

MATH 302 Discrete Mathematics (Spring 2012, Session 501)

Extra-credit Assignment

You must show your argument and computation to get points. Each problem counts 3 points. The maximum points that will be counted are 20.

1. Prove that $\sqrt{3}$ is irrational.
2. How many elements are there in the set $\mathcal{P}(\mathcal{P}(\mathcal{P}(\mathcal{P}(\{a, b\}))))$, where a and b are distinct?
3. The **symmetric difference** of A and B , denoted by $A \oplus B$, is the set containing those elements in either A or B , but not in both A and B . Show the identity $A \oplus B = (A - B) \cup (B - A)$.
4. Let x be a real number. The floor function $[x]$ assigns to x the largest integer that is less than or equal to x . Show that $[3x] = [x] + [x + \frac{1}{3}] + [x + \frac{2}{3}]$.
5. Let f_n be the n th Fibonacci number, i.e., defined by the initial conditions $f_0 = 0$, $f_1 = 1$, and the recurrence relation $f_n = f_{n-1} + f_{n-2}$, $n = 2, 3, \dots$. Prove that $f_0 f_1 + f_1 f_2 + \dots + f_{2n-1} f_{2n} = f_{2n}^2$, where n is a positive integer.
6. For $n \in \mathbb{Z}^+$, consider the following sum

$$\sum_{i=1}^n \frac{1}{(3i-1)(3i+1)}.$$

Make a conjecture about the value of the sum, and prove your conjecture.

7. Find the solution to the recurrent relation $a_n = 2a_{n-1} + 3a_{n-2}$ with the initial conditions $a_0 = 2$ and $a_1 = 10$.
8. Let n and r be positive integers. Show that

$$\sum_{k=0}^r \binom{n+k}{k} = \binom{n+r+1}{r}.$$

9. Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authority. Alice said “Carlos did it.” John said “I did not do it.” Carlos said “Diana did it.” Diana said “Carlos lied when he said that I did it.” Use logical expressions and truth tables to solve:

- (a) If the authority also knows that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.
- (b) If the authority also knows that exactly one is lying, who did it? Explain your reasoning.