

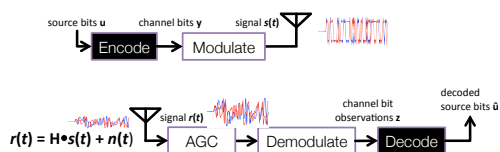
## Agenda

1. 9am Viterbi algorithm
2. 12pm Group presentations

## Applications of the Viterbi algorithm

- Communications systems
  - 802.11a/b/g, Zigbee, cellular
  - DSL, telephone modem, satellite
- Optical character recognition
- Speech recognition
- RADAR location system (GZ06 paper)

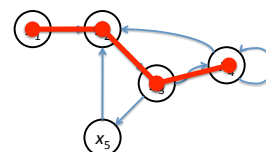
## Communications system model



## Model: discrete time Markov process

State at time  $k$   $x_k$  is one of a finite number  $M$  of **states**  $1, 2, \dots, M$

Example,  $M=5$ :

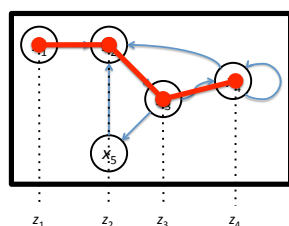


**Markov condition:**  $P(x_{k+1} | x_0, x_1, \dots, x_k) = P(x_{k+1} | x_k)$

## Model: hidden state Markov

We make  $K$  indirect observations  $z = \{z_1, \dots, z_K\}$  of the Markov process

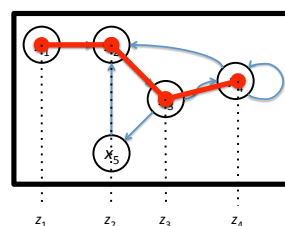
Example,  $K=4$ :



## What the Viterbi algorithm does

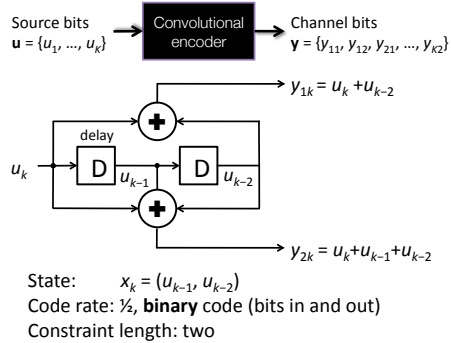
Estimate  $\mathbf{x} = \{x_1, \dots, x_K\}$  just from observing  $\mathbf{z} = \{z_1, \dots, z_K\}$

Example,  $K=4$ :



**Maximum a posteriori probability (MAP):**  $\mathbf{x}$  such that  $P(\mathbf{x} | \mathbf{z})$  is maximized

## Convolutional encoder



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## 802.11: adapt code rate, modulation

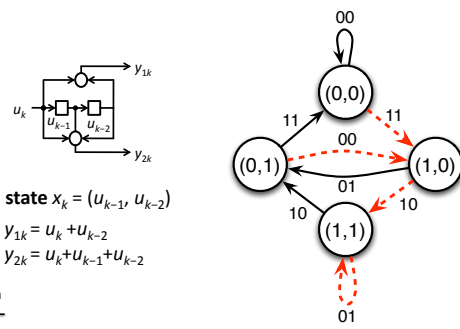
Bit-rate	802.11 Standards	DSSS or OFDM	Modulation	Bits per Symbol	Coding Rate	Mega-Symbols per second
1	b	DSSS	BPSK	1	1/11	11
2	b	DSSS	QPSK	2	1/11	11
5.5	b	DSSS	CCK	1	4/8	11
11	b	DSSS	CCK	2	4/8	11
6	a/g	OFDM	BPSK	1	1/2	12
9	a/g	OFDM	BPSK	1	3/4	12
12	a/g	OFDM	QPSK	2	1/2	12
18	a/g	OFDM	QPSK	2	3/4	12
24	a/g	OFDM	QAM-16	4	1/2	12
36	a/g	OFDM	QAM-16	4	3/4	12
48	a/g	OFDM	QAM-64	6	2/3	12
54	a/g	OFDM	QAM-64	6	3/4	12

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## From encoder to Markov model



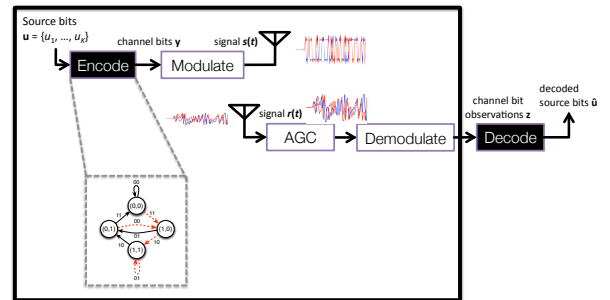
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## Matching reality with the model

Viterbi: estimate  $\mathbf{x} = \{x_1, \dots, x_k\}$  just from observing  $\mathbf{z} = \{z_1, \dots, z_k\}$

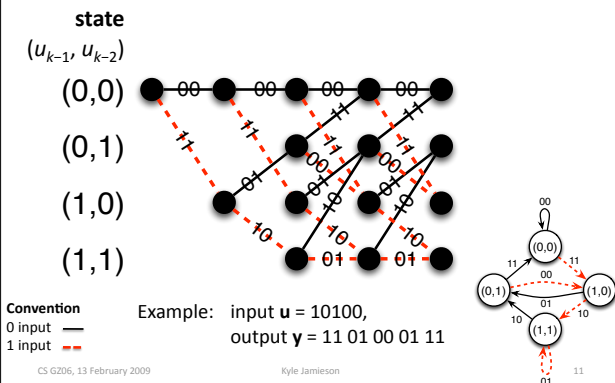


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## The Trellis

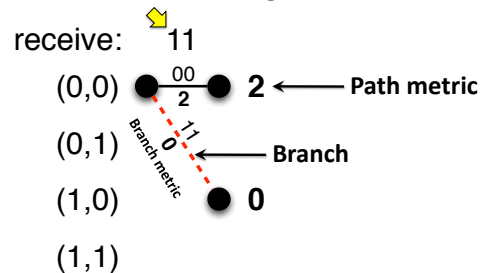


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## Viterbi decoding: receiver-side trellis



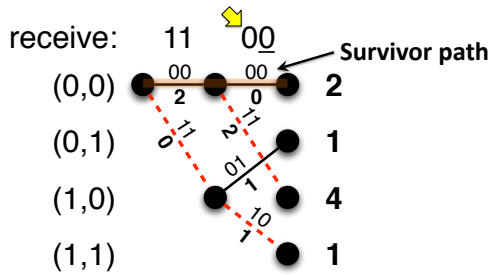
- Branch metric is Hamming distance between received and sent
- Path metric sums branch metrics

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### Viterbi decoding: receiver-side trellis



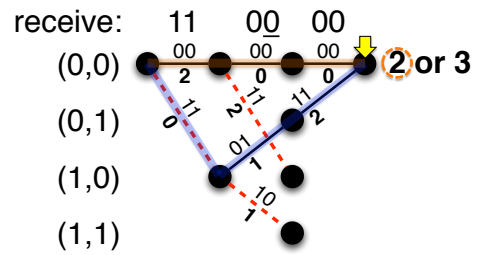
- Path metric sums branch metrics
  - What if we stopped decoding here?
- input  $u = 10100$

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### Viterbi decoding: receiver-side trellis



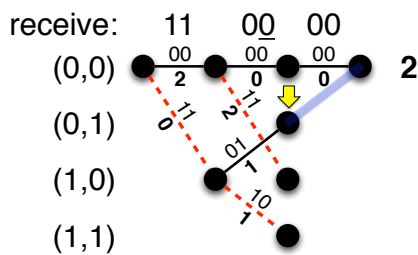
- Paths merge: key Viterbi dynamic programming step
- Prune just the incident branch for now

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### Viterbi decoding: receiver-side trellis



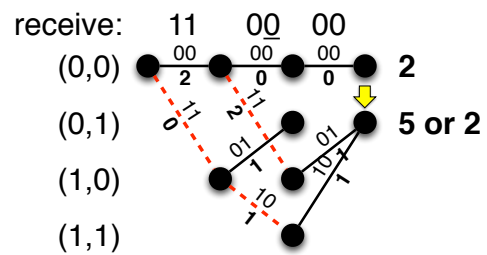
- Other survivor paths may go through remainder of losing path

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### Viterbi decoding: receiver-side trellis

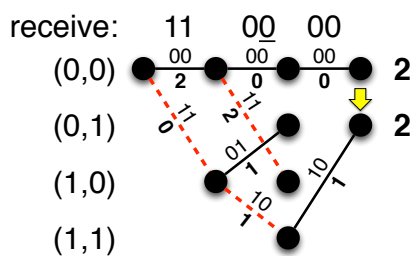


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### Viterbi decoding: receiver-side trellis

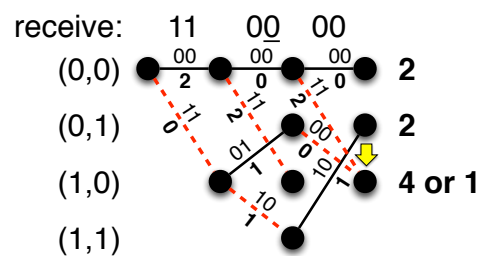


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### Viterbi decoding: receiver-side trellis

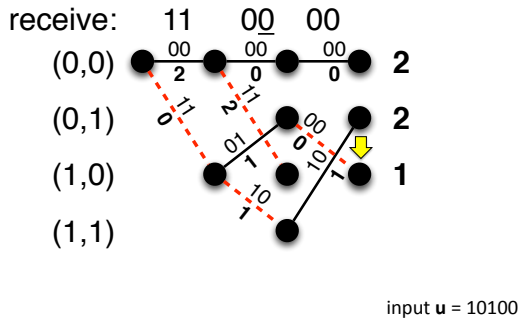


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### Viterbi decoding: receiver-side trellis

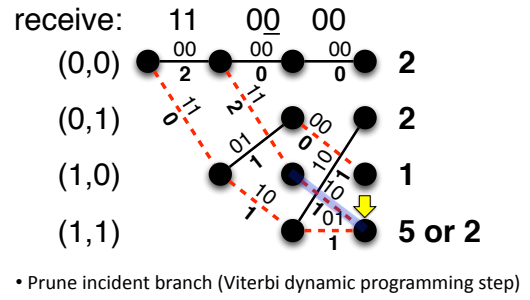


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### Viterbi decoding: receiver-side trellis

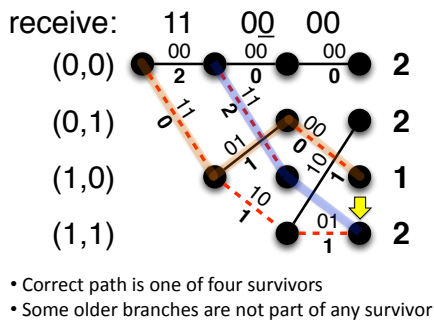


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### Viterbi decoding: receiver-side trellis

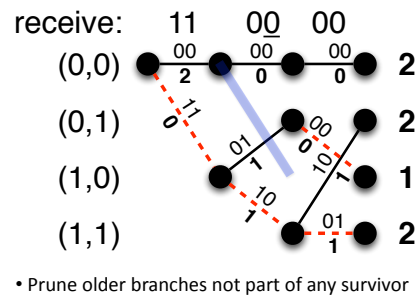


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### Viterbi decoding: receiver-side trellis

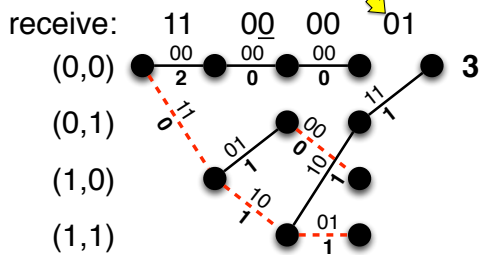


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### Viterbi decoding: receiver-side trellis

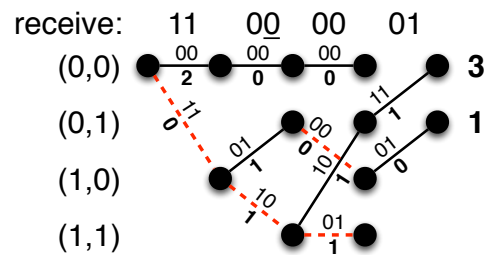


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### Viterbi decoding: receiver-side trellis

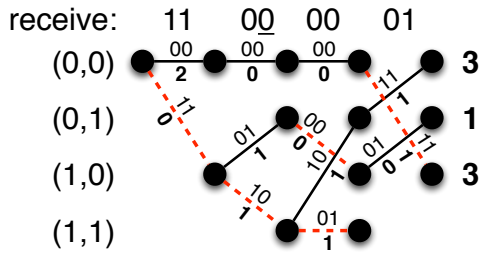


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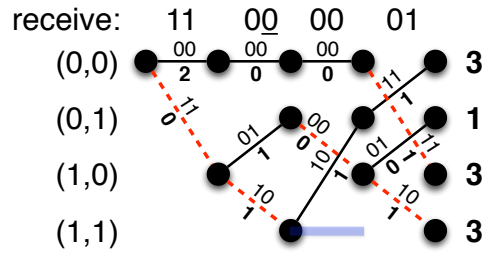
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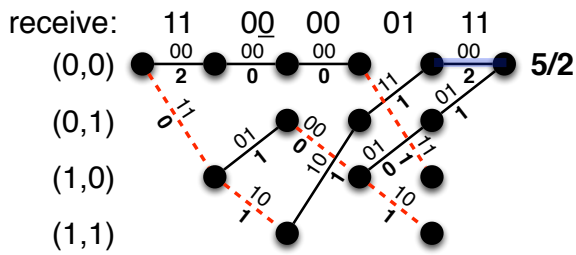
### Viterbi decoding: receiver-side trellis



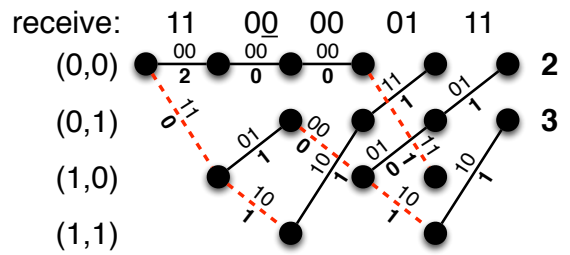
### Viterbi decoding: receiver-side trellis



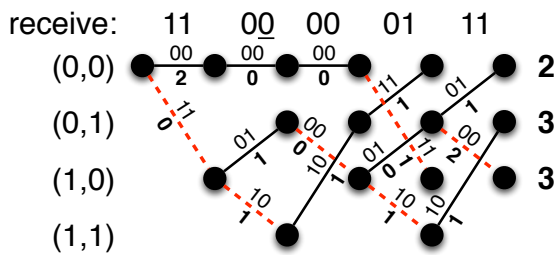
### Viterbi decoding: receiver-side trellis



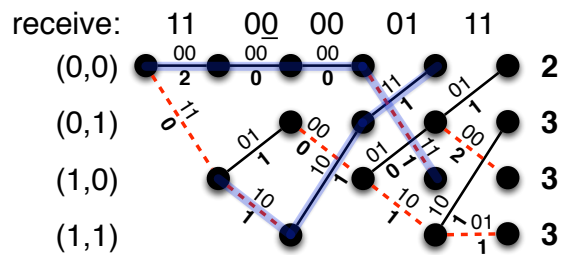
### Viterbi decoding: receiver-side trellis



### Viterbi decoding: receiver-side trellis



### Viterbi decoding: receiver-side trellis



• Complete path from beginning is a non-survivor

