

MATH 151 Engineering Mathematics (Fall 2012, practice only)

You must show necessary details in order to get full points.

1. Calculate the following limit, if it exists.

(a) $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{\sin x}$

(b) $\lim_{x \rightarrow \infty} \frac{\ln x^2}{x^{\frac{1}{3}}}$

(c) $\lim_{x \rightarrow 0} (1 - 2(x^2 + x))^{\frac{5}{x^3 - x^2}}$

2. Find y' if $-e^{y^2} + \tan xy = 0$.

3. Let $f''(x) = 2 \sin 2x + 3$, $f'(0) = 1$, $f(\pi) = 3$. Find $f(x)$.

4. Find the domain, range and derivative of the function $f(x) = \ln \cos^{-1} x$.

5. Let $f(x) = \frac{1+x^2}{1-x^2}$.

(a) Find all critical numbers of f .

(b) Find the intervals where f is increasing and decreasing.

(c) Find the intervals where f is concave upward and concave downward.

(d) Find the local maximum and minimum.

6. Find the area of the largest rectangle that can be inscribed inside a circle of radius 1 cm.

7. Use the fundamental theorem of calculus in the following problems.

(a) Let the function $f(x)$ be defined by $f(x) = \int_{\sqrt{x}}^0 (\tan t + t^2) dt$. Find $f'(x)$.

(b) Compute the definite integral $\int_{-2}^2 (x^2 - x + \cos x) dx$.

8. Find the values of c and d that make h continuous on \mathbb{R} :

$$h(x) = \begin{cases} 2x & \text{if } x < 1, \\ cx^2 + d & \text{if } 1 \leq x \leq 2, \\ 4x & \text{if } x > 2. \end{cases}$$