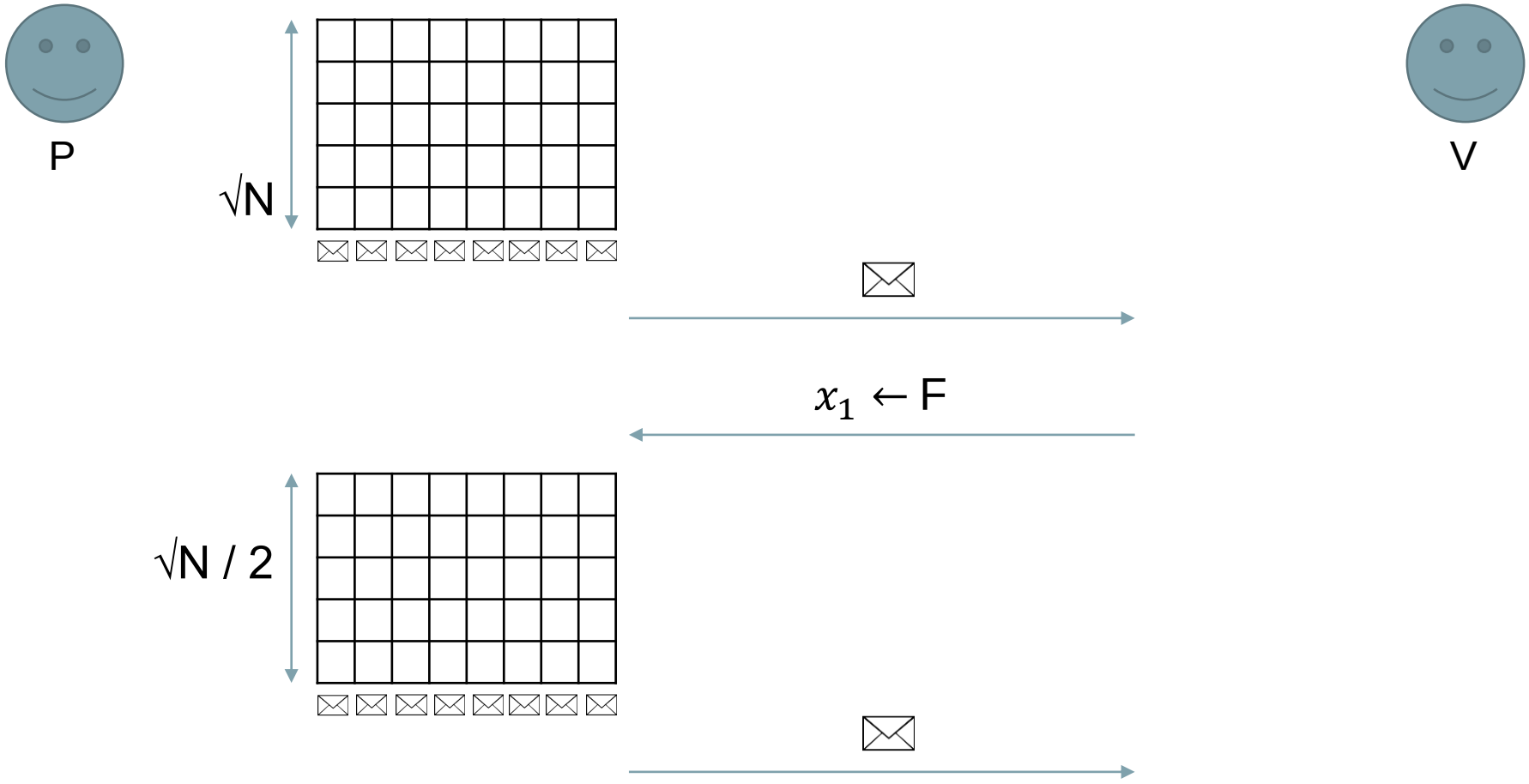
- Perfect Completeness
- Computational Soundness
- Statistical SHVZK

Proofs using [IKOS08]

- Binding commitments

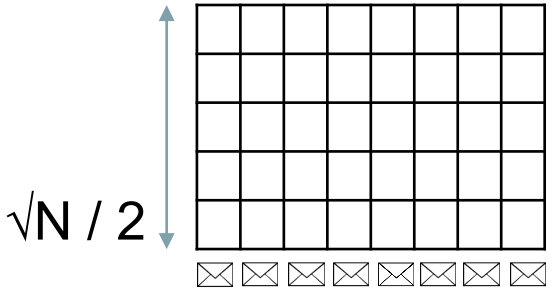
- Perfect Completeness
- Statistical Soundness
- Computational SHVZK

Protocol Flow



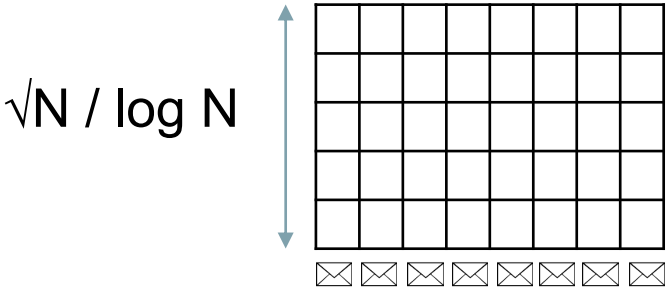
Protocol Flow

P

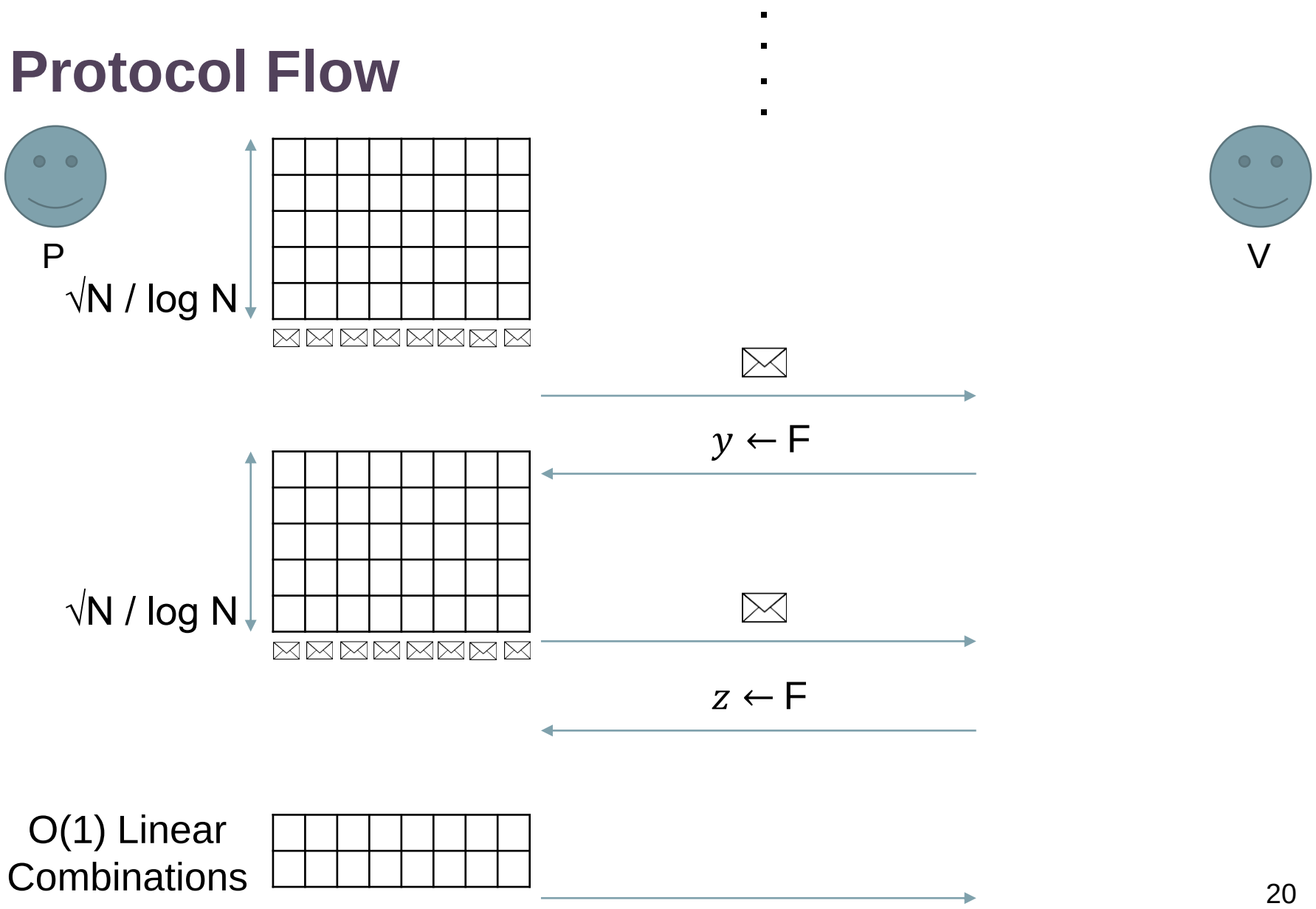


V

$x_{\log\log N} \leftarrow$ F



Protocol Flow

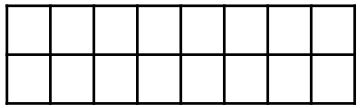


Protocol Flow

P

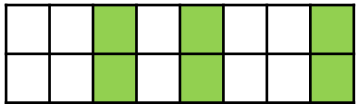
V

O(1) Linear Combinations



$I \subset \{1, \ldots, \sqrt{N}\}$

Open columns in I



Comparison for Arguments

Previous work	Prover	Verifier	Comm.	Assumption
[CD96]	O(λN) mult	O(λN) mult	O(N) elem	DLOG
[G09], [S09]	O(λN/log N) mult	O(λN) mult	O(√N) elem	DLOG
SNARKs	O(λN) mult	O(λ) mult	O(1) elem	KOE, qPDH
[BSCS16]	$O(N^{1+c})$ mult	$O(N^{1+c})$ mult	poly(λ) log N elem	CRHF
Ligero 2017	O(N log N) mult	O(N) mult	poly(λ)√N elem	CRHF
This work	O(N) mult	o(N) mult	poly(λ)√N elem	CRHF

Thanks!

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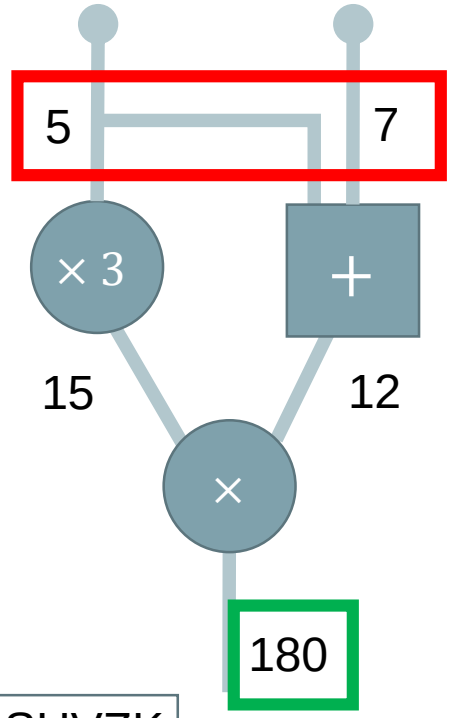

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