A new approach for the iterative regularization of nonlinear operator equations

Ronny Ramlau
Center for Industrial Mathematics
University of Bremen, Germany

We shall be concerned with the construction of Tikhonov–based iteration schemes for solving nonlinear operator equations. In particular, we are interested in algorithms for the computation of a minimizer of the Tikhonov functional. Besides the classical quadratic regularization term, we consider one–homogeneous (typically weighted ℓ_p) penalties on the coefficients (or isometrically transformed coefficients) of frame expansions of a function, and present a regularization result. In order to compute a minimizer of the Tikhonov functional, we introduce a replacement functional. It turns out that the replacement functional has much better properties than the original functional. Namely, it is globally convex and can be effectively minimized by a fixed point iteration. Based on the minimizers of the replacement functional, we introduce an iterative algorithm that converges towards a critical point of the Tikhonov functional, and under additional assumptions to the nonlinear operator F, to a global minimizer. Combining our iterative strategy with an appropriate parameter selection rule, we obtain convergence and convergence rates.

The performance of the resulting numerical schemes is demonstrated by solving the nonlinear inverse SPECT (Single Photon Emission Computerized Tomography) problem.