ERRATUM TO *A SUBSTRUCTURAL LOGIC FOR LAYERED GRAPHS*

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Abstract. Proposition 6.2 (2) and (3) of 'A substructural logic for layered graphs', by M. Collinson, K. McDonald, and D. Pym, *Journal of Logic and Computation* (2014) 24 (4): 953-988, are incorrect. We provide explanations of the failures of the intended proofs and specific counter-examples. The paper makes no further use of the claims and there are no consequences for the theory or examples that are presented.

1. Proposition 6.2 (2) and (3) Counter-examples

In Section 6 of [1], the following proposition is stated:

Proposition 6.2

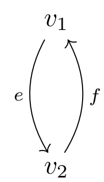
(1) ...
(2) If $[\![\phi]\!]_{\mathcal{E},\mathcal{F}} \subseteq [\![\phi]\!]_{\mathcal{E}}$ and $[\![\psi]\!]_{\mathcal{E}} \subseteq [\![\psi]\!]_{\mathcal{E},\mathcal{F}}$, then $[\![\phi \mathrel{-\!\!\blacktriangleright} \psi]\!]_{\mathcal{E}} \subseteq [\![\phi \mathrel{-\!\!\blacktriangleright} \psi]\!]_{\mathcal{E},\mathcal{F}}$.
(3) If $[\![\phi]\!]_{\mathcal{E}} \subseteq [\![\phi]\!]_{\mathcal{E},\mathcal{F}}$ and $[\![\psi]\!]_{\mathcal{E},\mathcal{F}} \subseteq [\![\psi]\!]_{\mathcal{E}}$, then $[\![\phi \mathrel{-\!\!\blacktriangleright} \psi]\!]_{\mathcal{E},\mathcal{F}} \subseteq [\![\phi \mathrel{-\!\!\blacktriangleright} \psi]\!]_{\mathcal{E}}$.

Attempting to prove these two claims, we encounter difficulties that suggest counter-examples.

For (2), suppose the antecedents are true. Let $G \vDash_{\mathcal{E}} \phi \mathrel{-\!\!\blacktriangleright} \psi$. We need $G \vDash_{\mathcal{E},\mathcal{F}} \phi \mathrel{-\!\!\blacktriangleright} \psi$, so suppose H is such that $G \,@_{\mathcal{E},\mathcal{F}}\, H\!\downarrow$ and $H \vDash_{\mathcal{E},\mathcal{F}} \phi$. From $[\![\phi]\!]_{\mathcal{E},\mathcal{F}} \subseteq [\![\phi]\!]_{\mathcal{E}}$, we have that $H \vDash_{\mathcal{E}} \phi$ and, since $G \,@_{\mathcal{E},\mathcal{F}}\, H\!\downarrow$ implies $G \,@_{\mathcal{E}}\, H\!\downarrow$, we can use $G \vDash_{\mathcal{E}} \phi \mathrel{-\!\!\blacktriangleright} \psi$ to conclude $G \,@_{\mathcal{E}} H \vDash_{\mathcal{E}} \phi$. Hence, by $[\![\psi]\!]_{\mathcal{E}} \subseteq [\![\psi]\!]_{\mathcal{E},\mathcal{F}}$, we may conclude $G \,@_{\mathcal{E}} H \vDash_{\mathcal{E},\mathcal{F}} \psi$. This doesn't, however, allow us to conclude the statement of the proposition: for that, we require $G \,@_{\mathcal{E},\mathcal{F}} H \vDash_{\mathcal{E},\mathcal{F}} \psi$, and we know that $G \,@_{\mathcal{E}} H \not\simeq G \,@_{\mathcal{E},\mathcal{F}} H$.

For (3), suppose the antecedents are true and let $G \vDash_{\mathcal{E},\mathcal{F}} \phi \mathrel{-\!\!\blacktriangleright} \psi$. Suppose we have H such that $G \,@_{\mathcal{E}} H\!\downarrow$ and $H \vDash_{\mathcal{E}} \phi$. We can use $[\![\phi]\!]_{\mathcal{E}} \subseteq [\![\phi]\!]_{\mathcal{E},\mathcal{F}}$ to conclude $H \vDash_{\mathcal{E},\mathcal{F}} \phi$. From here, we want to use the fact that $G \vDash_{\mathcal{E},\mathcal{F}} \phi \mathrel{-\!\!\blacktriangleright} \psi$ to obtain $G \,@_{\mathcal{E},\mathcal{F}} H \vDash \psi$, allowing us to use $[\![\psi]\!]_{\mathcal{E},\mathcal{F}} \subseteq [\![\psi]\!]_{\mathcal{E}}$ to conclude $G \,@_{\mathcal{E},\mathcal{F}} H \vDash_{\mathcal{E}} \psi$. However, we are unable to to use the first fact because we have no guarantee that $H \,@_{\mathcal{F}} G\!\downarrow$ and, even if we had such a guarantee, we should encounter the same problem as at the end of the argument above for (2), since $G \,@_{\mathcal{E}} H \not\simeq G \,@_{\mathcal{E},\mathcal{F}} H$.

These failed proof attempts suggest a simple counterexample that shows that neither (2) nor (3) is true. Let $\mathcal{G}$ be given by the graph



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Let $\mathcal{E} = \{e\}$, $\mathcal{F} = \{f\}$, $G_1 = \langle\{v_1\}, \emptyset\rangle$, $G_2 = \langle\{v_2\}, \emptyset\rangle$ and $G_3 = \langle\{v_1, v_2\}, \{e\}\rangle$. First consider a valuation $V_1 : Atoms \to P(Sg(\mathcal{G}))$ given by $V_1(p) = \{G_2\}$ and $V_1(q) = \{G_3\}$. Extending this for both $[\![-]\!]_{\mathcal{E}}$ and $[\![-]\!]_{\mathcal{E},\mathcal{F}}$ trivially satisfies $[\![p]\!]_{\mathcal{E},\mathcal{F}} \subseteq [\![p]\!]_{\mathcal{E}}$ and $[\![q]\!]_{\mathcal{E}} \subseteq [\![q]\!]_{\mathcal{E},\mathcal{F}}$. We claim $G_1 \in [\![p \blacktriangleright q]\!]_{\mathcal{E}}$ but $G_1 \notin [\![p \blacktriangleright q]\!]_{\mathcal{E},\mathcal{F}}$.

We have that only G_2 is such that $G_1 @_{\mathcal{E}} G_2 \downarrow$ with this composition equal to G_3. Since $G_2 \vDash_{\mathcal{E}} p$ and $G_3 \vDash_{\mathcal{E}} q$ we have $G_1 \vDash_{\mathcal{E}} p \blacktriangleright q$. However, only G_2 is such that $G_1 @_{\mathcal{E},\mathcal{F}} G_2 \downarrow$ and this composition equals $\mathcal{G}$. But $G_2 \vDash_{\mathcal{E},\mathcal{F}} p$ and $\mathcal{G} \nvDash_{\mathcal{E},\mathcal{F}} q$, so $G_1 \nvDash_{\mathcal{E},\mathcal{F}} p \blacktriangleright q$. This disproves (2).

Using the valuation $V_2 : Atoms \to P(Sg(\mathcal{G}))$ given by $V_2(p) = \{G_2\}$ and $V_2(q) = \{\mathcal{G}\}$,we give essentially the same argument to disprove (3).

No further use is made of Proposition 6.1 (2) or (3) and there are no consequences for the theory or examples that are presented.

References

[1] Matthew Collinson, Kevin McDonald, and David Pym. A substructural logic for layered graphs *Journal of Logic and Computation* (2014) 24 (4): 953–988. doi: 10.1093/logcom/exu002. First published online: February 18, 2014